Let (8,-6) Be A Point On The Terminal Side Of An Angle 0 In Standard Position. Find The Exact Value Of

Let (8, -6) Be A Point On The Terminal Side Of An Angle θ In Standard Position. Find The Exact Value Of

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Introduction

Understanding how to find the exact value of trigonometric functions for a given point on the terminal side of an angle in standard position is fundamental in mathematics, especially in trigonometry. When a point (x, y) lies on the terminal side of an angle θ in the coordinate plane, the goal is often to determine the exact values of the sine, cosine, tangent, and other related functions associated with θ. In this article, we will explore the process of finding these values step-by-step, using the specific point $(8, -6)$ as our example.

The Concept of Standard Position and Coordinates

What Is an Angle in Standard Position?

An angle θ is said to be in standard position when:

- Its vertex is at the origin $(0, 0)$.
- Its initial side lies along the positive x-axis.
- Its terminal side is determined by the ray passing through a point (x, y) in the coordinate plane.

Significance of the Point $(8, -6)$

- The point $(8, -6)$ lies somewhere in the fourth quadrant because:

- $x = 8 > 0$
- $y = -6 < 0$

- The coordinates give us the position of the terminal side of the angle θ in the coordinate system.

Visual Representation

Visualizing this point helps in understanding the angle's position:

- Draw the coordinate axes.
- Plot the point $(8, -6)$.
- Draw a line segment from the origin to this point.
- The angle θ is the measure between the positive x-axis and this line segment.

Step-by-Step Process to Find Exact Trigonometric Values

Step 1: Calculate the Distance from the Origin to the Point

This distance, called the radius or hypotenuse r, is essential for calculating trigonometric functions.

$$
r = \sqrt{x^2 + y^2}
$$

Applying the given coordinates:

$$
r = \sqrt{8^2 + (-6)^2} = \sqrt{64 + 36} = \sqrt{100} = 10
$$

Step 2: Determine the Basic Trigonometric Functions

Using the coordinates and the hypotenuse, the basic functions are:

- Sine $\sin \theta$:

$$
\sin \theta = \frac{y}{r} = \frac{-6}{10} = -\frac{3}{5}
$$

- Cosine $\cos \theta$:

$$
\cos \theta = \frac{x}{r} = \frac{8}{10} = \frac{4}{5}
$$

- Tangent $\tan \theta$:

$$
\tan \theta = \frac{y}{x} = \frac{-6}{8} = -\frac{3}{4}
$$

Step 3: Find the Reciprocal Functions

- Cosecant $\csc \theta$:

$$
\csc \theta = \frac{1}{\sin \theta} = \frac{1}{-\frac{3}{5}} = -\frac{5}{3}
$$

- Secant $\sec \theta$:

$$
\sec \theta = \frac{1}{\cos \theta} = \frac{1}{\frac{4}{5}} = \frac{5}{4}
$$

- Cotangent $\cot \theta$:

$$
\cot \theta = \frac{1}{\tan \theta} = \frac{1}{-\frac{3}{4}} = -\frac{4}{3}
$$

Step 4: Summarize the Exact Values

Function	Exact Value	Explanation
$\sin \theta$	$-\frac{3}{5}$	$y/r = -6/10$
$\cos \theta$	$\frac{4}{5}$	$x/r = 8/10$
$\tan \theta$	$-\frac{3}{4}$	$y/x = -6/8$
$\csc \theta$	$-\frac{5}{3}$	reciprocal of $\sin \theta$
$\sec \theta$	$\frac{5}{4}$	reciprocal of $\cos \theta$
$\cot \theta$	$-\frac{4}{3}$	reciprocal of $\tan \theta$

Understanding the Quadrant and Sign Conventions

Quadrant Location

Since the point $(8, -6)$ lies in the fourth quadrant:

- $\sin \theta$ is negative.
- $\cos \theta$ is positive.
- $\tan \theta$ is negative.

These signs are consistent with the computed values.

Significance for Trigonometric Functions

Knowing the quadrant helps to determine the signs of the trigonometric functions without ambiguity.

Additional Insights and Applications

Why Is Finding Exact Values Important?

- Precise calculations in engineering, physics, and mathematics.
- Simplification of expressions involving trigonometric functions.
- Solving equations where approximate decimal values are insufficient.

Common Applications

- Calculating angles in physics problems involving vectors.
- Analyzing periodic functions in signal processing.
- Solving geometry problems involving triangles and circles.

Practice Problems for Mastery

To reinforce understanding, try solving these similar problems:

1. Find the exact values of $\sin \theta$, $\cos \theta$, and $\tan \theta$ for the point $(-6, 8)$.
2. Given the point $(3, 4)$, determine the values of all six trigonometric functions.
3. For a point (x, y) in the second quadrant, with $x = -5$ and $y = 12$, compute the exact trigonometric functions of the corresponding angle θ.

