Ifsinx=-5/13 And 3pi/2 Is Less Than Or Equal To X Is Less Than Orequal To 2pi, Find Sin2x And Cos(x/2)

Ifsinx=-5/13 And 3pi/2 Is Less Than Or Equal To X Is Less Than Orequal To 2pi, Find Sin2x And Cos(x/2)

Understanding how to find the values of trigonometric functions such as Sin2x and Cos(x/2) given specific conditions is essential in advanced mathematics, especially in the context of calculus and analytical geometry. In this article, we will analyze the problem where $\sin x = -\frac{5}{13}$ and $\frac{3\pi}{2} \leq x \leq 2\pi$. We aim to find the values of $\sin 2x$ and $\cos \frac{x}{2}$. We will explore the problem systematically, applying fundamental trigonometric identities, understanding the quadrants involved, and step-by-step calculations to arrive at precise solutions.

Understanding the Given Conditions

Before diving into calculations, it is crucial to interpret the given information correctly.

Given Data

- $\sin x = -\frac{5}{13}$
- $\frac{3\pi}{2} \leq x \leq 2\pi$

The interval $\frac{3\pi}{2} \leq x \leq 2\pi$ lies in the fourth quadrant of the unit circle, where:

- Sine is negative.
- Cosine is positive.

The value $\sin x = -\frac{5}{13}$ confirms the sine's negativity in the specified interval, aligning with the quadrant's properties.

Step 1: Analyzing $\sin x = -\frac{5}{13}$

Given the sine value, we can determine other components related to angle x.

1.1. Recognizing the Triangle

- The sine value is given by $\sin x = \frac{\text{opposite}}{\text{hypotenuse}}$.
- Here, the opposite side is -5 (since sine is negative), and the hypotenuse is 13.

1.2. Finding $\cos x$

- Using the Pythagorean theorem:

$$
\cos x = \pm \sqrt{1 - \sin^2 x}
$$

- Substitute $\sin x = -\frac{5}{13}$:

$$
\cos x = \pm \sqrt{1 - \left(-\frac{5}{13}\right)^2} = \pm \sqrt{1 - \frac{25}{169}} = \pm \sqrt{\frac{169 - 25}{169}} = \pm \sqrt{\frac{144}{169}} = \pm \frac{12}{13}
$$

- Since x is in the fourth quadrant ($\frac{3\pi}{2} \leq x \leq 2\pi$), cosine is positive:

$$
\cos x = \frac{12}{13}
$$

1.3. Summary of Known Values

Quantity	Value	Significance
$\sin x$	$-\frac{5}{13}$	As given, in the fourth quadrant.
$\cos x$	$\frac{12}{13}$	Derived, positive in the fourth quadrant.

Step 2: Finding $\sin 2x$

The double-angle formula for sine is:

$$
\sin 2x = 2 \sin x \cos x
$$

2.1. Applying the Formula

- Substitute known values:

$$
\sin 2x = 2 \times \left(-\frac{5}{13}\right) \times \frac{12}{13}
$$

- Simplify numerator and denominator:

$$
\sin 2x = 2 \times -\frac{5 \times 12}{13 \times 13} = 2 \times -\frac{60}{169}
$$

$$
\sin 2x = -\frac{120}{169}
$$

2.2. Final Result for $\sin 2x$

$$
\boxed{
\sin 2x = -\frac{120}{169}
}
$$

This value indicates the sine of the double angle $2x$. Since $\sin 2x$ is negative, the angle $2x$ is either in the third or fourth quadrants, considering the range of x.

Step 3: Finding \(\cos \frac{x}{2}\)

The half-angle formula for cosine is:

\[

\cos \frac{x}{2} = \pm \sqrt{\frac{1 + \cos x}{2}}

\]

The sign depends on the quadrant of \(\frac{x}{2} \).

