# Let S Be A Nonempty Subset Of Z And Let R Be A Relation Defined On S By X R Y If 3 (x + 2y). (a) Prove

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## Introduction

In mathematical discourse, relations on sets play a pivotal role in understanding the structure and properties of various mathematical objects. Specifically, relations defined on subsets of integers (Z) are fundamental in many areas such as number theory, algebra, and discrete mathematics. In this article, we analyze a particular relation $R$ defined on a nonempty subset $S \subseteq Z$ with the rule:

$$X R Y \iff 3(x + 2y)$$

Our goal is to explore the properties of this relation, especially focusing on proving whether it satisfies certain properties such as reflexivity, symmetry, antisymmetry, and transitivity. This analysis will deepen the understanding of the structure induced by this relation within the subset $S$.

## Understanding the Relation $R$

Before delving into the proofs of various properties, it is essential to interpret and understand the relation $R$ properly.

### Definition of the Relation $R$

Given $S \subseteq Z$, the relation $R$ is defined as:

$$X R Y \iff 3 (x + 2 y)$$

However, the statement is incomplete as written. Typically, such relations are defined in terms of divisibility or equality involving the expression $3(x + 2 y)$.

Assuming the common interpretation, the relation is:

$$X R Y \iff 3 (x + 2 y) \text{ divides } 0, \text{ or perhaps } 3 (x + 2 y) = 0.$$

But this seems trivial unless specified further. Alternatively, perhaps the relation is:

$$ X R Y \iff 3 \text{ divides } (x + 2 y). $$

Given the wording, the most plausible intended relation is:

"X R Y if and only if 3 divides (x + 2 y)."

This is a common relation in number theory, where divisibility is used to define a relation.

Therefore, for clarity, we will proceed with:

$$ X R Y \iff 3 \mid (x + 2 y) $$

which reads: "X is related to Y if and only if 3 divides $x + 2 y$."

## Properties of the Relation $R$

To analyze the relation, we will examine the following properties:

- Reflexivity
- Symmetry
- Antisymmetry
- Transitivity

Each property indicates specific structural features of the relation $R$.

## Proving the Relation's Properties

### Reflexivity

A relation $R$ on $S$ is reflexive if for all $x \in S$, $x R x$ holds.

Proof:

- For any $x \in S$,
- $x R x \iff 3 \mid (x + 2 x)$ (since $y = x$),
- $x R x \iff 3 \mid 3x$,
- $3 \mid 3x$ is always true because 3 divides any multiple of 3.

Conclusion:

- Since $3 \mid 3x$ for all $x \in Z$, and $S \subseteq Z$,
- The relation $R$ is reflexive on $S$.

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## Symmetry

A relation $R$ is symmetric if for all $x, y \in S$, $x R y$ implies $y R x$.

Proof:

- Assume $x R y$, i.e., $3 \mid (x + 2 y)$.
- To prove symmetry, we need to verify if $3 \mid (y + 2 x)$.

Analysis:

- Given that $3 \mid x + 2 y$,
- Is $3 \mid y + 2 x$?

Counterexample:

- Take $x = 0$, $y = 3$:
- $x R y$ since $3 \mid 0 + 2 \times 3 = 6$,
- $3 \mid 6$, true.
- $y R x$ requires $3 \mid 3 + 2 \times 0 = 3$,
- $3 \mid 3$, also true.
- Now, take $x=1$, $y=0$:
- $x R y$ since $3 \mid 1 + 0 = 1$, but this is false because 3 does not divide 1.
- Alternatively, pick $x=1$, $y=1$:
- $1 R 1$ if $3 \mid 1 + 2 \times 1 = 3$,
- $3 \mid 3$, true.
- Is $1 R 1$ again? Yes, symmetric.

Counterexample for symmetry:

- Take $x=0$, $y=1$:
- $0 R 1$ if $3 \mid 0 + 2 \times 1 = 2$,
- $3 \nmid 2$, so $0 R 1$ is false.
- For the relation to be symmetric, the converse must also be true whenever $x R y$ is true.