Conclusion

Determining the exact values of trigonometric functions for a point on the terminal side of an angle in standard position is a critical skill in mathematics. Using the point $(8, -6)$, we demonstrated how to calculate the hypotenuse, then derive the sine, cosine, tangent, and their reciprocal functions. Remember, understanding the signs based on the quadrant enhances your ability to interpret these values correctly. Mastery of these concepts enables you to solve complex problems in geometry, physics, and engineering with confidence.

Frequently Asked Questions (FAQs)

Q1: Why do we calculate the hypotenuse \(r\) when finding trig functions?

A: The hypotenuse \(r\) represents the distance from the origin to the point, which normalizes the coordinates and allows us to compute ratios that define the trig functions.

Q2: How do I determine the signs of the trig functions in different quadrants?

A: Use the ASTC (All Students Take Calculus) mnemonic or the signs of \(x\) and \(y\) in each quadrant:

- Quadrant I: All positive
- Quadrant II: Sine positive
- Quadrant III: Tangent positive
- Quadrant IV: Cosine positive

Q3: Can these methods be used for points outside the coordinate axes?

A: Yes, these methods are applicable for any point (except at the origin), regardless of the quadrant or location.

By mastering the process of calculating exact trigonometric values from coordinate points, you enhance your problem-solving toolkit for a wide range of mathematical applications.

Frequently Asked Questions

Given the point (8, -6) on the terminal side of an angle θ in standard position, what is the exact value of sin θ?

sin θ = -6/10 = -3/5

For the point (8, -6) on the terminal side of θ, what is the exact value of cos θ?

cos θ = 8/10 = 4/5

Using the point (8, -6), what is the exact value of tan θ?

tan θ = -6/8 = -3/4

With the point (8, -6) on the terminal side, what is the exact value of csc θ?

csc θ = 1/sin θ = -5/3

Using the point (8, -6), what is the exact value of sec θ?

sec θ = 1/cos θ = 5/4

What is the exact value of cot θ for the point (8, -6) on the terminal side?

cot θ = 8/(-6) = -4/3

How do you find the hypotenuse r for the point (8, -6)?

$r = \sqrt{(8^2 + (-6)^2)} = \sqrt{(64 + 36)} = \sqrt{100} = 10$

Why is the sine of θ negative when the point is (8, -6)?

Because the y-coordinate is negative, indicating θ is in the fourth quadrant where sine is negative.

What quadrant does the point (8, -6) lie in, and how does it affect the signs of trigonometric functions?

The point lies in the fourth quadrant; in this quadrant, cosine and secant are positive, while sine, cosecant, tangent, and cotangent are negative.

How can you verify the exact values of the trigonometric functions for the point (8, -6)?

Calculate the hypotenuse r = 10, then use the definitions: sin θ = y/r = -6/10 = -3/5, cos θ = x/r = 8/10 = 4/5, etc., to verify the values.

Additional Resources

Let (8, -6) Be A Point On The Terminal Side Of An Angle θ In Standard Position. Find The Exact Value Of

When working with angles and their corresponding points on the coordinate plane, understanding how to find exact trigonometric values is fundamental. In this article, we focus on the problem: Let (8, -6) be a point on the terminal side of an angle θ in standard position. Find the exact value of—typically referring to sine, cosine, tangent, or other trigonometric functions associated with θ. This type of problem is common in trigonometry, especially when working with right triangles, unit circles, and coordinate geometry. It not only consolidates understanding of how points relate to angles but also emphasizes the importance of precise calculations over approximations.

Understanding the Problem

In the context of a point (x, y) on the terminal side of an angle θ in standard position:

- The angle θ is measured from the positive x-axis to the terminal side.
- The point (x, y) lies somewhere on this terminal side, which can be visualized as a ray extending from the origin.
- The goal is to find the exact value of a specific trigonometric function (like sin θ, cos θ, tan θ, or others) using the given point.

Given the point (8, -6), we can interpret that:

- The x-coordinate is 8.
- The y-coordinate is -6.
- The point lies in the fourth quadrant because $x > 0$ and $y < 0$.

Step 1: Visualize and Recognize the Coordinates

Before diving into calculations, visualize the coordinate plane:

- The point (8, -6) is located in the fourth quadrant.
- The terminal side of θ passes through this point, forming an angle with the positive x-axis.

Understanding the quadrant helps determine the signs of the trigonometric functions.

Step 2: Find the Radius (r)

The radius, or the hypotenuse of the right triangle formed, is essential for calculating the exact values of sine, cosine, etc. It is given by the distance from the origin to the point (8, -6):

$$ r = \sqrt{x^2 + y^2} $$

Calculating:

$$
r = \sqrt{8^2 + (-6)^2} = \sqrt{64 + 36} = \sqrt{100} = 10
$$

The radius is $r = 10$.

