Moment When The Car Spesi Car At That Moment. Question 4. By Using Derivative, Determine The Intervals

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Understanding the behavior of a moving vehicle, especially when analyzing its acceleration, deceleration, or constant speed phases, is crucial in physics and calculus. In particular, the question of when a car reaches specific speeds or changes its velocity significantly can be precisely answered using the tools of calculus—most importantly, derivatives. This article explores how to determine the intervals during which a car is speeding up, slowing down, or moving at a constant velocity by applying derivative concepts.

Introduction to Derivatives in Motion Analysis

The velocity of a car at any given moment is represented mathematically by the derivative of its position function with respect to time. If we denote the position of the car at time t as $s(t)$, then:

$$
v(t) = s'(t) = \frac{ds}{dt}
$$

Similarly, the acceleration is the derivative of velocity with respect to time:

$$
a(t) = v'(t) = s''(t)
$$

Using these derivatives, we can analyze the behavior of the car at different time intervals, understanding where it accelerates, decelerates, or maintains a constant speed.

Understanding the Key Concepts

Critical Points and Their Significance

A critical point occurs at a time t where the derivative $v(t)$ is zero or undefined:

- If $v(t) = 0$, the car is momentarily at rest.
- If $v(t)$ changes sign around t, the car switches from moving

forward to backward or vice versa.

Identifying these points helps us understand transition moments in the car's motion.

Intervals of Increase and Decrease

- Intervals where $v(t) > 0$: The car is moving forward and increasing its position.
- Intervals where $v(t) < 0$: The car is moving backward or decreasing its position.
- Intervals where $v(t) = 0$: The car is stationary or at a turning point.

Concavity and Inflection Points

While primarily used to analyze the shape of the position graph, the second derivative $a(t)$ indicates whether the velocity is increasing or decreasing.

- Positive $a(t)$: The velocity is increasing; the car is speeding up.
- Negative $a(t)$: The velocity is decreasing; the car is slowing down.

Step-by-Step Approach to Determine Intervals Using Derivatives

To analyze the intervals where the car speeds up or slows down, follow these systematic steps:

1. Obtain the Position Function $s(t)$

The problem typically provides a function $s(t)$, representing the position of the car at time t. For example:

$$
s(t) = at^3 + bt^2 + ct + d
$$

or any other differentiable function.

2. Find the Velocity Function $v(t) = s'(t)$

Differentiate $s(t)$ with respect to t:

$$
v(t) = \frac{ds}{dt}

\]

This gives the rate of change of position, or velocity at any moment.

3. Determine Critical Points of $v(t)$

Solve $v(t) = 0$ to find potential points where the car changes behavior:

$$
v(t) = 0
$$

Use algebraic methods or numerical techniques depending on the function's complexity.

4. Evaluate Sign of $v(t)$ Around Critical Points

Test points in each interval between critical points to determine whether $v(t)$ is positive or negative. This indicates the direction of motion.

5. Find the Acceleration Function $a(t) = v'(t)$

Differentiate $v(t)$:

$$
a(t) = \frac{dv}{dt} = s''(t)
$$

This helps determine where the car is speeding up or slowing down.

6. Analyze the Sign of $a(t)$ in Each Interval

- If $a(t) > 0$ and $v(t) > 0$, the car is speeding up.
- If $a(t) < 0$ and $v(t) > 0$, the car is slowing down.
- Similar logic applies when the velocity is negative.

Applying Derivatives: An Example Problem

Suppose the position function of a car is given by:

$$
s(t) = 2t^3 - 9t^2 + 12t
$$

where $s(t)$ is in meters, and t in seconds.

Step 1: Find $v(t)$

$$
v(t) = s'(t) = 6t^2 - 18t + 12
$$

Step 2: Determine critical points for $v(t) = 0$

$$
6t^2 - 18t + 12 = 0
$$

Divide through by 6:

$$
t^2 - 3t + 2 = 0
$$

Factor:

$$
(t - 1)(t - 2) = 0
$$

Critical points at:

$$
t = 1, \quad t = 2
$$

Step 3: Sign analysis of $v(t)$

- For $t < 1$, pick $t=0.5$:

$$
v(0.5) = 6(0.5)^2 - 18(0.5) + 12 = 6(0.25) - 9 + 12 = 1.5 - 9 + 12 = 4.5 > 0
$$

- For $1 < t < 2$, pick $t=1.5$:

$$
v(1.5) = 6(2.25) - 18(1.5) + 12 = 13.5 - 27 + 12 = -1.5 < 0
$$

- For $t > 2$, pick $t=3$:

$$
v(3) = 6(9) - 18(3) + 12 = 54 - 54 + 12 = 12 > 0
$$

Interpretation:

- The car moves forward (positive velocity) during $t \in [0,1)$.
- It moves backward (negative velocity) during $t \in (1,2)$.
- It moves forward again during $t > 2$.

