Let Set A Be The Set Of Integers. For All M And N In A, MRn> "m+nis Odd". Determine If The Relation

Let Set A Be The Set Of Integers. For All M And N In A, MRn > "m + n" is Odd. Determine If The Relation

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Introduction

In the realm of set theory and relations, understanding whether a specific relation possesses properties such as reflexivity, symmetry, antisymmetry, or transitivity is fundamental. The relation under consideration is defined on the set of integers, A, which includes all positive, negative, and zero integers. The relation states: for all M and N in A, MRn > "m + n" is odd.

This article aims to analyze this relation comprehensively, determine its properties, and conclude whether it qualifies as an equivalence relation, partial order, or another type of relation. To do so, we will first interpret the relation correctly, then examine its properties systematically.

Understanding the Relation

Clarifying the Definition

The given relation is: for all M and N in A, MRn > "m + n" is odd.

Interpreting this statement, it appears to involve a relation R between M and N, with the condition that the product M N (or M R N) is greater than the sum M + N, and this value (the product) is odd.

However, the exact phrasing suggests that the relation is characterized by a property involving the product M N being greater than the sum M + N, with the additional condition that the product M N is odd.

In more formal terms, the relation R can be defined as:

For all $M, N \in A$, $M R N \Leftrightarrow M N > M + N$ and M N is odd.

This interpretation aligns with the typical structure of such problems, where a relation is defined via specific algebraic constraints.

Formal Definition of the Relation

Based on the interpretation above, we define:

Relation R on A: For $M, N \in A$,

$$
M R N \iff (M \times N) > (M + N) \quad \text{and} \quad (M \times N) \text{ is odd}
$$

Our goal is to analyze whether R is:

- Reflexive
- Symmetric
- Antisymmetric
- Transitive

And to understand the nature of R, including whether it partitions A or forms an equivalence or partial order.

Analyzing the Properties of the Relation

1. Is R Reflexive?

A relation R on A is reflexive if for every $M \in A$, $M R M$ holds.

Let's examine the condition for $M R M$:

$$
M \times M > M + M \quad \text{and} \quad M^2 \text{ is odd}
$$

which simplifies to:

$$
M^2 > 2M \quad \text{and} \quad M^2 \text{ is odd}
$$

Step 1: When is M^2 odd?

- M^2 is odd if and only if M is odd.
- Reason: The square of an integer is odd if and only if the integer itself is odd.

Step 2: When does $M^2 > 2M$?

- Rearranged as:

$$
M^2 - 2M > 0
$$

- Factor:

$$
M(M - 2) > 0
$$

- Sign analysis:

- If M > 2, then $M > 0$ and $M - 2 > 0$, so $M(M - 2) > 0$.
- If M < 0, then $M < 0$, and $M - 2 < 0$, so the product is positive ($negative \times negative = positive$).

- For M = 0 or 1 or 2:

- M=0: $0^2=0$, $0 > 0$? No.
- M=1: $1^2=1$, $1 > 2$? No.
- M=2: $4 > 4$? No, equality.

- For M=3: $9 > 6$? Yes, so M=3 satisfies.

Step 3: Combining the conditions for reflexivity:

- M must be odd (for M^2 to be odd).
- M must satisfy $M(M-2) > 0$.

Testing odd M:

- M=1: $1(1-2) = 1 \times (-1) = -1$, which is less than 0 → No.
- M=3: $3(3-2) = 3 \times 1=3 > 0$ → Yes.
- M=5: $5(3) = 15 > 0$ → Yes.
- M=-1: $-1(-1-2) = -1(-3) = 3 > 0$ → Yes.
- M=-3: $-3(-3-2) = -3(-5)=15 >0$ → Yes.

But note that:

- For M=1: $M^2=1$ (odd), but $M^2 > 2M$? $1 > 2$? No.

Similarly, for M=1, the condition fails.

Thus, the only M for which M R M is true are:

- M odd and $M(M-2) > 0$

which is true for:

- $M \geq 3$ and M odd, or
- $M \leq -1$ and M odd.

Conclusion:

The reflexivity condition holds only for M where:

- M is odd,
- $M \geq 3$ or $M \leq -1$,
- and satisfies $M(M-2) > 0$.

In particular, for M=3, M R M holds; for M=-1, M R M holds.

Result:

- The relation is not reflexive on the entire set A, because M=1 or M=0 do not satisfy the condition.
- It is partially reflexive on a subset of A, specifically odd integers where $M \geq 3$ or $M \leq -1$.

---

2. Is R Symmetric?

A relation R is symmetric if for all M, N ∈ A, whenever M R N, then N R M.

Check whether the condition:

$$
(M \times N) > (M + N) \quad \text{and} \quad (M \times N) \text{ is odd}
$$

implies:

$$
(N \times M) > (N + M) \quad \text{and} \quad (N \times M) \text{ is odd}
$$

Since multiplication is commutative:

$$
M \times N = N \times M
$$

and

$$
M + N = N + M
$$

Therefore, the conditions are symmetric in M and N.

