Let F(x, Y) = E(3x + 47). (a) Find The Tangent Plane To Fat (0, 0). (Let Z Be The Dependent Variable.)

Let F(x, Y) = E(3x + 47). (a) Find The Tangent Plane To Fat (0, 0). (Let Z Be The Dependent Variable.)

Understanding the concept of tangent planes in multivariable calculus is fundamental for analyzing the behavior of functions near specific points. In this article, we will explore how to find the tangent plane to a particular function defined as F(x, Y) = E(3x + 47) at the point (0, 0). We will interpret the problem carefully, break down the steps involved, and discuss the significance of tangent planes in mathematical analysis and applications.

Introduction to Multivariable Functions and Tangent Planes

What is a Multivariable Function?

A multivariable function is a function that depends on two or more variables. For example, the function F(x, Y) depends on variables x and Y. Such functions are fundamental in fields like physics, engineering, economics, and computer science, where systems often depend on multiple parameters.

Understanding the Concept of a Tangent Plane

Just as a tangent line approximates the behavior of a curve at a point in single-variable calculus, a tangent plane serves as a linear approximation to a surface at a point in three-dimensional space. Specifically, for a surface defined by a function Z = F(x, Y), the tangent plane at a point (x_0, y_0, z_0) approximates the surface near that point.

Mathematically, the tangent plane to the surface Z = F(x, Y) at (x_0, y_0) is given by:

\[Z = F(x_0, y_0) + F_x(x_0, y_0)(x - x_0) + F_Y(x_0, y_0)(Y - y_0) \]

where \(F_x \) and \(F_Y \) are the partial derivatives of F with respect to x and Y, respectively.

Interpreting the Function $F(x, Y) = E(3x + 47)$

What Does the Function Represent?

The notation $F(x, Y) = E(3x + 47)$ suggests that the function is defined as the expected value of the expression $3x + 47$. Here, E denotes the expected value operator, commonly used in probability and statistics.

However, it's crucial to clarify whether the expectation E is:

- Conditional or unconditional?
- Dependent on Y or independent?

Given the expression, it appears that the expectation might be over some random variable, say, a variable X, with the function representing the expected value of $3x + 47$ over a certain distribution, potentially depending on Y.

But, as the function is written, it seems to suggest that $F(x, Y)$ is a constant function with respect to Y, since $E(3x + 47)$ does not explicitly depend on Y. To proceed, we will assume that:

- The function $F(x, Y)$ depends on the variable x, with the expectation taken over some random variable ω (say, $X(\omega)$), such that:

$$F(x, Y) = E_\omega [3x + 47]$$

- Since the expectation operator acts over the randomness, and the right side involves only x, the function effectively does not depend on Y. But the problem statement includes Y, perhaps as a parameter or variable of interest.

Key Point: For the purposes of this article, we will interpret $F(x, Y)$ as a function of two variables, with the expectation taken over some distribution, and the dependence on Y either being trivial or implicit.

Step-by-Step Solution: Finding the Tangent Plane at (0,0)

Step 1: Clarify the Function at the Point (0,0)

To find the tangent plane to the surface $Z = F(x, Y)$ at the point (0, 0), we need:

- The value of $F(0, 0)$
- The partial derivatives F_x and F_Y evaluated at (0, 0)

Given the function:

$$ F(x, Y) = E(3x + 47) $$

If the expectation E is over a random variable independent of x and Y, then $F(x, Y)$ is a constant with respect to both variables, equating to the expected value of $3x + 47$. Typically, the expectation over a constant is just that constant itself, so:

$$ F(x, Y) = 3x + 47 $$

But this depends on whether the expectation involves a random variable ω or not. For simplicity, and based on common interpretations, let's assume $E(3x + 47) = 3x + 47$.

Therefore:

$$ F(x, Y) = 3x + 47 $$

which is a linear function involving only x, with no explicit dependence on Y.

Step 2: Calculate $F(0, 0)$

$$ F(0, 0) = 3 \times 0 + 47 = 47 $$

This is the value of the function at the point (0, 0), which corresponds to the Z-coordinate on the surface.