Analyzing Acceleration to Determine Speeding Up and Slowing Down

Step 4: Find $a(t) = v'(t)$

$$
a(t) = \frac{d}{dt}(6t^2 - 18t + 12) = 12t - 18
$$

Step 5: Find when $a(t) = 0$

$$
12t - 18 = 0 \implies t = 1.5
$$

Step 6: Sign analysis of $a(t)$

- For $t < 1.5$, say $t=1$:

$$
a(1) = 12(1) - 18 = -6 < 0
$$

- For $t > 1.5$, say $t=2$:

$$
a(2) = 24 - 18 = 6 > 0
$$

Interpretation:

- When $a(t) < 0$, the velocity is decreasing; the car is slowing down.
- When $a(t) > 0$, the velocity is increasing; the car is speeding up.

Summarizing the Intervals of Speeding Up and Slowing Down

Based on the above analysis:

Interval	Velocity $v(t)$	Acceleration $a(t)$	Motion Status	Speeding Up or Slowing Down
$t \in [0,1)$	Positive	Negative ($a(t)<0$)	Moving forward	Slowing down (velocity decreasing)
$t \in (1,1.5)$	Negative	Negative	Moving backward	Slowing down
$t \in (1.5,2)$	Negative	Positive	Moving backward	Speeding up (velocity decreasing in magnitude)
$t > 2$	Positive	Positive	Moving forward	Speeding up

Practical Implications and Real-World Applications

Understanding the intervals when a car accelerates or decelerates has multiple practical applications:

- Safety Analysis: Detecting when a vehicle is speeding up or slowing down helps in accident prevention.
- Performance Optimization: Car engineers analyze these intervals to improve

Frequently Asked Questions

What does it mean when a car 'spesi' (spins) at a certain moment, and how can we analyze this using derivatives?

When a car 'spesi' or spins at a specific moment, it indicates a sudden change in direction or velocity. Using derivatives, we can analyze the rate of change of the car's position or velocity to determine when such spinning occurs by finding where the derivative indicates a change in motion patterns.

How do you find the intervals during which a car's speed is increasing or decreasing using derivatives?

To find intervals where the car's speed is increasing or decreasing, take the derivative of the velocity function. If the derivative is positive over an interval, the speed is increasing; if negative, the speed is decreasing. These intervals can be identified by analyzing the sign of the derivative.

What is the significance of the critical points in analyzing the car's motion at the moment it spins?

Critical points, where the derivative equals zero or is undefined, indicate potential local maxima, minima, or points of inflection. Analyzing these points helps determine moments when the car's acceleration or velocity changes direction, which could correspond to spinning or skidding.

Using derivatives, how can we determine the exact intervals where the car is spinning or losing control?

By deriving the position or velocity function and analyzing where the second derivative (acceleration) changes sign or reaches critical points, we can identify the intervals where the car's motion indicates spinning or loss of control. Specifically, a change in the sign of the second derivative suggests a change in acceleration behavior, marking potential moments of instability.

Can you explain the process of using derivatives to

analyze the moment when the car spins, focusing on interval determination?

Yes. First, derive the relevant motion function (position, velocity, or acceleration). Then, find the critical points by setting the derivative equal to zero. Next, analyze the sign of the derivative around those points to determine increasing or decreasing intervals. These intervals reveal when the car is stable or spinning, allowing us to pinpoint the specific moments or durations of spinning behavior.

Additional Resources

Moment When The Car Spesi Car At That Moment. Question 4. By Using Derivative, Determine The Intervals

In the world of calculus and physics, understanding the motion of objects such as cars hinges on analyzing how their position changes over time. Whether you're a student tackling a calculus problem or a vehicle enthusiast interested in the nuances of motion, the question often boils down to determining when a vehicle speeds up, slows down, or momentarily comes to a stop. This is where derivatives become powerful tools — enabling us to scrutinize the behavior of a function that models the car's position and extract meaningful insights about its motion.

This article delves into the process of analyzing a car's motion using derivatives, specifically focusing on how to determine the intervals where the car accelerates, decelerates, or remains stationary. We'll navigate through the foundational concepts, apply systematic methods, and clarify the significance of these intervals in real-world contexts, all while maintaining a clear and engaging narrative suitable for both students and enthusiasts.