Conclusion:

- If M R N holds, then N R M also holds.
- Thus, R is symmetric.

---

3. Is R Antisymmetric?

A relation R is antisymmetric if for all M, N ∈ A:

$$
\text{if } M R N \text{ and } N R M, \text{ then } M = N
$$

Since R is symmetric, the only way for both $M R N$ and $N R M$ to hold is when M and N satisfy the relation simultaneously.

Let's examine whether two distinct elements can satisfy the relation:

Suppose M ≠ N, and both M R N and N R M hold.

Given the symmetry, this is possible unless M=N.

To verify, pick specific pairs:

- M=3, N=5:

Compute:

$$
M \times N = 15 \quad (\text{odd}),
$$
$$
M + N = 8,
$$
$$
15 > 8 \quad \text{and } 15 \text{ is odd} \Rightarrow M R N \text{ holds}
$$

Similarly,

\[
N \times M = 15,
\]
\[
N + M=8,
\]
\[
15 > 8,
\]

so N R M also holds.

But M ≠ N, so the relation is symmetric but not antisymmetric.

Conclusion:

- The relation R is not antisymmetric because there exist distinct M and N such that both \(M R N \) and \(N R M \) hold.

---

4. Is R Transitive?

A relation R is transitive if whenever M R N and N R P, then M R P.

Given the properties, we must check:

- Does \(M R N \) and \(N R P \) imply \(M R P \)?

Recall:

\[
M R N \Rightarrow M N > M + N \quad \text{and} \quad M N \text{ is odd}
\]
\[
N R P \Rightarrow N P > N + P \quad \text{and}

Frequently Asked Questions

What is the set A in the given relation?

The set A is the set of all integers.

What is the relation defined on set A?

The relation R is defined by: for all M and N in A, MRN > m + n, and the relation holds when this inequality is true.

Is the relation symmetric?

No, the relation is not necessarily symmetric because the inequality involves M and N in a way that does not guarantee symmetry.

Is the relation reflexive?

To determine if it's reflexive, check if for all M in A, MR M > m + m. Since MR M simplifies to M R M, which depends on R, the relation's reflexivity depends on the specific definition of R, but generally, it's unlikely to be reflexive.

Is the relation transitive?

The transitivity of the relation depends on whether the inequality holds through the chain of relations, which is unlikely given the inequality involves products and sums. A specific analysis is required.

Does the relation depend on the parity of the integers?

Yes, since the relation specifies that MRn > m + n is odd, the parity of the resulting product and sum plays a role in whether the relation holds.

Can the relation be true for all integers M and N?

No, because the relation involves an inequality that depends on M and N, and not all pairs will satisfy MRn > m + n.

How does the condition 'MRn > m + n' relate to properties like positivity or parity?

It suggests that the product of M and N must be greater than the sum of M and N, which is influenced by their signs and parity; for example, if M and N are positive, the inequality is more likely to hold.

What is the significance of the 'odd' condition in the relation?

The relation requires that MRn > m + n be an odd number, adding a parity constraint that influences which pairs of integers satisfy the relation.

Based on the given condition, can we classify the relation as a function?

No, because the relation relates pairs of integers without assigning a unique output to each input; it's a binary relation, not necessarily a function.

Additional Resources

Let Set A Be The Set Of Integers. For All M And N In A, MRn > "m + n" is Odd – this statement introduces a fascinating relation within the set of integers that invites a thorough examination. Understanding the nature of this relation can deepen our grasp on properties such as reflexivity, symmetry, transitivity, and the overall classification of the relation. In this comprehensive guide, we will analyze the relation step-by-step, breaking down its components, exploring various cases, and ultimately determining whether it qualifies as an equivalence relation, partial order, or falls into another classification.

Understanding the Relation: "For All M and N in A, MRn > 'm + n' is Odd"

Before diving into the properties, let's clarify the statement itself. The phrase "Let Set A be the set of integers" sets the universe of discourse. The relation in question involves two elements, M and N, both belonging to A, and a statement involving M, N, and their sum.

The key part of the relation is:

> For all M, N ∈ A, MRn > "m + n" is Odd.

At face value, this can be interpreted as:

- The relation involves the expression MRn (which we need to interpret—likely meaning some operation involving M and N).
- The condition involves the sum m + n.
- The relation perhaps imposes a condition that MRn is greater than m + n, and that this difference (or the relation itself) reflects an odd number.

However, the statement as written is somewhat ambiguous. To proceed meaningfully, we need to clarify the intended meaning:

Assumption and Clarification

- It appears that MRn might be a typo or shorthand for a relation involving M and N, perhaps M N or some operation.
- The phrase "MRn > 'm + n'" suggests that MRn is an expression comparable to m + n, and the inequality involves their difference being odd.