3.1. Determine the Range of \(\frac{x}{2} \)

- Since \(x\) is between \(\frac{3\pi}{2}\) and \(2\pi\):

\[

\frac{3\pi}{2} \leq x \leq 2\pi

\]

- Dividing through by 2:

\[

\frac{3\pi}{4} \leq \frac{x}{2} \leq \pi

\]

- The interval \(\left[\frac{3\pi}{4}, \pi \right] \) lies in the second quadrant \(\left(\frac{\pi}{2} < \frac{x}{2} < \pi\right)\) and the boundary at \(\pi\).

- In the second quadrant, cosine is negative, so:

\[

\cos \frac{x}{2} = - \sqrt{\frac{1 + \cos x}{2}}

\]

3.2. Calculate \(\cos \frac{x}{2}\)

- Substitute \(\cos x = \frac{12}{13}\):

\[

\cos \frac{x}{2} = - \sqrt{\frac{1 + \frac{12}{13}}{2}}

\]

- Simplify numerator:

\[

1 + \frac{12}{13} = \frac{13}{13} + \frac{12}{13} = \frac{25}{13}

\]

- Plug back into the formula:

\[

\cos \frac{x}{2} = - \sqrt{\frac{\frac{25}{13}}{2}} = - \sqrt{\frac{25}{13} \times \frac{1}{2}} = -

\sqrt{\frac{25}{26}}

\]

- Simplify the square root:

\[

\cos \frac{x}{2} = - \frac{\sqrt{25}}{\sqrt{26}} = - \frac{5}{\sqrt{26}}

\]

- Rationalize the denominator:

$$
\cos \frac{x}{2} = - \frac{5}{\sqrt{26}} \times \frac{\sqrt{26}}{\sqrt{26}} = - \frac{5 \sqrt{26}}{26}
$$

3.3. Final Result for $\cos \frac{x}{2}$

$$
\boxed{
\cos \frac{x}{2} = - \frac{5 \sqrt{26}}{26}
}
$$

Summary of Results

To encapsulate the findings:

Expression	Value	Remarks
$\sin 2x$	$-\frac{120}{169}$	Double angle sine, negative indicating third/fourth quadrants.
$\cos \frac{x}{2}$	$- \frac{5 \sqrt{26}}{26}$	Half-angle cosine, negative in the second quadrant.

Additional Insights and Applications

Understanding these calculations is fundamental in various branches of mathematics and physics, including:

- Trigonometric Identity Applications: Simplifications involving double and half-angle formulas.
- Wave and Oscillation Analysis: Calculating phase shifts and amplitudes.
- Engineering Fields: Signal processing, where phase and amplitude are critical.
- Geometry and Navigation: Calculating distances and directions on the circle.

Moreover, mastering these techniques enhances problem-solving skills in calculus, especially in integration and differentiation involving trigonometric functions.

Conclusion

Given $\sin x = -\frac{5}{13}$ within the interval $\frac{3\pi}{2} \leq x \leq 2\pi$, the key steps involved determining $\cos x$, applying double and half-angle formulas, and considering the quadrant-specific signs for the functions. The results show that:

$$
\boxed{
\sin 2x = -\frac{120}{169}
}
$$
and

\[

\boxed{

\cos \frac{x}{2} = - \frac{5 \sqrt{26}}{26}

}

\]

These solutions

Frequently Asked Questions

Given that sin x = -5/13 and 3□/2 □ x □ 2□, how do we find sin 2x?

Since sin x = -5/13 and x is in the fourth quadrant (between 3□/2 and 2□), cos x = □(1 - sin^2 x) = □(1 - (-5/13)^2) = □(1 - 25/169) = □(144/169) = 12/13. Because x is in the fourth quadrant, cos x > 0. Then, sin 2x = 2 sin x cos x = 2 (-5/13) (12/13) = -120/169.

How do we compute cos(x/2) given the value of sin x = -5/13?

First, find sin(x/2) using the half-angle formula: sin(x/2) = ±□[(1 - cos x)/2]. Since cos x = 12/13, then sin(x/2) = ±□[(1 - 12/13)/2] = ±□[(1/13)/2] = ±□[1/26] = ±1/□26. Because x is in the fourth quadrant, x/2 is in the second quadrant (□/2 to □), where sin(x/2) > 0 and cos(x/2) < 0. Therefore, cos(x/2) = -□[(1 + cos x)/2] = -□[(1 + 12/13)/2] = -□[(25/13)/2] = -□[25/26] = -5/□26.

