# Let Sine Of Theta Equals The Quantity 2 Times Radical 2 End Quantity Over 5 And Pi Over 2 Is Less Than

**Let Sine Of Theta Equals The Quantity 2 Times Radical 2 End Quantity Over 5 And Pi Over 2 Is Less Than** and explore the comprehensive understanding of this trigonometric expression. This article delves into the mathematical concepts involved, including solving for theta, understanding the sine function, interpreting the inequality involving pi over 2, and applying these ideas to real-world problems. Whether you're a student preparing for exams or a mathematics enthusiast seeking clarity, this guide aims to provide detailed insights into this intriguing trigonometric statement.

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## Understanding the Given Expression

The statement "Let sine of theta equals the quantity 2 times radical 2 over 5 and pi over 2 is less than" indicates a focus on a specific sine value and an inequality involving the angle theta. To fully grasp this, we need to break down the components:

- Sine of Theta (sin θ): A fundamental trigonometric function representing the ratio of the length of the side opposite the angle to the hypotenuse in a right-angled triangle.
- The given value: \( \sin \theta = \frac{2 \sqrt{2}}{5} \)
- The inequality: \( \frac{\pi}{2} < \theta \)

Our goal is to interpret what these mean, find solutions for theta, and understand the implications of the inequality.

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## Deciphering the Value of sin θ

### Analyzing the Expression \( \sin \theta = \frac{2 \sqrt{2}}{5} \)

This value involves both a rational number and a radical, and is approximately calculable:

$$
\sin \theta = \frac{2 \sqrt{2}}{5}
$$

Calculating the approximate decimal value:

$$
\sqrt{2} \approx 1.4142
$$

$$
2 \times 1.4142 \approx 2.8284
$$

$$
\frac{2.8284}{5} \approx 0.56568
$$

Thus:

$$
\sin \theta \approx 0.56568
$$

Key observations:

- Since $\sin \theta$ is positive and less than 1, $\theta$ is in the first or second quadrants.
- The sine function reaches this value at two points within the range $0$ to $2\pi$.

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# Finding the General Solutions for Theta

## Principal Value of Theta

To find the specific angles $\theta$ satisfying $\sin \theta = \frac{2 \sqrt{2}}{5}$, we use the inverse sine function:

$$
\theta_0 = \arcsin \left( \frac{2 \sqrt{2}}{5} \right)
$$

Calculating this:

\[

\theta_0 \approx \arcsin(0.56568) \approx 34.4^\circ
\]

In radians:

\[
\theta_0 \approx 0.6 \text{ radians}
\]

Note: The inverse sine function yields a principal value in the interval \([-\frac{\pi}{2}, \frac{\pi}{2}]\).

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## General Solutions for \(\sin \theta = \frac{2 \sqrt{2}}{5}\)

Since sine is positive in the first and second quadrants:

\[
\theta = \theta_0 + 2 \pi n
\]

and

\[
\theta = \pi - \theta_0 + 2 \pi n
\]

where \(n\) is any integer.

Expressed explicitly:

\[
\boxed{
\theta = 0.6 + 2 \pi n \quad \text{or} \quad \theta = \pi - 0.6 + 2 \pi n
}
\]

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## Interpreting the Inequality \( \frac{\pi}{2} < \theta \)

This inequality restricts the solutions to those where:

$$
\theta > \frac{\pi}{2}
$$

Since $\frac{\pi}{2} \approx 1.5708$ radians, the solutions must be greater than this value.

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## Finding the Relevant Solutions

Given the general solutions:

$$
\theta_1 = 0.6 + 2 \pi n
$$
$$
\theta_2 = \pi - 0.6 + 2 \pi n
$$

We want all $\theta$ satisfying:

$$
\theta > 1.5708
$$

Let's examine these solutions for different integer values of $n$:

- For $n=0$:

- $\theta_1 = 0.6$ (which is less than 1.57, discard)
- $\theta_2 = \pi - 0.6 \approx 3.1416 - 0.6 = 2.5416$ (which is greater than 1.57, include)

- For $n=1$:

- $\theta_1 = 0.6 + 2\pi \approx 0.6 + 6.2832 = 6.8832$ (> 1.57, include)
- $\theta_2 = 2.5416 + 2\pi \approx 2.5416 + 6.2832 = 8.8248$ (> 1.57, include)

- For $n=-1$:

- $\theta_1 = 0.6 - 2\pi \approx 0.6 - 6.2832 = -5.6832$ (< 1.57, discard)
- $\theta_2 = 2.5416 - 6.2832 = -3.7416$ (< 1.57, discard)

Similarly, for other negative $n$, solutions will be less than $\frac{\pi}{2}$.

