Let F(x,y,z)=1z2xi+(13y3+tan(z))j+(1x2z+3y2)k. Use The Divergence Theorem To Evaluate SF DS Where S Is

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Introduction

In vector calculus, surface integrals are fundamental tools used to evaluate fluxes of vector fields across surfaces. When dealing with complex surfaces, direct computation of these integrals can often be challenging. Fortunately, the Divergence Theorem provides a powerful method to simplify such calculations by converting surface integrals into volume integrals.

This article aims to explore the application of the Divergence Theorem to evaluate the flux integral of a given vector field \(\mathbf{F}(x,y,z) \) across a specified surface \(S \). We will dissect the process step-by-step, beginning with defining the vector field, understanding the theorem, and then applying it to compute the flux integral.

Understanding the Vector Field \(\mathbf{F}(x,y,z) \)

The vector field provided is:

\[
\mathbf{F}(x,y,z) = 1z^2 x\, \mathbf{i} + (13 y^3 + \tan z)\, \mathbf{j} + (1 x^2 z + 3 y^2)\, \mathbf{k}
\]

Breaking Down \(\mathbf{F} \)

- \(\mathbf{i} \)-component: \(z^2 x \)
- \(\mathbf{j} \)-component: \(13 y^3 + \tan z \)
- \(\mathbf{k} \)-component: \(x^2 z + 3 y^2 \)

Understanding the structure of \(\mathbf{F} \) is crucial for calculating its divergence and understanding its behavior within a volume.

The Divergence Theorem: Concept and Significance

What Is the Divergence Theorem?

The Divergence Theorem, also known as Gauss's theorem, states that for a vector field $\mathbf{F}$ defined on a closed surface S that encloses a volume V:

$$
\iint_{S} \mathbf{F} \cdot \mathbf{n}\, dS = \iiint_{V} (\nabla \cdot \mathbf{F})\, dV
$$

Where:
- $\mathbf{n}$ is the outward unit normal vector on the surface S,
- dS is the surface element,
- $\nabla \cdot \mathbf{F}$ is the divergence of $\mathbf{F}$,
- dV is the volume element.

Why Use the Divergence Theorem?

- Simplifies the calculation of flux integrals over complex surfaces.
- Converts a surface integral into a volume integral, often easier to evaluate.
- Useful in physics and engineering applications, such as fluid flow, electromagnetism, and heat transfer.

Conditions for Applying the Theorem

- $\mathbf{F}$ must be continuously differentiable in the volume V.
- The surface S must be closed and oriented outward.

Step 1: Defining the Surface S

The problem specifies evaluating $\iint_{S} \mathbf{F} \cdot d\mathbf{S}$ over a surface S. To apply the Divergence Theorem, the surface must enclose a bounded volume V.

Typical choices for S:

- A sphere
- A cube
- A cylinder
- Any closed surface

Since the problem statement does not specify S, for demonstration purposes, assume S is the boundary of the cube $0 \leq x \leq a$, $0 \leq y \leq b$, $0 \leq z \leq c$.

Step 2: Calculating the Divergence $\nabla \cdot \mathbf{F}$

The divergence of $\mathbf{F}$ is:

$$
\nabla \cdot \mathbf{F} = \frac{\partial}{\partial x} (z^2 x) + \frac{\partial}{\partial y} (13 y^3 + \tan z) + \frac{\partial}{\partial z} (x^2 z + 3 y^2)
$$

Let's compute each term separately:

1. $\frac{\partial}{\partial x} (z^2 x)$

- z^2 is treated as a constant with respect to x,
- Derivative: z^2

2. $\frac{\partial}{\partial y} (13 y^3 + \tan z)$

- $13 y^3$ derivative: $39 y^2$,
- $\tan z$ is independent of y, derivative: 0,
- Total: $39 y^2$

3. $\frac{\partial}{\partial z} (x^2 z + 3 y^2)$

- $x^2 z$ derivative: x^2,
- $3 y^2$ independent of z: derivative 0,
- Total: x^2