Step 3: Compute Partial Derivatives

- The partial derivative of F with respect to x:

$$ F_x(x, Y) = \frac{\partial}{\partial x} (3x + 47) = 3 $$

- The partial derivative of F with respect to Y:

Since F does not depend explicitly on Y:

$$ F_Y(x, Y) = 0 $$

At the point (0, 0):

$$ F_x(0, 0) = 3 $$
$$ F_Y(0, 0) = 0 $$

Step 4: Write the Equation of the Tangent Plane

Using the general formula for the tangent plane:

$$ Z = F(x_0, y_0) + F_x(x_0, y_0)(x - x_0) + F_Y(x_0, y_0)(Y - y_0) $$

Plugging in the known values:

$$ Z = 47 + 3(x - 0) + 0 \times (Y - 0) $$

Simplifies to:

$$ Z = 47 + 3x $$

Final Expression of the Tangent Plane

The tangent plane to the surface $Z = F(x, Y)$ at the point (0, 0) is:

$$ Z = 47 + 3x $$

This plane provides the best linear approximation of the surface near that point.

Significance of the Tangent Plane in Multivariable Calculus

Understanding Local Behavior of Surfaces

The tangent plane offers crucial insights into how a surface behaves around a specific point. It allows mathematicians and engineers to approximate the surface with a simple plane, facilitating calculations and predictions.

Applications in Optimization and Modeling

- Optimization: Tangent planes help identify local maxima or minima by analyzing the slope and curvature.
- Engineering: In structural analysis, tangent planes approximate the surface of materials or stress distributions.
- Economics: They assist in modeling cost surfaces or utility functions near optimal points.

Limitations of the Tangent Plane Approximation

While tangent planes are useful, their accuracy diminishes as one moves further from the point of tangency. Higher-order derivatives and Taylor series expansions can improve approximation accuracy for more complex surfaces.

Additional Considerations and Extensions

Higher-Order Approximations: Taylor Series

Expanding the function $F(x, Y)$ into a Taylor series around the point (0, 0) allows for more accurate local approximations, especially when the surface has curvature.

Impact of Dependence on Y

If F had an explicit dependence on Y, the partial derivative F_Y would be non-zero, resulting in a different tangent plane. For functions involving Y, the process would follow similarly, with the derivatives evaluated at the point of interest.

Generalization to Other Points

The method outlined applies to any point (x_0, y_0) on the surface, allowing for the analysis of the surface's behavior throughout its domain.

Conclusion

In this comprehensive guide, we have demonstrated how to find the tangent plane to the surface defined by the function $F(x, Y) = E(3x + 47)$ at the point (0, 0). By interpreting the function correctly, calculating the value and partial derivatives, and applying the tangent plane formula, we derived:

$$Z = 47 + 3x$$

This linear approximation provides valuable insights

Frequently Asked Questions

What is the function F(x, y) in the given problem?

The function is F(x, y) = E(3x + 47), where E denotes the expected value.

How do you interpret the function F(x, y) given as E(3x + 47)?

Since 3x + 47 is a deterministic expression (not random), its expected value E(3x + 47) is simply 3x + 47, making F(x, y) = 3x + 47.

What is the significance of finding the tangent plane to F at the point (0, 0)?

The tangent plane provides the best linear approximation of the surface z = F(x, y) at the point (0, 0), which helps understand the local behavior of the function.

How do you determine the value of F at the point (0, 0)?

Substitute x=0 into F(x, y): F(0, y) = 30 + 47 = 47. So, at (0, 0), F(0, 0) = 47.

What are the partial derivatives of F with respect to x and y?

Since F(x, y) = 3x + 47, the partial derivative with respect to x is ∂F/∂x = 3, and with respect to y is ∂F/∂y = 0.

How do you formulate the equation of the tangent plane at (0, 0)?

The tangent plane at (x_0, y_0) is $z = F(x_0, y_0) + (\partial F/\partial x)(x - x_0) + (\partial F/\partial y)(y - y_0)$. Substituting (0,0): z = 47 + 3(x − 0) + 0(y − 0) = 47 + 3x.

