Line S Has An Equation Of Y2= 4(x+1). Line T Includes The Point (1,6) And Is Parallel To Line S. What

Line S Has An Equation Of Y2= 4(x+1). Line T Includes The Point (1,6) And Is Parallel To Line S. What

Understanding the fundamentals of coordinate geometry and the properties of parallel lines is essential for solving problems involving equations of lines. In this guide, we will analyze the given information about Line S and Line T, explore how to determine the equation of Line T, and provide comprehensive steps to arrive at the solution. This will not only clarify the problem at hand but also reinforce key concepts related to slope, point-slope form, and parallel lines in coordinate geometry.

Analyzing the Given Data

Before jumping into calculations, it's important to interpret the problem statement carefully. The details provided are:

- Line S Has an Equation Of Y2= 4(x+1).
- Line T Includes The Point (1,6).
- Line T Is Parallel To Line S.

Let's examine each piece of information:

Understanding Line S's Equation

The equation given is Y2= 4(x+1). However, this appears to be a typographical error or misstatement. Likely, the intended equation is:

- Y = 4(x + 1)

or equivalently,

- Y = 4x + 4

This is a linear equation in slope-intercept form, where the slope (m) and y-intercept (b) can be identified.

Key points:

- The slope of Line S, m_s = 4
- The y-intercept of Line S, (0, 4)

Line T's Characteristics

- It passes through the point (1, 6).
- It is parallel to Line S, which implies it has the same slope as Line S.

Understanding these points allows us to determine the equation of Line T.

Determining the Equation of Line T

Since Line T is parallel to Line S, it shares the same slope. The steps to find Line T's equation are:

1. Identify the slope of Line T

2. Use the point-slope form with the given point

3. Convert to slope-intercept form for clarity

Let's break down each step.

Step 1: Find the slope of Line T

- Because Line T is parallel to Line S,

- $m_t = m_s = 4$

Step 2: Use the point-slope form

- The point-slope form of a line's equation is:

- $Y - y_1 = m (X - x_1)$

- Plugging in the point (1, 6) and the slope 4:

- Y - 6 = 4 (X - 1)

Step 3: Convert to slope-intercept form

- Simplify the equation:

- Y - 6 = 4X - 4

- Add 6 to both sides:

- $Y = 4X - 4 + 6$

- $Y = 4X + 2$

Result: The equation of Line T is $Y = 4X + 2$.

Understanding the Geometric Relationship

Knowing both equations, we can analyze the relationship between the two lines.

Parallel Lines and Their Properties

- Parallel lines have identical slopes.
- They never intersect.
- They can be distinguished by their different y-intercepts.

In our case:

- Line S: $Y = 4X + 4$
- Line T: $Y = 4X + 2$

The y-intercepts are different (4 and 2 respectively), confirming they are parallel but distinct lines.

Graphical Representation

Plotting both lines helps visualize their relationship:

- Line S crosses the y-axis at (0, 4)
- Line T crosses the y-axis at (0, 2)
- Both lines have the same slope of 4, indicating they run parallel with the same steepness.

Additional Insights and Related Concepts

Beyond solving the specific problem, this scenario introduces several key concepts in coordinate geometry.

Understanding Slope and Its Significance

- The slope (m) indicates the rate of change of y with respect to x.
- A positive slope (like 4) means the line rises as x increases.
- The magnitude of the slope determines the steepness.

Point-Slope Form of a Line

- Useful when a point on the line and the slope are known.
- Formula: $Y - y_1 = m (X - x_1)$
- Ideal for deriving the equation of a line passing through a specific point with a known slope.

Parallel Lines and Their Equations

- Parallel lines have equal slopes.
- Their equations differ only in their y-intercept term.
- Recognizing this property simplifies the process of finding equations of parallel lines.

Applications in Real-World Contexts

- Designing parallel roadways or railway tracks.
- Engineering structures requiring parallel supports.
- Graphing data trends that follow parallel behaviors.

Practical Example: Applying the Concepts

Suppose you are given a line with an equation and need to find a parallel line passing through a certain point. Here's a step-by-step approach:

1. Identify the slope of the given line.
2. Use this slope with the point to create the point-slope form.
3. Simplify to slope-intercept form if needed.
4. Verify the parallelism by ensuring the slopes are identical.

In our problem:

- Given: $Y = 4X + 4$ (Line S)
- Find: Equation of a line passing through (1, 6) and parallel to Line S.

