In A Material For Which O=5.0 S/m And E, =1 The Electric Field Intensity Is E- 250 Sin 10" (V/m). Find

In A Material For Which O=5.0 S/m And E_0 = 1 The Electric Field Intensity Is E = 250 Sin 10^4 t (V/m). Find

---

Introduction

Understanding electric fields within conductive materials is fundamental in electromagnetism and electrical engineering. The problem presented involves a material with specific properties—conductivity (O) and a constant (E_0)—and a time-varying electric field. The task is to analyze this information to find key parameters such as the current density, power dissipation, and possibly the phase relationship within the material.

This article will delve into the principles governing electric fields in conductors, interpret the given data, and step through the calculation process methodically. We will examine the concepts of conduction current density, Ohm's law in the differential form, and how sinusoidal electric fields influence current and power transfer in the material.

Understanding the Given Data

Material Conductivity (σ)

- Given as σ = 5.0 S/m (siemens per meter).
- This value indicates how easily current can pass through the material.

Constant E_0 = 1

- The mention of E_0 = 1 suggests a normalized or reference value, possibly indicating the amplitude or a phase constant in the electric field expression.

Electric Field Expression

- $E(t) = 250 \sin(10^4 t)$ V/m
- This sinusoidal form describes a time-varying electric field oscillating at angular frequency $\omega = 10^4$ rad/sec.

Analyzing the Electric Field Expression

Time Domain Representation

- The electric field varies sinusoidally with time:

$E(t) = E_0 \sin(\omega t)$

- With $E_0 = 250$ V/m and $\omega = 10^4$ rad/sec, the field oscillates between +250 V/m and -250 V/m.

Significance of Frequency

- The angular frequency $\omega = 10^4$ rad/sec indicates a high-frequency electromagnetic wave or signal within the material, affecting how the current responds due to the material's conductivity.

Applying Ohm’s Law in Differential Form

Fundamental Relation

- The conduction current density J relates to the electric field E via:

$J = \sigma E$

- Since both J and E are time-varying, their sinusoidal nature must be considered in the analysis.

Calculating Current Density

- At any instant:

$J(t) = \sigma E(t) = \sigma \times 250 \sin(10^4 t)$ A/m²

- Substituting $\sigma = 5.0$ S/m:

$J(t) = 5.0 \times 250 \sin(10^4 t) = 1250 \sin(10^4 t)$ A/m²

Result: The current density oscillates sinusoidally with amplitude 1250 A/m².

Power Dissipation in the Material

Instantaneous Power Density

- The power dissipated per unit volume (power density) due to conduction is given by:

$p(t) = J(t) \cdot E(t)$

- Since J and E are in the same direction:

$p(t) = J(t) E(t)$

- Substituting the expressions:

$p(t) = [1250 \sin(10^4 t)] \times [250 \sin(10^4 t)] = (1250 \times 250) \sin^2(10^4 t)$

$p(t) = 312{,}500 \sin^2(10^4 t)$ W/m³

Average Power Dissipation

- Over a full cycle, the average power density is:

$P_{avg} = (1/T) \int_0^T p(t)\, dt$

- Recall that:

$\langle \sin^2(\omega t) \rangle = 1/2$ over one cycle

- Therefore:

$P_{avg} = 312{,}500 \times 1/2 = 156{,}250$ W/m³

Interpretation: The material dissipates an average power of 156,250 watts per cubic meter due to conduction losses.

Phase Relationship and Reactance

Inclusion of Material Permittivity and Inductive Effects

- The problem primarily considers conduction (ohmic) effects, but at high frequencies, displacement currents and inductance could influence the phase relationship between E and J.

Assumption of Resistive Behavior

- Given the high conductivity and sinusoidal electric field, the current is assumed to be in phase with the electric field, typical for good conductors at high frequencies when dielectric effects are negligible.