Putting it all together:

$$
\nabla \cdot \mathbf{F} = z^2 + 39 y^2 + x^2
$$

Step 3: Setting Up the Volume Integral

Using the Divergence Theorem:

$$
\iint_{S} \mathbf{F} \cdot d\mathbf{S} = \iiint_{V} (z^2 + 39 y^2 + x^2) \, dV
$$

Assuming V is the cube $0 \leq x \leq a$, $0 \leq y \leq b$, $0 \leq z \leq c$, the volume integral becomes:

$$
\int_0^a \int_0^b \int_0^c (z^2 + 39 y^2 + x^2) \, dz\, dy\, dx

\]

Step 4: Computing the Volume Integral

Because the integrand separates into sums of functions of individual variables, the triple integral simplifies into sums of products of single-variable integrals:

\[
\iiint_{V} (z^2 + 39 y^2 + x^2) \, dV = \int_0^a \int_0^b \int_0^c z^2 \, dz\, dy\, dx + \int_0^a \int_0^b \int_0^c 39 y^2 \, dz\, dy\, dx + \int_0^a \int_0^b \int_0^c x^2 \, dz\, dy\, dx
\]

Calculating each separately:

1. \(\int_0^a \int_0^b \int_0^c z^2 \, dz\, dy\, dx \)

- \(z \)-integral: \(\int_0^c z^2 \, dz = \frac{c^3}{3} \)
- \(y \)-integral: \(\int_0^b dy = b \)
- \(x \)-integral: \(\int_0^a dx = a \)

Total:

\[
a \times b \times \frac{c^3}{3} = \frac{abc^3}{3}
\]

2. \(\int_0^a \int_0^b \int_0^c 39 y^2 \, dz\, dy\, dx \)

- \(z \)-integral: \(\int_0^c dz = c \)
- \(y \)-integral: \(\int_0^b 39 y^2 dy = 39 \times \frac{b^3}{3} = 13 b^3 \)
- \(x \)-integral: \(\int_0^a dx = a \)

Total:

\[
a \times c \times 13 b^3 = 13 a c b^3
\]

3. \(\int_0^a \int_0^b \int_0^c x^2 \, dz\, dy\, dx \)

- \(z \)-integral: \(\int_0^c dz = c \)
- \(y \)-integral: \(\int_0^b dy = b \)
- \(x \)-integral: \(\int_0^a x^2 dx = \frac{a^3}{3} \)

Total:

\[
c \times b \times \frac{a^3}{3} = \frac{a^3 b c}{3}
\]

Final Volume Integral Result

Adding all parts:

\[
\boxed{
\iiint_{V} (\nabla \cdot \mathbf{F})\, dV = \frac{abc^3}{3} + 13 a c b^3 +
\frac{a^3 b c}{3}

Frequently Asked Questions

What is the divergence of the vector field F(x, y, z) = 1z2xi + (13y3 + tan(z))j + (1x2z + 3y2)k?

The divergence div F = ∂/∂x (1z2x) + ∂/∂y (13y3 + tan(z)) + ∂/∂z (1x2z + 3y2). Calculating each term: ∂/∂x (1z2x) = 1z2, ∂/∂y (13y3 + tan(z)) = 39y2, ∂/∂z (1x2z + 3y2) = 1x2. Therefore, div F = 1z2 + 39y2 + 1x2.

How does the Divergence Theorem simplify the evaluation of the surface integral over a closed surface S for the vector field F?

The Divergence Theorem states that the flux of F across a closed surface S is equal to the triple integral of the divergence of F over the volume enclosed by S. Mathematically, ∭_V div F dV = ∬_S F · n dS. This allows us to evaluate the surface integral by computing a volume integral instead of directly integrating over S.

Given the divergence of F, what is the general approach to compute the flux over surface S using the Divergence Theorem?

First, determine the volume V enclosed by surface S. Next, set up the triple integral of div F over V, i.e., ∭_V (1z2 + 39y2 + 1x2) dV. Finally, evaluate this volume integral, which gives the total flux of F through S.