Solution:

- Slope of Line S: 4
- Use point (1, 6):

- $Y - 6 = 4 (X - 1)$

- $Y = 4X + 2$

This confirms that the line passing through (1, 6) parallel to Line S is $Y = 4X + 2$.

Summary and Key Takeaways

- The equation Y2= 4(x+1) is likely meant to be Y = 4(x + 1).
- The slope of Line S is 4.
- Line T passes through (1,6) and is parallel to Line S, so it shares the same slope.
- The equation of Line T is Y = 4X + 2.
- Parallel lines have identical slopes but different y-intercepts.
- Understanding these properties simplifies solving related problems.

Conclusion

Deciphering the relationship between lines based on their equations and points requires a solid grasp of slope, the point-slope form, and the properties of parallel lines. This problem exemplifies how to approach such tasks systematically, ensuring clarity and accuracy. By mastering these concepts, you can confidently analyze similar geometric problems and apply these principles in various mathematical and real-world contexts.

Remember: Always verify your calculations and double-check the properties of the lines to ensure consistency and correctness in your solutions.

Frequently Asked Questions

What is the equation of Line S given that Y^2 = 4(x + 1)?

The equation of Line S is Y^2 = 4(x + 1), which represents a parabola opening sideways, not a line.

How do you find the slope of Line S based on its equation Y^2 = 4(x + 1)?

Since Y^2 = 4(x + 1) is a parabola, it does not have a single slope. However, at specific points, the slope can be found by implicit differentiation.

What is the slope of Line T which is parallel to Line S?

Since Line T is parallel to Line S, it must have the same slope as Line S at the point of interest, but because Line S is a parabola, the slope varies. For a line parallel to a parabola, the slope is determined at a given point.

Given that Line T passes through the point (1,6) and is parallel to Line S, what is the equation of Line T?

Because Line T is a straight line passing through (1,6) and parallel to the tangent of the parabola at that point, its slope is equal to the derivative of Y^2 = 4(x + 1) at that point, which can be calculated to find the line's equation.

How do you find the slope of the tangent line to the parabola Y^2 = 4(x + 1) at the point (1,6)?

Differentiate implicitly: 2Y(dY/dx) = 4, so dY/dx = 2 / Y. At (1,6), the slope is 2 / 6 = 1/3.

What is the equation of Line T passing through (1,6) with slope 1/3?

Using point-slope form: y - 6 = (1/3)(x - 1), so the equation is y = (1/3)x + (17/3).

Additional Resources

Line S Has An Equation Of Y^2 = 4(x + 1). Line T Includes The Point (1, 6) And Is Parallel To Line S. What

Understanding the relationships between lines and their equations is fundamental in coordinate geometry, especially when exploring their slopes, directions, and positions relative to each other. In this article, we delve into the specific problem involving two lines: Line S, defined by the equation \(y^2 = 4(x + 1) \), and Line T, which passes through the point (1, 6) and is parallel to Line S. We will analyze the characteristics of Line S, derive the equation of Line T, and explore the geometric implications of their parallelism.

Analyzing Line S: The Parabola \(y^2 = 4(x + 1) \)

Understanding the Equation of Line S

The given equation \(y^2 = 4(x + 1) \) is a standard form of a parabola that opens to the right. It can be rewritten as:

\[
y^2 = 4(x + 1)
\]

which is similar to the general form \(y^2 = 4ax \), shifted horizontally. Here:

- The parabola's vertex is at \((-1, 0)\).
- The focus is located at \((0, 0)\), since for \(y^2 = 4ax \), the focus is at \((a, 0)\). Comparing, \(4a = 4\), so \(a=1\).

Features of Line S:

- It is a right-opening parabola with its vertex at \(-1, 0\).

- The axis of symmetry is the horizontal line $y=0$.
- The parabola is symmetric with respect to the $y=0$ line.

Pros and Cons:

- Pros: The parabola's equation is straightforward, allowing for easy derivation of slopes of tangent lines at various points.
- Cons: The parabola is not a line but a curve, so parallelism with lines involves understanding tangent lines or specific lines passing through points.

Finding the Slope of the Tangent Line to Parabola S

To analyze the slope of lines parallel to Line S, we need to understand the slopes of tangent lines to the parabola at various points.

Differentiating $y^2 = 4(x + 1)$ implicitly with respect to x:

$$
2y \frac{dy}{dx} = 4
$$

$$
\frac{dy}{dx} = \frac{2}{y}
$$

This derivative gives the slope of the tangent line at any point (x, y) on the parabola:

$$
m_{tangent} = \frac{2}{y}
$$

Implication:

- The slope of the tangent lines varies depending on the point on the parabola.
- Lines parallel to Line S are tangent lines with the same slope as the tangent line at some point.