Summary of solutions satisfying the inequality:

$$
\boxed{
\theta \in \left[ 2.5416 + 2 \pi n, \quad 6.8832 + 2 \pi n \right], \quad
\text{for all integers } n \text{ with } n \geq 0
}
$$

Or, explicitly:

$$
\theta \in \left\{
\begin{aligned}
& 2.5416 + 2 \pi n, \\
& 6.8832 + 2 \pi n
\end{aligned}
\right.,
\quad n \geq 0
$$

Visualizing the Solutions

A helpful method is to plot these solutions on the unit circle or graph $\sin \theta$ to see where the sine curve exceeds the value $0.56568$ for $\theta > 1.5708$.

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# Applications of the Solution

Understanding solutions to sine equations with inequalities has numerous applications:

- Engineering: Signal processing often involves phase angles where certain sine values are critical.
- Physics: In wave mechanics, oscillations depend on angles satisfying specific sine conditions.
- Mathematics: Trigonometric inequalities help in solving real-world problems involving angles and periodic functions.

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# Additional Considerations and Tips

- Range of $\arcsin$: Remember that $\arcsin$ returns a value in $[-\frac{\pi}{2}, \frac{\pi}{2}]$. When solving for $\theta$, consider the quadrants where sine is positive.

- Periodicity of sine: The sine function repeats every $2\pi$, so solutions are infinite but can be restricted based on the domain.
- Simplification of radical expressions: When dealing with radicals, simplifying or approximating can provide better intuition about the values involved.

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## Summary of the Key Steps

1. Identify the given value of $\sin \theta$: $\frac{2 \sqrt{2}}{5}$, approximately 0.56568.
2. Find the principal value: $\theta_0 = \arcsin \left( \frac{2 \sqrt{2}}{5} \right) \approx 0.6$ radians.
3. Determine the general solutions: $\theta = \theta_0 + 2 \pi n$ and $\theta = \pi - \theta_0 + 2 \pi n$.
4. Apply the inequality: $\theta > \frac{\pi}{2}$, which restricts the solutions to those with $n \geq 0$ and $\theta$ in the appropriate ranges.
5. Express solutions explicitly: As shown, solutions are at $\theta \approx 2.54 + 2 \pi n$ and $\theta \approx 6.88 + 2 \pi n$.

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## Conclusion

Understanding the equation $\sin \theta = \frac{2 \sqrt{2}}{5}$ with the inequality $\frac{\pi}{2} < \theta$ involves analyzing the sine function, calculating inverse sine

## Frequently Asked Questions

### What is the value of theta when sin(theta) equals $(2\sqrt{2})/5$?

Theta can be found using inverse sine: theta = arcsin($(2\sqrt{2})/5$). Since sine is positive, theta lies between 0 and $\pi/2$, approximately 0.588 radians or 33.69 degrees.

### Is theta in the interval $(\pi/2, \pi)$ when sin(theta) = $(2\sqrt{2})/5$?

No, since sin(theta) = $(2\sqrt{2})/5$ is positive and less than 1, and given that π/2 < theta, the value of theta would be in the first quadrant, i.e., between 0 and π/2.

## How do I find all solutions for theta given sin(theta) = (2√2)/5?

Since sine is positive, solutions are theta = arcsin((2√2)/5) + 2πk and theta = π - arcsin((2√2)/5) + 2πk, where k is any integer.

## Given that π/2 < theta, is the solution for theta unique?

No, because sine is periodic; there are multiple solutions. But if you specify π/2 < theta < 3π/2, then theta = π - arcsin((2√2)/5).

## What is the approximate numerical value of theta when sin(theta) = (2√2)/5?

Calculating arcsin((2√2)/5) gives approximately 0.588 radians or about 33.69 degrees.

## How does the interval π/2 < theta < π affect the solutions?

In that interval, the solution for theta is theta = π - arcsin((2√2)/5), which is approximately 2.553 radians or 146.31 degrees.

## Can sin(theta) = (2√2)/5 be true for theta outside the interval (π/2, π)?

Yes, since sine is positive in both the first and second quadrants, solutions can occur in (0, π/2) and (π/2, π).