What information about surface S is necessary to

apply the Divergence Theorem in this problem?

You need to know the shape and boundaries of the surface S to define the volume V it encloses. This includes the limits for x, y, and z coordinates and whether S is a standard shape like a sphere, cube, or more complex surface. Without this, you cannot set up the volume integral accurately.

Are there any common challenges when applying the Divergence Theorem to vector fields like F(x, y, z) in practice?

Yes, challenges include accurately identifying the volume V enclosed by S, setting up the correct limits of integration, and simplifying the divergence expression. Additionally, complex geometries may require advanced techniques or coordinate transformations to evaluate the volume integral effectively.

Additional Resources

Let $F(x,y,z) = 1z^2xi + (13y^3 + \tan(z))j + (1x^2z + 3y^2)k$. Use The Divergence Theorem To Evaluate SF DS Where S Is

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Introduction

In vector calculus, surface integrals and divergence theorems serve as fundamental tools to analyze fluxes across surfaces and the behavior of vector fields within volumes. The divergence theorem, also known as Gauss’s theorem, provides a powerful method to convert a surface integral over a closed surface into a triple volume integral over the region it encloses. This approach simplifies complex calculations, especially when the divergence of the vector field is easier to compute than directly evaluating the surface integral.

In this detailed exploration, we examine a vector field:

$$
\mathbf{F}(x, y, z) = z^2 x \mathbf{i} + (13 y^3 + \tan z) \mathbf{j} + (x^2 z + 3 y^2) \mathbf{k}
$$

and evaluate the flux integral

$$
\iint_{S} \mathbf{F} \cdot d\mathbf{S}
$$

where S is a closed surface bounding a region V. We will utilize the divergence theorem to transform this surface integral into a volume integral, simplifying the calculation and gaining insights into the behavior of the vector field within the volume.

Understanding the Divergence Theorem

Theoretical Framework

The divergence theorem states that for a vector field $\mathbf{F}$ continuously differentiable on a region V:

$$
\iint_{S} \mathbf{F} \cdot d\mathbf{S} = \iiint_{V} (\nabla \cdot \mathbf{F}) \, dV
$$

where:

- S is the closed surface bounding V,
- $d\mathbf{S}$ is the outward pointing surface element,
- $\nabla \cdot \mathbf{F}$ is the divergence of $\mathbf{F}$,
- V is the volume enclosed by S.

This theorem simplifies flux calculations, especially when evaluating over complicated surfaces, by reducing the problem to finding the divergence and integrating over the volume.

Conditions for Application

- The vector field must be continuously differentiable within V.
- The surface S must be closed and oriented outward.

Decomposition of the Vector Field

Given:

$$
\mathbf{F}(x, y, z) = z^2 x \mathbf{i} + (13 y^3 + \tan z) \mathbf{j} + (x^2 z + 3 y^2) \mathbf{k}
$$

we note the components:

- $F_x = z^2 x$
- $F_y = 13 y^3 + \tan z$
- $F_z = x^2 z + 3 y^2$

Symmetry and Behavior

Before computing divergence, observe the structure:

- F_x involves x and z^2,
- F_y involves y raised to odd powers and $\tan z$,
- F_z involves $x^2 z$ and quadratic y terms.

Understanding these components helps anticipate the nature of the divergence and the potential symmetry or simplifications.

Computing the Divergence

Step-by-Step Derivation

The divergence is:

$$
\nabla \cdot \mathbf{F} = \frac{\partial F_x}{\partial x} + \frac{\partial F_y}{\partial y} + \frac{\partial F_z}{\partial z}
$$

Calculate each term:

1. $\frac{\partial F_x}{\partial x} = \frac{\partial (z^2 x)}{\partial x} = z^2$
2. $\frac{\partial F_y}{\partial y} = \frac{\partial (13 y^3 + \tan z)}{\partial y} = 39 y^2 + 0 = 39 y^2$
3. $\frac{\partial F_z}{\partial z} = \frac{\partial (x^2 z + 3 y^2)}{\partial z} = x^2 + 0 = x^2$

Thus, the divergence simplifies to:

$$
\nabla \cdot \mathbf{F} = z^2 + 39 y^2 + x^2
$$

Interpretation

The divergence is a quadratic form in x, y, z, indicating symmetric contributions from the squared terms and a z-dependent term.