Understanding Line T: Point (1, 6) and Parallel to Line S

What Does Parallelism Mean in This Context?

Since Line S is a parabola, it isn't a straight line but a curve with a family of tangent lines at various points. When the problem states that "Line T includes the point (1, 6) and is parallel to Line S," it suggests that Line T is a straight line parallel to some tangent line of the parabola.

Key Interpretation:

- The line T is a straight line passing through $(1, 6)$.
- It is parallel to the tangent line to parabola S at some point (x_0, y_0).

Alternatively, since the wording might be ambiguous, it could also imply:

- Line T is parallel to the axis of the parabola (which is horizontal), or
- Line T is parallel to the parabola's directrix or focal line.

However, typically, "parallel to a parabola" is interpreted as being parallel to a tangent line at some point on the parabola.

Finding the Equation of Line T

Given the point $(1, 6)$, and the fact that Line T is parallel to some tangent line of the parabola $y^2 = 4(x + 1)$, we proceed as follows:

- The slope of the tangent line at some point (x_0, y_0) on the parabola is:

$$
m = \frac{2}{y_0}
$$

- For Line T, passing through $(1, 6)$, the equation is:

$$
y - 6 = m (x - 1)
$$

- Since Line T is parallel to the tangent line at some point (x_0, y_0), and the tangent line's slope at that point is $m = 2 / y_0$, then:

$$
m = \frac{2}{y_0}
$$

- The point (x_0, y_0) lies on the parabola:

$$
y_0^2 = 4(x_0 + 1)
$$

- The tangent line at (x_0, y_0) has slope $2 / y_0$, and its equation:

\[

$$
Y - y_0 = \frac{2}{y_0} (X - x_0)
$$

Determining the Specific Tangent Line and Line T's Equation

Relating the Point (1, 6) to the Tangent Line

Since Line T is parallel to the tangent line at some point (x_0, y_0), and passes through $(1, 6)$, the line's equation is:

$$
y - 6 = \frac{2}{y_0} (x - 1)
$$

But for this line to be tangent to the parabola at (x_0, y_0), it must pass through that point, so:

$$
y_0 = y_0
$$

which is trivial. The key is to find the y_0 such that the tangent line at (x_0, y_0) passes through $(1, 6)$.

Plugging $(x, y) = (1, 6)$:

$$
6 - y_0 = \frac{2}{y_0} (1 - x_0)
$$

Rearranged:

$$
(6 - y_0) y_0 = 2 (1 - x_0)
$$

But note that $y_0^2 = 4 (x_0 + 1) \Rightarrow x_0 = \frac{y_0^2}{4} - 1$.

Substitute x_0:

$$
(6 - y_0) y_0 = 2 \left(1 - \left(\frac{y_0^2}{4} - 1 \right) \right)
$$

Simplify RHS:

$$
2 \left(1 - \frac{y_0^2}{4} + 1 \right) = 2 \left(2 - \frac{y_0^2}{4} \right) = 4 - \frac{y_0^2}{2}
$$

LHS:

$$
(6 - y_0) y_0 = 6 y_0 - y_0^2
$$

Set equal:

$$
6 y_0 - y_0^2 = 4 - \frac{y_0^2}{2}
$$

Multiply both sides by 2 to clear denominators:

$$
2 (6 y_0 - y_0^2) = 2 \times 4 - y_0^2
$$

$$
12 y_0 - 2 y_0^2 = 8 - y_0^2
$$

Bring all to one side:

$$
12 y_0 - 2 y_0^2 - 8 + y_0^2 = 0
$$

Simplify:

$$
(12 y_0) - (2 y_0^2 - y_0^2) - 8 = 0
$$

$$
12 y_0 - y_0^2 - 8 = 0
$$

Rearranged as quadratic in y_0:

$$
- y_0^2 + 12 y_0 - 8 = 0
$$

Multiply through by -1:

\[
y_0^2 - 12 y_0 + 8 = 0
\]

Solve for \(y_0 \):

\[
y_0 = \frac{12 \pm \sqrt{144 - 4 \times 1 \times 8}}{2} = \frac{12 \pm \sqrt{144 - 32}}{2} =
\frac{12 \pm \sqrt{112}}{2}
\]

\[
\sqrt{112} = \sqrt{16 \times 7} = 4 \sqrt{7}
\]

Therefore:

\[

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