In which quadrant does x lie based on sin x = -5/13 and the given interval?

Since sin x = -5/13 and x is between 3□/2 and 2□, x lies in the fourth quadrant, where sine is negative and cosine is positive.

What is the value of sin 2x if sin x = -5/13 and x is in the fourth quadrant?

sin 2x = -120/169, calculated as 2 sin x cos x = 2 (-5/13) (12/13).

How is the sign of cos(x/2) determined in this problem?

Since x is in the fourth quadrant, x/2 is in the second quadrant (□/2 to □), where cosine values are negative. Therefore, cos(x/2) = -5/□26.

Why do we use half-angle formulas for cos(x/2) in this problem?

Because the problem asks for cos(x/2), and half-angle formulas allow us to find this value directly from known values of cos x or sin x, especially when x is in a specific quadrant.

Summarize the steps to find sin 2x and cos(x/2) given sin x = -5/13 and the interval 3□/2 □ x □ 2□.

First, determine the quadrant of x and find cos x using Pythagoras. Then, compute sin 2x using the double-angle formula: 2 sin x cos x. Next, apply half-angle formulas to find cos(x/2), considering the quadrant of x/2 to determine the correct sign. This process yields sin 2x = -120/169 and cos(x/2) = -5/□26.

Additional Resources

Understanding the Trigonometric Problem: Finding Sin 2x and Cos (x/2) given Sin x = -5/13 within the interval \(\frac{3\pi}{2} \leq x \leq 2\pi \)

Introduction and Overview

This problem involves evaluating two key trigonometric functions—Sin 2x and Cos (x/2)—based on a given sine value and a specified interval for x. It's a classic example of applying fundamental trigonometric identities, understanding the geometric implications of the sine value, and considering the constraints of the interval to determine the correct quadrants and signs.

The core tasks are:

- Extracting the value of x from $\sin x = -\frac{5}{13}$ within the interval $\frac{3\pi}{2} \leq x \leq 2\pi$.
- Calculating $\sin 2x$ using the double-angle formula.
- Calculating $\cos \frac{x}{2}$ using half-angle formulas or related identities.

This comprehensive exploration will guide you through each step, incorporating geometric reasoning, algebraic calculations, and the use of identities to arrive at accurate solutions.

Understanding the Given Data and Interval

1. The Given Sine Value: $\sin x = -\frac{5}{13}$

The sine value is negative, which indicates that x lies either in the third or fourth quadrants of the Cartesian plane, where sine is negative:

- Quadrant III: $\pi < x < \frac{3\pi}{2}$
- Quadrant IV: $\frac{3\pi}{2} < x < 2\pi$

However, the interval provided is:

$$
\frac{3\pi}{2} \leq x \leq 2\pi
$$

which covers the entire fourth quadrant and the endpoint at 2π. Since the sine is negative and x lies in this interval, the possible range for x is in the fourth quadrant, possibly including the boundary at 2π.

Key Point:

- $\sin x = -\frac{5}{13}$ is a specific value, and the negative sign confirms the position in the fourth quadrant within the given interval.

---

2. The Interval: $\frac{3\pi}{2} \leq x \leq 2\pi$

Let's understand this interval:

- $\frac{3\pi}{2}$ corresponds to 270°, or in radians, 270°.
- 2π corresponds to 360°, or 360°.

Thus, the interval spans from 270° to 360°, which is the entire fourth quadrant, where:

- Sine (sin x): Negative
- Cosine (cos x): Positive
- Tangent (tan x): Negative

Given the sine value and the interval, the problem confirms that x is somewhere in the fourth quadrant, specifically between 270° and 360°.