## What are the general solutions for theta when sin(theta) = (2√2)/5?

The general solutions are theta = arcsin((2√2)/5) + 2πk and theta = π - arcsin((2√2)/5) + 2πk, for any integer k.

## Is the value of sin(theta) = (2√2)/5 common in trigonometry problems?

Yes, it’s a typical value when dealing with angles that produce specific sine values, often involving radicals, and is useful for practice with inverse functions.

## How do I verify that sin(theta) = (2√2)/5 is a valid sine value?

Since (2√2)/5 is approximately 0.5657, and sine values range between -1 and 1, this value is valid and within the possible range for sine.

# Additional Resources

Let Sine Of Theta Equal The Quantity 2 Times Radical 2 End Quantity Over 5 And Pi Over 2 Is Less Than

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Introduction

In the realm of trigonometry, the exploration of functions and their properties is fundamental to understanding the geometric and algebraic relationships that underpin mathematics. Among these, the sine function stands out due to its periodic nature and widespread applications in physics, engineering, and mathematics itself. One particularly intriguing problem involves analyzing the behavior and solutions of the equation:

Let sine of theta equal the quantity 2 times radical 2 over 5, and pi over 2 is less than theta.

This statement, rich with mathematical nuance, invites a comprehensive exploration of the sine function, inequalities involving angles, and the solutions that satisfy such conditions. This review delves into the depths of this formulation, examining its theoretical basis, solving strategies, and implications in both pure and applied contexts.

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The Mathematical Foundations

Understanding the Sine Function

The sine function, denoted as sin(θ), maps an angle θ (measured in radians or degrees) to a real number between -1 and 1. Its graph is periodic with a period of 2π, oscillating smoothly between these bounds.

Key properties include:

- Range: $-1 \leq \sin(\theta) \leq 1$
- Periodicity: $\sin(\theta + 2\pi) = \sin(\theta)$
- Symmetry: $\sin(\pi - \theta) = \sin(\theta)$, and $\sin(-\theta) = -\sin(\theta)$
- Special Angles: $\sin(0) = 0$, $\sin(\pi/2) = 1$, $\sin(\pi) = 0$, $\sin(3\pi/2) = -1$, $\sin(2\pi) = 0$

Any solution involving $\sin(\theta)$ must respect these bounds, which is crucial in interpreting the given equation.

The Expression: $2\sqrt{2} / 5$

The quantity $2\sqrt{2} / 5$ can be simplified or approximated for better intuition:

- $\sqrt{2} \approx 1.4142$
- $2\sqrt{2} \approx 2\ 1.4142 \approx 2.8284$
- Therefore, $2\sqrt{2} / 5 \approx 2.8284 / 5 \approx 0.5657$

This value is significant because it is within the sine function's range, confirming that the equation has real solutions.

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Formulating the Equation

Given the statement, the primary equation under consideration is:

$\sin(\theta) = 2\sqrt{2} / 5 \approx 0.5657$

with the additional constraint:

$\pi/2 < \theta$

This inequality restricts the solutions to angles greater than $\pi/2$ radians (90 degrees).

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Analyzing the Solutions

Step 1: Finding the Reference Angle

The first step is to identify the reference angle, $\theta_0$, such that:

$\sin(\theta_0) = 0.5657$

Since sine is positive in both the first and second quadrants, $\theta_0$ will typically be in the first quadrant (0 to $\pi/2$). Using inverse sine:

$\theta_0 = \arcsin(0.5657)$

Calculating $\theta_0$:

- $\theta_0 \approx \arcsin(0.5657) \approx 0.601$ radians (approximate)

This angle in degrees:

- $\theta_0 \approx 0.601\ (180/\pi) \approx 34.43°$

Step 2: General Solution for $\sin(\theta) = 0.5657$

The sine function's symmetry grants us two primary solutions within $[0, 2\pi)$:

- $\theta = \theta_0 + 2\pi k$
- $\theta = \pi - \theta_0 + 2\pi k$

for any integer k.

In the principal period $[0, 2\pi)$:

- First solution: $\theta \approx 0.601$ radians
- Second solution: $\theta \approx \pi - 0.601 \approx 2.540$ radians

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Incorporating the Constraint: $\theta > \pi/2$

The problem constrains θ to be greater than $\pi/2$ ($\approx 1.5708$ radians). Therefore:

- The solution $\theta \approx 0.601$ radians (about 34.43°) is discarded because it is less than $\pi/2$.
- The relevant solution within $[0, 2\pi)$ is $\theta \approx 2.540$ radians (about 145.57°).