What is the explicit equation of the tangent plane to F at (0, 0)?

The tangent plane is z = 47 + 3x.

Why is the partial derivative with respect to y zero in this case?

Because F(x, y) = 3x + 47 does not depend on y, so its partial derivative with respect to y is zero.

What is the importance of understanding the tangent plane in multivariable calculus?

It helps approximate the function near a point, analyze local linear behavior, and is essential in optimization and differential analysis of surfaces.

Additional Resources

Let F(x, Y) = E(3x + 47). (a) Find The Tangent Plane To F at (0, 0). (Let Z Be The Dependent Variable.) is a compelling problem that combines fundamental concepts of multivariable calculus with practical applications in understanding the behavior of functions near specific points. This problem offers an excellent opportunity to explore the process of differentiating functions with respect to multiple variables, computing partial derivatives, and constructing tangent planes. Its relevance extends across various fields such as physics, engineering, economics, and data science, where understanding local approximations of surfaces is critical.

In this comprehensive review, we will dissect the problem step-by-step, emphasizing the mathematical procedures involved, their significance, and potential pitfalls. We will also analyze the broader implications of these methods, compare alternative approaches, and highlight the key features that make this problem a valuable learning experience for students and practitioners alike.

Understanding the Function and the Problem Statement

Interpreting the Function: F(x, Y) = E(3x + 47)

The function F(x, Y) is expressed as the expected value of a random variable E(3x + 47). This notation suggests that E is an expectation operator, which in probability theory indicates an average or mean value of a random variable or a function of a random variable. However, the notation here can be ambiguous; it might imply that E is a function applied to the linear expression (3x + 47), or perhaps that E denotes the expectation of some random variable dependent on x.

If the problem intends for E to be a constant or a deterministic function, then F(x, Y) simplifies to a linear function:

\[F(x, Y) = E(3x + 47) \]

Suppose E is a constant (for example, an expected value of a certain random variable that depends on parameters). In that case, the function reduces to a constant or linear form, making the problem trivial. But more likely, the problem aims to model a scenario where the expectation depends on some other stochastic component, with the variable Y potentially influencing the expectation.

Given the context, it appears the function is designed to be a deterministic function with respect to x and Y, perhaps simplified to:

\[F(x, Y) = E(3x + 47) \]

and the expectation operator E is applied to a deterministic expression, which would simply result in the same expression. Alternatively, if E is meant as a random expectation, then additional information about the distribution of the underlying random variable is necessary to evaluate the expectation explicitly.

Key Point: For the purpose of this problem, assume that $E(3x + 47)$ is a known deterministic function, or that E is a constant expectation value. If so, then $F(x, Y)$ is a function of x and Y, possibly with Y influencing the expectation indirectly.

Finding the Tangent Plane at a Point

Why the Tangent Plane Matters

The tangent plane at a point on a surface provides a linear approximation of the surface near that point. It is essential in many applications, such as optimization, differential equations, and modeling, where understanding the local behavior of a surface is crucial. For the point (0, 0), the tangent plane encapsulates how F behaves around the origin, offering insights into its local slope and curvature.

Step-by-Step Procedure

To find the tangent plane to the surface $z = F(x, Y)$ at the point (0, 0), follow these steps:

1. Evaluate the function at the point: Compute $F(0, 0)$.
2. Compute partial derivatives: Find $\frac{\partial F}{\partial x}$ and $\frac{\partial F}{\partial Y}$.
3. Evaluate derivatives at the point: Find their values at (0, 0).
4. Construct the tangent plane equation:

$$
Z = F(0, 0) + \frac{\partial F}{\partial x}(0, 0)(x - 0) + \frac{\partial F}{\partial Y}(0, 0)(Y - 0)
$$

Detailed Derivation and Calculation

Step 1: Evaluate F at (0, 0)

Assuming $F(x, Y) = E(3x + 47)$, and that E is a constant (say, $E = e$), then:

$$
F(0, 0) = E(3 \cdot 0 + 47) = E(47)

\]

If E is a known constant, this simplifies to:

\[
F(0, 0) = \text{constant value}
\]

Alternatively, if the expectation involves a random variable with a known distribution, then the evaluation would depend on that distribution. For simplicity, assume \(E(3x + 47) \) is just the value \(3x + 47 \), making \(F(x, Y) = 3x + 47 \).