Additional Parameters to Find

Potentially Required Calculations

- The problem asks to "Find" a certain quantity, which may include:

- Current density amplitude
- Average power loss
- Skin depth, if relevant
- Impedance of the material

Calculating the RMS Values

- RMS (Root Mean Square) values are often used in AC analysis:

$E_{rms} = E_0 / \sqrt{2} = 250 / 1.414 \approx 177$ V/m

$J_{rms} = \sigma E_{rms} \approx 5.0 \times 177 \approx 885$ A/m^2

- Power dissipation in RMS terms:

$P_{rms} = J_{rms} \times E_{rms} = (J_{rms})^2 / \sigma$

Alternatively, using power formula:

$P_{avg} = \sigma E_{rms}^2$

$P_{avg} \approx 5.0 \times (177)^2 \approx 5.0 \times 31{,}329 \approx 156{,}645$ W/m^3

which aligns with previous calculations, confirming consistency.

Implications and Practical Applications

Material Heating

- The high power density indicates significant heating within the material due to resistive losses.
- In engineering design, this necessitates considerations for cooling or selecting materials with lower conductivity for specific applications.

Electromagnetic Wave Propagation

- The high-frequency electric field suggests the wave may experience skin effect, where current tends to concentrate near the surface.
- Skin depth δ can be estimated as:

$$\delta = \sqrt{2 / (\omega \mu \sigma)}$$

Assuming free space permeability $\mu \approx \mu_0 = 4\pi \times 10^{-7}$ H/m:

$$\delta = \sqrt{2 / (10^4 \times 4\pi \times 10^{-7} \times 5.0)} \approx \sqrt{2 / (10^4 \times 6.2832 \times 10^{-7} \times 5.0)}$$

Simplify numerator:

$$\delta \approx \sqrt{2 / (10^4 \times 3.1416 \times 10^{-6})}$$

Calculate denominator:

$$10^4 \times 3.1416 \times 10^{-6} = 0.031416$$

Therefore:

$$\delta \approx \sqrt{2 / 0.031416} \approx \sqrt{63.66} \approx 7.98 \text{ meters}$$

- The large skin depth indicates that at this frequency and conductivity, the wave penetrates deeply, behaving more like a bulk conductor rather than surface-confined.

Summary and Conclusions

- The sinusoidal electric field in the material induces a sinusoidal current density directly proportional to the conductivity.
- The current density amplitude is 1250 A/m², oscillating in phase with the electric field.
- The average power dissipated per unit volume is approximately 156,250 W/m³, indicating substantial resistive heating.
- RMS values provide a simplified view of the power and current characteristics, confirming the high energy dissipation.
- Skin depth calculations suggest that the wave penetrates deeply into the material at the given

frequency, affecting how the material would behave in applications involving high-frequency electromagnetic fields.
- These analyses are critical in designing electronic components, RF systems, and understanding material heating and electromagnetic wave propagation.

Final Remarks

The analysis of electric fields in conductors with sinusoidal excitation offers vital insights into power dissipation, current flow, and electromagnetic behavior. Proper understanding ensures effective material selection, system design, and thermal management in practical engineering applications.

Frequently Asked Questions

What is the significance of the given conductivity O=5.0 S/m in the context of electromagnetic wave propagation?

The conductivity O=5.0 S/m indicates the material's ability to conduct electric current, affecting the attenuation and penetration of electromagnetic waves within the material. Higher conductivity results in greater attenuation of the wave.

How do you interpret the electric field intensity expression E = 250 sin(10 t) V/m in analyzing the behavior of the electromagnetic wave?

This expression describes a sinusoidal time-varying electric field with an amplitude of 250 V/m and an angular frequency of 10 rad/sec, representing an alternating electric field propagating through the material.

Given the electric field E = 250 sin(10 t), how do you calculate the instantaneous electric field at a specific time t?

You substitute the value of t into the expression: E(t) = 250 sin(10 t). For example, at t=0, E=0; at t=π/20, E=250 sin(10 π/20)=250 sin(π/2)=250 V/m.

What additional parameters are needed to determine the power loss or attenuation in the material with the given properties?

To determine power loss or attenuation, you need the wave's propagation constant (including attenuation coefficient), phase constant, and the frequency of the wave, as well as the material's permeability. These allow calculation of the attenuation factor within the material.

How would you compute the average power transmitted through the material given the electric field intensity E = 250 sin(10 t)?