Conclusion:

- Since the divisibility condition depends on both $x$ and $y$, and the expressions $x + 2 y$ and $y + 2 x$ are not necessarily divisible by 3 simultaneously, the relation $R$ is not necessarily symmetric.

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## Antisymmetry

A relation $R$ is antisymmetric if for all $x, y \in S$, whenever $x R y$ and $y R x$, then $x = y$.

Analysis:

- Suppose $x R y$ and $y R x$,
- i.e., $3 \mid x + 2 y$ and $3 \mid y + 2 x$.

Implication:

- $x + 2 y \equiv 0 \pmod{3}$,
- $y + 2 x \equiv 0 \pmod{3}$.

Adding these two congruences:

$$ (x + 2 y) + (y + 2 x) \equiv 0 + 0 \pmod{3} $$
$$ x + 2 y + y + 2 x = 3 x + 3 y \equiv 0 \pmod{3} $$

which simplifies to:

$$ 3(x + y) \equiv 0 \pmod{3} $$

This is always true, so no contradiction arises.

Now, consider the difference:

$$ (x + 2 y) - (y + 2 x) = x + 2 y - y - 2 x = -x + y $$

If both are divisible by 3, then:

$$ 3 \mid -x + y $$
$$ \Rightarrow x \equiv y \pmod{3} $$

Thus, whenever $x R y$ and $y R x$, then $x \equiv y \pmod{3}$.

But does $x = y$ necessarily follow? Not necessarily, since $x \equiv y \pmod{3}$ does not mean $x = y$.

Counterexample:

- Take $x=0$, $y=3$:
- $3 \mid 0 + 2 \times 3 = 6$,
- $3 \mid 6$, so $0 R 3$,
- $3 \mid 3 + 2 \times 0 = 3$,
- $3 \mid 3$, so $3 R 0$.

- But $x=0 \neq 3$, so the relation is not antisymmetric unless $x = y$.

Conclusion:

- Since there are pairs \( (x, y) \) with \( x R y \) and \( y R x \) but \( x \neq y \), the relation \( R \) is not antisymmetric.

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## Transitivity

A relation \( R \) is transitive if for all \( x, y, z \in S \), whenever \( x R y \) and \( y R z \), then \( x R z \).

Assuming: \( x R y \) and \( y R z \), i.e.,

- \( 3 \mid x + 2 y \),
- \( 3 \mid y + 2 z \).

To prove: \( 3 \mid x + 2 z

# Frequently Asked Questions

## What is the relation R defined on a subset S of Z by X R Y if 3 divides (x + 2y)?

The relation R is defined such that for any elements X and Y in S, X R Y holds if and only if the integer 3 divides the sum x + 2y, i.e., 3 | (x + 2y).

## How can we prove that the relation R is reflexive on S?

To prove R is reflexive, show that for any x in S, x R x holds. This requires verifying that 3 divides (x + 2x) = 3x, which is true for all x in Z, hence R is reflexive.

## How do we demonstrate that R is symmetric given the relation defined by 3 | (x + 2y)?

Assuming x R y (3 | x + 2y), we need to check if y R x (3 | y + 2x). Since 3 divides x + 2y, then 3 divides y + 2x if and only if the difference (x + 2y) - (y + 2x) is divisible by 3. Simplifying, this difference is (x - y), so symmetry depends on the properties of x - y.

## What is the main step to prove that R is transitive on S?

To prove transitivity, assume x R y and y R z (i.e., 3 | x + 2y and 3 | y + 2z). Then, show that 3 divides x + 2z by combining these divisibility conditions, typically through linear combinations or modular arithmetic.

## What properties of divisibility by 3 are useful for analyzing the relation R?

Properties such as closure under addition and scalar multiplication modulo 3 are useful. Specifically, understanding that if 3 divides two expressions, their sum or difference may also be divisible by 3 helps in proving relation properties like transitivity and symmetry.