Determining the Exact Value of x

1. Using the Pythagorean Identity for $\sin x$

Given:

$$
\sin x = -\frac{5}{13}
$$

We can find $\cos x$ using the Pythagorean identity:

$$
\sin^2 x + \cos^2 x = 1
$$

Substitute the known value:

$$
\left(-\frac{5}{13}\right)^2 + \cos^2 x = 1
$$

$$
\frac{25}{169} + \cos^2 x = 1
$$

$$
\cos^2 x = 1 - \frac{25}{169} = \frac{169}{169} - \frac{25}{169} = \frac{144}{169}
$$

Taking the square root:

$$
\cos x = \pm \frac{12}{13}
$$

Determining the Sign:

- Since x is in the fourth quadrant ($\frac{3\pi}{2} \leq x \leq 2\pi$), cosine is positive there.
- Therefore:

$$
\cos x = \frac{12}{13}
$$

---

2. Identifying the Reference Angle θ

The reference angle θ in the context of $\sin x = -\frac{5}{13}$ is an acute angle in the first quadrant with:

$$
\sin \theta = \frac{5}{13}
$$

This is a standard 5-12-13 right triangle, often used as a Pythagorean triple:

Side Opposite θ	5
Side Adjacent to θ	12
Hypotenuse	13

The reference angle θ satisfies:

$$
\sin \theta = \frac{5}{13}
$$

$$
\cos \theta = \frac{12}{13}
$$

In our case, since x is in the fourth quadrant, x can be expressed as:

$$
x = 2\pi - \theta
$$

where θ is the positive acute angle with $\sin \theta = 5/13$.

Calculating $\sin 2x$: Double-Angle Formula

1. Recall the Double-Angle Identity for $\sin 2x$

$$
\sin 2x = 2 \sin x \cos x
$$

Plugging in the known values:

$$
\sin 2x = 2 \times \left(-\frac{5}{13}\right) \times \frac{12}{13}
$$

Calculations:

$$
\sin 2x = 2 \times -\frac{5}{13} \times \frac{12}{13} = -2 \times \frac{5 \times 12}{13 \times 13}
$$

Simplify numerator and denominator:

$$
\sin 2x = -2 \times \frac{60}{169} = - \frac{120}{169}
$$

Result:

$$
\boxed{
\sin 2x = - \frac{120}{169}
}
$$

This value is consistent with expectations for the double angle in the fourth quadrant, where sine remains negative.

Calculating $\cos \frac{x}{2}$: Half-Angle Formula

Calculating $\cos \frac{x}{2}$ is more nuanced because the half-angle formulas depend on the quadrant of $\frac{x}{2}$, which is half of x.

---

1. Expressing x and $\frac{x}{2}$

Given:

$$
x \in \left[\frac{3\pi}{2}, 2\pi \right]
$$

Dividing through by 2:

$$
\frac{x}{2} \in \left[\frac{3\pi}{4}, \pi \right]
$$

So:

- $\frac{x}{2}$ ranges from:

- $\frac{3\pi}{4}$ (135°) to π (180°).

- This interval includes the second quadrant ($\frac{\pi}{2}$ to π) and the boundary at π.

- In the second quadrant: $\cos \frac{x}{2}$ is negative.

- At $\frac{x}{2} = \pi$, $\cos \pi = -1$.

- At $\frac{x}{2} = \frac{3\pi}{4}$, $\cos \frac{3\pi}{4} = -\frac{\sqrt{2}}{2}$.

Conclusion:

- $\cos \frac{x}{2}$ is negative throughout this interval.

---

2. Using the Half-Angle Formula for $\cos \frac{x}{2}$

The half-angle formula for cosine is:

$$
\cos \frac{x}{2} = \pm \sqrt{\frac{1 + \cos x}{2}}
$$

- The sign depends on the quadrant of $\frac{x}{2}$.

- Since $\frac{x}{2} \in \left[\frac{3\pi}{4}, \pi \right]$, where cosine is negative, we take the negative root.

Calculate $\cos x$:

\[

\cos x = \frac{12}{13}

\]

Substitute:

\[

\cos \frac{x}{2} = - \sqrt{\frac{1 + \frac{12}{

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