Assuming a known magnetic field or using the intrinsic impedance of the material, the average power can be calculated using $P_{avg} = (1/2) \mathrm{Re}(E H)$, or by applying the Poynting vector. Additional information about the magnetic field or wave impedance is required for precise calculation.

Additional Resources

Comprehensive Analysis of Electric Field Intensity in a Material with Conductivity O=5.0 S/m and Permittivity E=1

Introduction

Understanding the behavior of electric fields within different materials is fundamental to electromagnetics, electrical engineering, and materials science. When dealing with conductive materials, specific parameters, such as conductivity and permittivity, influence how electromagnetic waves propagate and how electric fields are established and maintained within the medium.

In this detailed analysis, we focus on a particular scenario: a material characterized by a conductivity $\sigma = 5.0\, \text{S/m}$ and a permittivity $\varepsilon = 1$ (in appropriate units), subjected to an electric field described by the sinusoidal function $E(t) = 250 \sin 10^n t \, \text{V/m}$. Our goal is to interpret, analyze, and understand the implications of this electric field within the material, factoring in its electrical properties.

Understanding the Material Parameters

Conductivity (σ)

- Definition: Conductivity quantifies how well a material can conduct electric current.
- Value in this context: $\sigma = 5.0\, \text{S/m}$.
- This is a moderate value, typical for materials like saltwater or certain ceramics.
- High conductivity materials (e.g., metals) typically have conductivities on the order of $10^7\, \text{S/m}$, whereas insulators have conductivities near zero.

- Implications:
- The material allows significant conduction currents when subjected to an electric field.
- Energy dissipation occurs via Joule heating, proportional to σE^2.

Permittivity (ε)

- Definition: Permittivity measures a material’s ability to permit electric field penetration.
- Value in this context: $\varepsilon = 1$.
- This suggests an arbitrary or normalized unit system, or a simplified scenario.

- In SI units, permittivity of free space is $\varepsilon_0 \approx 8.854 \times 10^{-12} \, \text{F/m}$.
- For the purpose of this analysis, we'll assume the units are normalized for simplicity.

- Implications:
- The permittivity influences the capacitance and the propagation speed of electromagnetic waves in the medium.
- A low permittivity indicates a less polarizable medium, affecting wave propagation characteristics.

The Electric Field Expression

The electric field is described as:

$$
E(t) = 250 \sin(10^{n} t) \, \text{V/m}
$$

where:

- Amplitude E_0: 250 V/m
- Angular frequency $\omega = 10^{n} \, \text{rad/sec}$, with n being an unspecified exponent.

This sinusoidal variation indicates that the electric field is alternating, oscillating with time. Understanding the behavior of this field involves analyzing:

- The amplitude
- The frequency (determined by 10^{n})
- The phase (here, zero phase shift assumed)

Clarifying the Parameters

Before proceeding, it's important to clarify:

- What is the value of n?
- Since it is unspecified, we can analyze general cases for varying n.
- Is the field uniform or spatially varying?
- The given expression suggests a uniform time-varying field, not spatially dependent.

Analyzing the Electric Field in the Material

Sinusoidal Electric Field: Physical Significance

- The sinusoidal form implies a time-varying electric field, common in AC circuits, electromagnetic wave propagation, and RF applications.

- The behavior of the field within the material depends on frequency, which influences phenomena such as:

- Skin effect
- Attenuation
- Wave propagation speed

Frequency-Dependent Effects

1. Low-frequency regime:
- The material behaves more like a resistor.
- The conduction currents dominate.
- The electric field penetrates deeply (large skin depth).

2. High-frequency regime:
- Displacement current becomes significant.
- The wave may be partially reflected or attenuated.
- Skin effect causes the current to concentrate near the surface.

Calculating Key Parameters

1. Angular Frequency (ω)

Given $\omega = 10^{n}$:

- For example:
- If $n = 3$, then $\omega = 10^{3} = 1000 \, \text{rad/sec}$.
- If $n = 6$, then $\omega = 10^{6} = 1,000,000 \, \text{rad/sec}$.

The choice of n dramatically influences the electromagnetic response.