## How does modular arithmetic assist in proving that the relation R is an equivalence relation?

Modular arithmetic allows us to show that the relation is reflexive, symmetric, and transitive by working within the congruence classes modulo 3. Demonstrating that x R y corresponds to x ≡ -2y (mod 3), and verifying these properties satisfy the criteria for an equivalence relation.

# Additional Resources

Let S Be A Nonempty Subset Of Z And Let R Be A Relation Defined On S By X R Y If 3 (x + 2y). (a) Prove

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Investigating the Relation \( R \) on Subset \( S \subseteq \mathbb{Z} \): An Analytical Approach

In the realm of algebraic structures, relations serve as foundational building blocks that reveal intricate connections between elements of a set. When the set in question is a subset of integers, \(\mathbb{Z}\), and the relation is defined through algebraic expressions, the resultant analysis uncovers fundamental properties that influence broader mathematical theories, including orderings, equivalence, and partial orderings.

This article undertakes a comprehensive examination of the relation \( R \) defined on a nonempty subset \( S \subseteq \mathbb{Z} \) by:

\[
X R Y \iff 3(x + 2y) \text{ holds some specified property}
\]

Specifically, the relation is defined such that for elements \( x, y \in S

\):

\[
X R Y \iff 3(x + 2 y) \quad \text{(with some property to be clarified)}
\]

The focus of our investigation is to rigorously analyze the properties of this relation, provide proofs of its characteristics (such as reflexivity, symmetry, transitivity), and understand its implications within the context of algebraic relations.

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Clarification of the Relation Definition

Before delving into the properties, it is crucial to interpret the precise meaning of the relation \( R \). The original statement appears to be incomplete; typically, relations defined on \(\mathbb{Z}\) involve a condition such as:

\[
X R Y \iff 3(x + 2 y) \text{ is divisible by some integer}
\]

or

\[
X R Y \iff 3(x + 2 y) = 0
\]

or similar forms.

Assumption for the Analysis:

For the purpose of this investigation, we assume the relation \( R \) is defined as:

\[
X R Y \iff 3(x + 2 y) \text{ is divisible by } 3,
\]

which simplifies to:

\[
X R Y \iff (x + 2 y) \text{ is an integer multiple of } 1,
\]

but since \( x, y \in \mathbb{Z} \), the divisibility by 3 is the natural candidate for the property.

Therefore, the relation is:

$$
X R Y \iff 3 \mid 3(x + 2 y),
$$

which simplifies to:

$$
X R Y \iff 3 \mid 3(x + 2 y) \Rightarrow 3 \mid 3(x + 2 y),
$$

but since $3 \mid 3(x + 2 y)$ is always true, unless the relation involves the expression $x + 2 y$ itself being divisible by 3.

Hence, a more precise relation is:

$$
X R Y \iff 3 \mid (x + 2 y),
$$

which is a common form in algebraic relations involving divisibility.

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Formal Definition of the Relation

Definition:

Let $S \subseteq \mathbb{Z}$ be nonempty, and define the relation $R$ on $S$ by:

$$
\boxed{
X R Y \iff 3 \mid (x + 2 y),
}
$$

where $x, y \in S$.

This relation states that an ordered pair $(x,y) \in S \times S$ is related if and only if the sum $x + 2 y$ is divisible by 3.

---

Fundamental Properties of the Relation $R$

A comprehensive analysis of the relation $R$ involves examining classical properties such as:

- Reflexivity
- Symmetry
- Antisymmetry

- Transitivity
- Equivalence relation criteria
- Partial ordering criteria

This section systematically explores each property, providing proofs where applicable.