Step 3: Recall the Definitions of Trigonometric Functions

Using the point and the radius, we can define the six basic trigonometric functions:

- Sine: $\sin \theta = \frac{y}{r}$
- Cosine: $\cos \theta = \frac{x}{r}$
- Tangent: $\tan \theta = \frac{y}{x}$
- Cosecant: $\csc \theta = \frac{r}{y}$
- Secant: $\sec \theta = \frac{r}{x}$
- Cotangent: $\cot \theta = \frac{x}{y}$

Step 4: Calculate the Exact Values

Let's compute the exact values for the primary functions:

Sine:

$$
\sin \theta = \frac{y}{r} = \frac{-6}{10} = -\frac{3}{5}
$$

Cosine:

$$
\cos \theta = \frac{x}{r} = \frac{8}{10} = \frac{4}{5}
$$

Tangent:

$$
\tan \theta = \frac{y}{x} = \frac{-6}{8} = -\frac{3}{4}
$$

Corresponding reciprocal functions:

Cosecant:

$$
\csc \theta = \frac{r}{y} = \frac{10}{-6} = -\frac{5}{3}
$$

Secant:

$$
\sec \theta = \frac{r}{x} = \frac{10}{8} = \frac{5}{4}
$$

Cotangent:

$$
\cot \theta = \frac{x}{y} = \frac{8}{-6} = -\frac{4}{3}
$$

Step 5: Confirm the Quadrant and Significance

Since the point is in the fourth quadrant:

- Sine is negative (which aligns with $-\frac{3}{5}$)
- Cosine is positive (aligns with $\frac{4}{5}$)
- Tangent is negative (aligns with $-\frac{3}{4}$)

This confirms the consistency of the calculations.

Additional Considerations:

Depending on the specific question, you might be asked for:

- The exact value of $\sin \theta$, $\cos \theta$, or $\tan \theta$.
- The value of the other trigonometric functions.
- The reference angle θ_{ref}.
- The angle θ itself in degrees or radians.

Step 6: Find the Reference Angle

The reference angle θ_{ref} is the acute angle between the terminal side and the x-axis. It can be found using:

$$
\theta_{ref} = \arccos \left(\frac{x}{r} \right) \quad \text{or} \quad \arcsin \left(\frac{y}{r} \right)
$$

Using the cosine:

$$
\theta_{ref} = \arccos \left(\frac{4}{5} \right) = \arccos 0.8
$$

The exact value of $\arccos 0.8$ is not a standard special angle, but for exact calculations, it's often left as $\arccos \frac{4}{5}$. Alternatively, using the sine:

$$
\theta_{ref} = \arcsin \left(-\frac{3}{5} \right)
$$

Since $\sin \theta_{ref}$ is negative, and the point is in the fourth quadrant, the reference angle is:

$$
\theta_{ref} = \arcsin \left(\frac{3}{5} \right) = \arcsin \left(0.6 \right)
$$

which is approximately 36.87°, but for exact values, it's best to leave it in terms of inverse trigonometric functions.

Summary of the Exact Values:

Function	Exact Value	Sign in Quadrant IV
$\sin \theta$	$-\frac{3}{5}$	Negative
$\cos \theta$	$\frac{4}{5}$	Positive
$\tan \theta$	$-\frac{3}{4}$	Negative
$\csc \theta$	$-\frac{5}{3}$	Negative
$\sec \theta$	$\frac{5}{4}$	Positive
$\cot \theta$	$-\frac{4}{3}$	Negative

Practical Applications

Understanding how to derive these values from a point is essential in various fields, including physics, engineering, and computer graphics, where angles and coordinates intersect. The ability to transition seamlessly from a coordinate point to exact trigonometric values allows for precise modeling, analysis, and problem-solving.

Final Remarks

When working with points on the terminal side of an angle in standard position, always:

- Determine the radius $r = \sqrt{x^2 + y^2}$.
- Use the definitions of trigonometric functions in terms of x, y, r.
- Consider the quadrant to assign correct signs.
- Recognize that some values are best left in exact fractional form, especially when they relate to special triangles or Pythagorean triples.

This structured approach ensures accuracy and clarity in solving similar problems involving points and angles in the coordinate plane.

In conclusion, given the point (8, -6), the exact values of the trigonometric functions are:

- $\sin \theta = -\frac{3}{5}$
- $\cos \theta = \frac{4}{5}$

- $\tan \theta = -\frac{3}{4}$

These values provide a complete picture of the angle's trigonometric profile in standard position, enabling further analysis or application with confidence.

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