---

2. Skin Depth (δ)

The skin depth indicates how deeply the electromagnetic wave penetrates into a conducting medium and is given by:

$$
\delta = \sqrt{\frac{2}{\omega \mu \sigma}}
$$

- Assumptions:
- Magnetic permeability μ is approximately $\mu_0 = 4\pi \times 10^{-7} \, \text{H/m}$ (free space permeability).
- If the material is non-magnetic, μ remains μ_0.

- Calculation example:

Suppose $n = 3$, so $\omega = 1000\, \text{rad/sec}$.

$$
\delta = \sqrt{\frac{2}{(1000) \times (4\pi \times 10^{-7}) \times 5}}
$$

Calculations:

$$
\delta = \sqrt{\frac{2}{1000 \times 4\pi \times 10^{-7} \times 5}}
$$

$$
\delta = \sqrt{\frac{2}{ (1000 \times 5) \times 4\pi \times 10^{-7}}}
$$

$$
\delta = \sqrt{\frac{2}{5000 \times 4\pi \times 10^{-7}}}
$$

$$
\delta = \sqrt{\frac{2}{5000 \times 1.2566 \times 10^{-6}}}
$$

$$
\delta = \sqrt{\frac{2}{6.283 \times 10^{-3}}}
$$

$$
\delta \approx \sqrt{318.31} \approx 17.84 \, \text{meters}
$$

This indicates that at this frequency, the electromagnetic wave penetrates approximately 18 meters into the material, a relatively large depth, implying minimal attenuation.

Similarly, for higher n, skin depth decreases, leading to more surface-concentrated currents.

Power Dissipation and Joule Heating

The power dissipated per unit volume due to conduction is:

$$
P = \sigma E^2
$$

Given $E(t) = 250 \sin(\omega t)$, the average power over a cycle is:

$$
\langle P \rangle = \frac{1}{T} \int_0^T \sigma E^2(t) dt

\]

$$
\langle P \rangle = \sigma \times \frac{1}{T} \int_0^T 250^2 \sin^2(\omega t) dt
$$

$$
\langle P \rangle = \sigma \times 62,500 \times \frac{1}{T} \int_0^T \sin^2(\omega t) dt
$$

Since the average value of $\sin^2$ over a period is $1/2$:

$$
\langle P \rangle = \sigma \times 62,500 \times \frac{1}{2} = \sigma \times 31,250 \, \text{W/m}^3
$$

Substituting $\sigma = 5.0 \, \text{S/m}$:

$$
\langle P \rangle = 5 \times 31,250 = 156,250\, \text{W/m}^3
$$

This substantial power density indicates significant heating effects within the material, especially at higher amplitudes or frequencies.

Electromagnetic Wave Propagation Analysis

Maxwell's Equations and Wave Equation

In a conducting medium, the electric field’s behavior adheres to the wave equation derived from Maxwell's equations:

$$
\nabla^2 E - \mu \varepsilon \frac{\partial^2 E}{\partial t^2} - \mu \sigma \frac{\partial E}{\partial t} = 0
$$

This equation illustrates the combined effects of wave propagation, dielectric response, and conduction losses.

Wave Solutions

The general solution involves exponential decay due to conduction:

$$
E(z,t) \sim E_0 e^{-\alpha z} e^{j (\omega t - \beta z)}
$$

Where:

- \(\alpha \) is the attenuation constant.
- \(\beta \) is the phase constant.

For a good conductor or high-frequency scenario:

\[
\alpha \

In A Material For Which O 5 0 S M And E 1 The Electric Field Intensity Is E 250 Sin 10 V M Find

In A Material For Which O 5 0 S M And E 1 The Electric Field Intensity Is E 250 Sin 10 V M Find

Related Articles

- If The Weeks Of Supply Is 10 And The Average Aggregate Inventory Value Is $1,000,000, What Is The Cost
- Jaylo Bought A Television Priced At $349. She Received A 20% Discount And Paid A 7% Sales Tax. What Is
- How Does Nappis Decision To Adapt The Original Story By Including Images Of Books And Words Affect The

Back to Home