---

Reflexivity

Definition:

A relation $R$ on $S$ is reflexive if for all $x \in S$:

$$
x R x
$$

Proof:

For $R$ to be reflexive, we need:

$$
3 \mid (x + 2x) = 3x,
$$

which simplifies to whether $3 \mid 3x$. Since $3x$ is always divisible by 3 for any integer $x$, the relation is reflexive on any subset $S \subseteq \mathbb{Z}$.

Conclusion:

$$
\boxed{
\text{The relation } R \text{ is reflexive on } S \text{ for any } S
\subseteq \mathbb{Z}.
}
$$

---

Symmetry

Definition:

$R$ is symmetric if for all $x, y \in S$:

$$
x R y \Rightarrow y R x
$$

which translates to:

$$
3 \mid (x + 2 y) \Rightarrow 3 \mid (y + 2 x).
$$

Proof:

Suppose $x R y$, i.e.,

$$
3 \mid (x + 2 y).
$$

This means:

$$
x + 2 y \equiv 0 \pmod{3}.
$$

We want to determine whether this implies:

$$
3 \mid (y + 2 x),
$$

or equivalently,

$$
y + 2 x \equiv 0 \pmod{3}.
$$

Analysis:

From the initial condition:

$$
x + 2 y \equiv 0 \pmod{3}.
$$

Note that:

$$
x + 2 y \equiv 0 \pmod{3}
\Rightarrow x \equiv -2 y \pmod{3}.
$$

Since modulo 3, $-2 y \equiv y$ (because $-2 \equiv 1 \pmod{3}$), we get:

$$
x \equiv y \pmod{3}.

\]

Now, check $y + 2 x$:

$$
y + 2 x \equiv y + 2 y \equiv 3 y \equiv 0 \pmod{3}.
$$

Thus, if $x R y$, then $y R x$ because the divisibility condition is symmetric under this derivation.

Conclusion:

$$
\boxed{
\text{The relation } R \text{ is symmetric on } S \text{ for any } S \subseteq \mathbb{Z}.
}
$$

---

Transitivity

Definition:

$R$ is transitive if for all $x, y, z \in S$:

$$
x R y \text{ and } y R z \Rightarrow x R z.
$$

This translates to:

$$
3 \mid (x + 2 y) \text{ and } 3 \mid (y + 2 z) \Rightarrow 3 \mid (x + 2 z).
$$

Proof:

Suppose:

$$
x + 2 y \equiv 0 \pmod{3},
$$
$$
y + 2 z \equiv 0 \pmod{3}.
$$

From the first:

$$
x \equiv -2 y \pmod{3},
$$

and from the second:

$$
y \equiv -2 z \pmod{3}.
$$

Substitute $y \equiv -2 z$ into the first:

$$
x \equiv -2 (-2 z) = 4 z \equiv 4 z \pmod{3}.
$$

Since $4 \equiv 1 \pmod{3}$, this simplifies to:

$$
x \equiv z \pmod{3}.
$$

Now, evaluate $x + 2 z$:

$$
x + 2 z \equiv z + 2 z = 3 z \equiv 0 \pmod{3}.
$$

Thus,

$$
3 \mid (x + 2 z),
$$

which confirms $x R z$.

Conclusion:

$$
\boxed{
\text{The relation } R \text{ is transitive on } S \text{ for any } S
\subseteq \mathbb{Z}.
}
$$

---

Is $R$ an Equivalence Relation?

Given the above properties: reflexivity, symmetry, transitivity, the relation $R$ qualifies as an equivalence relation on $S \subseteq \mathbb{Z}$.

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Hence:

\[
\boxed{
\text{The relation } R \text{ is an equivalence relation on } S.
}
\]

---

Equivalence Classes Induced by \( R \)

Having established that \( R \) is an equivalence relation, it is natural to
analyze the structure of the equivalence classes.

Characterization of Equivalence Classes

Recall:

\[
x R y \iff 3 \mid (x + 2 y).
\]

From earlier derivations, the
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## [Let S Be A Nonempty Subset Of Z And Let R Be A Relation Defined On S By X R Y If 3 X 2y A Prove]

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