Given the context, a plausible interpretation is:

"For all M, N ∈ A, the difference between M N and (M + N) is odd."

Expressed mathematically:

> For all M, N ∈ A, (M N) - (M + N) is odd.

This interpretation makes sense because:

- Both M and N are integers.

- The difference between their product and sum can be analyzed.
- The property of being odd or even is straightforward to determine.

Restatement of the Relation

Relation R on A is defined as:

> For all M, N ∈ A, (M N) - (M + N) is odd.

Our goal: Determine if R is a valid relation, and analyze its properties.

Step 1: Formal Definition

Let's formalize the relation:

R = { (M, N) ∈ A × A | (M N) - (M + N) is odd }

In words:

> The pair (M, N) is in the relation R if and only if the difference between their product and sum is odd.

Step 2: Simplify the Condition

To analyze the relation, let's examine the parity of (M N) - (M + N).

Recall:

- The parity of an integer is either even or odd.
- The sum or product's parity can be deduced from the parity of the operands.

Key facts:

- Even + Even = Even
- Even + Odd = Odd
- Odd + Odd = Even

- Even Even = Even
- Even Odd = Even
- Odd Odd = Odd

Step 3: Analyze the Parity of the Difference

Let’s denote:

- M and N are integers; each is either even or odd.
- We'll analyze all possible cases based on the parity of M and N.

Case 1: Both M and N are even

- M: even
- N: even

Compute:

- M N: even even = even
- M + N: even + even = even

Difference:

- (M N) - (M + N): even - even = even

Result: The difference is even.

Conclusion: For M and N both even, the pair (M, N) is not in R (since the difference is not odd).

Case 2: M is even, N is odd

- M: even
- N: odd

Compute:

- M N: even odd = even
- M + N: even + odd = odd

Difference:

- even - odd = odd

Result: The difference is odd.

Conclusion: For M even, N odd, the pair (M, N) is in R.

Case 3: M is odd, N is even

- M: odd
- N: even

Compute:

- M N: odd even = even
- M + N: odd + even = odd

Difference:

- even - odd = odd

Result: The difference is odd.

Conclusion: For M odd, N even, the pair (M, N) is in R.

Case 4: Both M and N are odd

- M: odd
- N: odd

Compute:

- M N: odd odd = odd
- M + N: odd + odd = even

Difference:

- odd - even = odd

Result: The difference is odd.

Conclusion: For both odd, the pair (M, N) is in R.

Summary of Parity Analysis

M parity	N parity	Difference (MN) - (M+N)	In R?
Even	Even	Even	No
Even	Odd	Odd	Yes
Odd	Even	Odd	Yes
Odd	Odd	Odd	Yes

Note: The only case where the pair is not in R is when both M and N are even.

Key Insight

- The relation R includes all pairs where the parity of M and N are not both even.

- Pairs where both M and N are even are not in R.

Step 4: Characterize the Relation R

R can be described as:

> The set of all pairs (M, N) in A × A such that not both M and N are even.

Or, equivalently:

> R = {(M, N) | at least one of M or N is odd}

Step 5: Formal Properties of R

Now, we analyze the properties of the relation:

1. Reflexivity

A relation R on A is reflexive if:

> For every M ∈ A, (M, M) ∈ R.

Check:

- For M ∈ A, is (M, M) ∈ R?

Recall:

- (M, M) ∈ R if the difference (M M) - (M + M) is odd.

Compute:

- M M: M^2
- M + M: 2M

Difference:

- M^2 - 2M

Parity of M^2 - 2M:

- M^2: same parity as M squared
- 2M: always even (since 2 times any integer is even)

Since:

- If M is even:

- M^2: even

- 2M: even

- Difference: even - even = even → (M, M) not in R

- If M is odd:

- M^2: odd

- 2M: even

- Difference: odd - even = odd → (M, M) in R

Conclusion:

> R is reflexive only for odd M.

Since the relation is not reflexive for all M (because for even M, (M, M) is not in R), R is not reflexive on the entire set A.

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2. Symmetry

Check whether:

> If (M, N) ∈ R, then (N, M) ∈ R.

From the characterization:

- (M, N) ∈ R if at least one of M or N is odd.
- The pair (N, M) depends on the same parity of N and M.

Since parity is symmetric with respect to order:

- If at least one of M or N is odd, then the same applies for N and M.

Conclusion:

> R is symmetric.

---

3. Transitivity

Check whether:

> If (M, N) ∈ R and (N, P) ∈ R, then (M, P) ∈ R.

Using the characterization:

- (M, N) ∈ R implies at least one of M or N is odd.
- (N, P) ∈ R implies at least one of N or P is odd.

Possible scenarios:

- N is odd:

- Then both (M, N) and (N, P) are in R regardless of M and P's parity.

- For (M, P):

- If M or P is odd, then (M, P) ∈ R.

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