Specifying the Region V and Surface S

Choosing a Suitable Volume

To proceed with the volume integral, we need to specify the region V.

Common choices include:

- A sphere centered at the origin,
- A rectangular box,
- A cylinder.

For the purpose of this analysis, assume V is a unit cube:

$$
V = \{(x, y, z) \mid -1 \leq x, y, z \leq 1\}
$$

and S is its boundary surface.

Justification

- The cube simplifies the limits of integration.
- The symmetry about the origin aids in evaluating integrals involving odd functions.
- The boundary surface S comprises six faces, each flat, making the application of the divergence theorem straightforward.

Setting up the Volume Integral

Applying the divergence theorem, the flux becomes:

$$
\iint_{S} \mathbf{F} \cdot d\mathbf{S} = \iiint_{V} (z^2 + 39 y^2 + x^2) \, dV
$$

For the cube $\{-1 \leq x, y, z \leq 1\}$:

$$
\iiint_{V} (z^2 + 39 y^2 + x^2) \, dx\,dy\,dz = \int_{-1}^{1} \int_{-1}^{1} \int_{-1}^{1} (z^2 + 39 y^2 + x^2) \, dx\,dy\,dz
$$

Separability of the Integrand

Since the integrand is a sum of functions each in a single variable, the triple integral splits into the sum of three separate integrals:

$$
\iiint_{V} z^2 \, dV + 39 \iiint_{V} y^2 \, dV + \iiint_{V} x^2 \, dV
$$

which can be expressed as:

\[
\left(\int_{-1}^1 z^2 \, dz \right) \left(\int_{-1}^1 dy \right) \left(\int_{-1}^1 dx \right) + 39 \left(\int_{-1}^1 y^2 \, dy \right) \left(\int_{-1}^1 dx \right) \left(\int_{-1}^1 dz \right) + \left(\int_{-1}^1 x^2 \, dx \right) \left(\int_{-1}^1 dy \right) \left(\int_{-1}^1 dz \right)
\]

Computing the Integrals

Each integral over \([-1, 1]\):

- For the quadratic term:

\[
\int_{-1}^1 t^2 \, dt = \left[\frac{t^3}{3} \right]_{-1}^1 = \frac{1}{3} - \left(-\frac{1}{3} \right) = \frac{2}{3}
\]

- For the linear term:

\[
\int_{-1}^1 dt = 2
\]

Thus:

1. \(z^2\) integral:

\[
\left(\frac{2}{3} \right) \times 2 \times 2 = \frac{2}{3} \times 4 = \frac{8}{3}
\]

2. \(39 y^2\) integral:

\[
39 \times \frac{2}{3} \times 2 \times 2 = 39 \times \frac{2}{3} \times 4 = 39 \times \frac{8}{3} = 39 \times \frac{8}{3} = \frac{312}{3} = 104
\]

3. \(x^2\) integral:

\[
\frac{2}{3} \times 2 \times 2 = \frac{8}{3}
\]

Summing the results

Total flux:

\[

```
\frac{8}{3} + 104 + \frac{8}{3} = \left( \frac{8}{3} + \frac{8}{3} \right) +
104 = \frac{16}{3} + 104
\]

Expressed with common denominator:

\[
= \frac{16}{3} + \frac{312}{3} = \frac{328}{3}
\]

---

Final Result

\[
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```

Let F X Y Z 1z2xi 13y3 Tan Z J 1x2z 3y2 K Use The Divergence Theorem To Evaluate Sf Ds Where S Is

Let F X Y Z 1z2xi 13y3 Tan Z J 1x2z 3y2 K Use The Divergence Theorem To Evaluate Sf Ds Where S Is

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