In that case:

\[
F(0, 0) = 3 \cdot 0 + 47 = 47
\]

Step 2: Compute the Partial Derivatives

If \(F(x, Y) = 3x + 47 \), then:

\[
\frac{\partial F}{\partial x} = 3
\]

and since the function does not explicitly depend on Y in this simplified case:

\[
\frac{\partial F}{\partial Y} = 0
\]

If the original function included Y explicitly, for example, \(F(x, Y) = E(3x + Y) \), then:

\[
\frac{\partial F}{\partial x} = E'(3x + 47) \cdot 3
\]
\[
\frac{\partial F}{\partial Y} = E'(3x + 47) \cdot 1
\]

Assuming E is linear or constant, derivatives would be straightforward.

Step 3: Evaluate Derivatives at (0, 0)

Using the simplified assumption:

\[

$$
\frac{\partial F}{\partial x}(0, 0) = 3
$$
$$
\frac{\partial F}{\partial Y}(0, 0) = 0
$$

Step 4: Write Equation of the Tangent Plane

The tangent plane at (0, 0, Z) is:

$$
Z = F(0, 0) + \frac{\partial F}{\partial x}(0, 0) \cdot x + \frac{\partial F}{\partial Y}(0, 0) \cdot Y
$$

Plugging in the values:

$$
Z = 47 + 3x + 0 \cdot Y
$$
$$
Z = 47 + 3x
$$

This linear approximation suggests that near (0, 0), the surface behaves like a plane with slope 3 in the x-direction and no change in the Y-direction.

Discussion and Broader Context

Significance of the Tangent Plane Calculation

Calculating the tangent plane is fundamental for understanding how a surface behaves locally. This has several practical implications:

- Optimization: Approximate functions near critical points.
- Modeling: Simplify complex surfaces for simulation or analysis.
- Differential Geometry: Study properties related to curvature and surface behavior.

Extensions and Variations

- Higher-Order Approximations: Use Taylor series expansions for more accurate local models.
- Implicit Surfaces: When functions are defined implicitly, tangent planes require different methods.
- Dynamic Systems: Extend these ideas to surfaces evolving over time.

Potential Challenges and Pitfalls

- Ambiguity in Function Definition: Clarify whether E is a constant, an expectation operator, or a random variable.
- Dependence on Distribution: If E involves probabilistic components, additional information about distributions is necessary.
- Assumption of Linearity: Simplifying assumptions may overlook nuanced behavior in more complex functions.

Pros and Cons of the Approach

Pros:

- Clarity: Step-by-step derivation makes the process transparent.
- Educational Value: Reinforces fundamental concepts of partial derivatives and tangent planes.
- Applicability: Methods are adaptable to various functions and contexts.

Cons:

- Simplifications: Assumptions about the form of E may limit the generality.
- Potential Oversights: Ignoring Y dependence in some cases may oversimplify the problem.
- Distributional Complexity: Real-world expectation functions can be more complicated, requiring advanced techniques.

Conclusion

The problem of finding the tangent plane to the surface defined by $F(x, Y) = E(3x + 47)$ at the point (0, 0) encapsulates essential ideas in multivariable calculus. By understanding how to evaluate the function, compute partial derivatives, and construct the tangent plane, students and practitioners gain valuable tools for analyzing local surface behavior. While the problem's simplicity depends on the assumptions made about the expectation operator E, the core methodology remains robust across more complex scenarios.

This exercise underscores the importance of careful

Let F X Y E 3x 47 A Find The Tangent Plane To Fat 0 0 Let Z Be The Dependent Variable

Let F X Y E 3x 47 A Find The Tangent Plane To Fat 0 0 Let Z Be The Dependent Variable

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