This is the only solution in the primary cycle satisfying the inequality.

Step 3: Extending to All Possible Solutions

Because sine is periodic, solutions recur every $2\pi$ radians. The general solutions for θ are:

- $\theta \approx 0.601 + 2\pi k$
- $\theta \approx 2.540 + 2\pi k$

for any integer k.

Applying the constraint $\theta > \pi/2$:

- For the first set:

$\theta > 1.5708$

Since $0.601 + 2\pi k > 1.5708$

- For $k = 0$: $0.601 > 1.5708$? No, discard.
- For $k = 1$: $0.601 + 2\pi \approx 0.601 + 6.283 \approx 6.884$, which is $> 1.5708$. Valid.
- For $k = -1$: $0.601 - 2\pi \approx 0.601 - 6.283 \approx -5.682$, discard.

- For the second set:

$\theta \approx 2.540 + 2\pi k$

- For k = 0: 2.540 > 1.5708? Yes, valid.
- For k = -1: 2.540 - 6.283 ≈ -3.743, discard.
- For k = 1: 2.540 + 6.283 ≈ 8.823 > 1.5708, valid.

Thus, the solutions satisfying the inequality are:

- $\theta \approx 2.540$ (k=0)
- $\theta \approx 6.884$ (k=1)
- $\theta \approx 8.823$ (k=1), and so on, adding $2\pi$ each time.

Expressed generally:

- $\theta = 2.540 + 2\pi k$, where $k \geq 0$

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Implications and Applications

Mathematical Significance

This analysis underscores the importance of understanding the periodicity and symmetry of trigonometric functions. It also demonstrates how inequalities refine the set of solutions, which is crucial in practical applications such as physics, engineering, and computer science.

Real-world Contexts

- Wave Phenomena: The solutions correspond to specific points in wave cycles where certain conditions are met, like phase shifts or amplitude thresholds.
- Signal Processing: Filtering signals based on phase angles often involves solving similar trigonometric equations with inequalities.
- Engineering Design: Mechanical systems with oscillatory components may require solutions for angles exceeding certain thresholds to optimize performance.

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Summary of Key Findings

- The equation $\sin(\theta) = 2\sqrt{2} / 5 \approx 0.5657$ has solutions at $\theta \approx 0.601$ radians and $\theta \approx 2.540$ radians within $[0, 2\pi)$.
- The constraint $\pi/2 < \theta$ restricts solutions to $\theta \approx 2.540$ radians plus any $2\pi$ multiples.
- The general solution set, respecting the inequality, is:

$\theta = 2.540 + 2\pi k$, where k is any integer such that $\theta > \pi/2$.

- For k=0, the principal solution is approximately 145.57° (or 2.540 radians).

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## Advanced Considerations

### Multiple-Period Solutions

In scenarios requiring solutions over extended domains, the periodic nature of sine ensures infinitely many solutions of the form:

$$\theta = \arcsin(0.5657) + 2\pi k \text{ and } \theta = \pi - \arcsin(0.5657) + 2\pi k$$

with the inequality constraint applied accordingly.

### Numerical Methods and Precision

While approximate values (e.g., $\theta \approx 0.601$ radians) are sufficient for most practical purposes, precise solutions often require computational tools such as:

- Graphing calculators
- Computer algebra systems (CAS)
- Numerical solvers

to enhance accuracy, especially in sensitive engineering calculations.

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## Conclusion

The problem of solving $\sin(\theta) = 2\sqrt{2} / 5$ with the condition $\pi/2 < \theta$ encapsulates core principles of trigonometry: periodicity, symmetry, and inequalities. By methodically deriving the reference angle, recognizing the range constraints, and extending solutions across multiple periods, we gain a comprehensive understanding of the solution space.

This exploration not only reinforces foundational mathematical concepts but also highlights their practical relevance across scientific disciplines. Whether modeling wave behaviors or designing oscillatory systems, the insights gleaned from such trigonometric analyses serve as essential tools for scientists and engineers alike.

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## References


- Stewart, J. (2015). Precalculus: Mathematics for Calculus. Cengage Learning.
- Larson, R., & Edwards, B. H. (2017). Precalculus. Cengage Learning.
- Trigonometry.org. (n.d.). Basic Trigonometric Functions and Identities. Retrieved from https://www.trigonometry.org

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Note: All approximate numerical values are rounded to
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