Understanding the Foundation: Position, Velocity, and Acceleration

Before diving into derivatives and intervals, it's essential to grasp the basic relationships among position, velocity, and acceleration:

- Position function, $s(t)$: Describes the location of the car at any given time t.
- Velocity, $v(t)$: The rate at which the position changes, calculated as the first derivative of position: $v(t) = s'(t)$.
- Acceleration, $a(t)$: The rate at which velocity changes, computed as the derivative of velocity: $a(t) = v'(t) = s''(t)$.

In analyzing the car's motion, the key objective often revolves around understanding the behavior of $v(t)$ and $a(t)$:

- When $v(t) > 0$, the car moves forward.
- When $v(t) < 0$, the car moves backward.
- When $v(t) = 0$, the car is momentarily at rest or stationary.
- The sign of $a(t)$ indicates whether the car is speeding up (positive acceleration) or slowing down (negative acceleration).

The Core Question: Using Derivatives to Determine Critical Intervals

Suppose we are given a position function $s(t)$, perhaps defined by a polynomial or another differentiable function, representing the car's position over time. The goal is to determine the specific time intervals during which the car is accelerating or decelerating.

This involves several steps:

1. Find the velocity function $v(t)$: Compute the first derivative $s'(t)$.
2. Identify critical points of $v(t)$: Find where $v(t) = 0$ or $v(t)$ is undefined, as these points partition the timeline into intervals.
3. Determine the sign of $v(t)$ on each interval: To establish whether the car is moving forward or backward.
4. Find the acceleration $a(t)$: Compute $s''(t)$ to analyze the change in velocity.
5. Determine the sign of $a(t)$: To see whether the car is speeding up or slowing down on each interval.

By analyzing these signs, we can classify each interval as one of the following:

- Accelerating: When the velocity and acceleration have the same sign.
- Decelerating: When the velocity and acceleration have opposite signs.
- At rest: When $v(t) = 0$.

Step-by-Step Approach: Applying Derivatives to Find Intervals

Let's walk through a systematic approach, using an example to illustrate each step.

1. Given Position Function

Suppose the position of the car is modeled by:

$$s(t) = t^3 - 6t^2 + 9t$$

defined for $t \geq 0$.

2. Calculate Velocity $v(t)$

Differentiate $s(t)$:

$$v(t) = s'(t) = 3t^2 - 12t + 9$$

This quadratic function describes how fast and in which direction the car is moving at any time t.

3. Find Critical Points of $v(t)$

Set $v(t) = 0$:

$$3t^2 - 12t + 9 = 0$$

Divide both sides by 3:

$$t^2 - 4t + 3 = 0$$

Factor:

$(t - 1)(t - 3) = 0$

Solutions:

$t = 1, 3$

These are the critical points where the velocity is zero, indicating potential moments of change in motion.

4. Sign of v(t) in Intervals

Choose test points in the intervals:

- $t \in (0,1)$: $t=0.5$
- $t \in (1,3)$: $t=2$
- $t \in (3, \infty)$: $t=4$

Calculate v(t):

- $v(0.5) = 3(0.5)^2 - 12(0.5) + 9 = 3(0.25) - 6 + 9 = 0.75 - 6 + 9 = 3.75 > 0$
- $v(2) = 3(4) - 12(2) + 9 = 12 - 24 + 9 = -3 < 0$
- $v(4) = 3(16) - 12(4) + 9 = 48 - 48 + 9 = 9 > 0$

Interpretation:

- Car moves forward ($v > 0$) in $(0,1)$ and $(3, \infty)$.
- Car moves backward ($v < 0$) in $(1,3)$.

5. Calculate Acceleration a(t)

Differentiate v(t):

$a(t) = v'(t) = 6t - 12$

Find acceleration at critical points:

- $a(1) = 6(1) - 12 = -6 < 0$
- $a(3) = 6(3) - 12 = 6 > 0$

Sign of a(t):

- For $t < 2$: $a(t) = 6t - 12$; at $t=1$, $a(1) = -6$, negative, so acceleration is negative.
- For $t > 2$: $a(t)$ positive.

6. Determine Accelerating or Decelerating Intervals

- Interval $(0,1)$: $v > 0$, $a < 0$ → car moves forward but slowing down (decelerating).
- Interval $(1,3)$: $v < 0$, $a < 0$ (at $t=2$, $a<0$), car moves backward and slowing down.
- Interval $(3, \infty)$: $v > 0$, $a > 0$ → car moves forward and speeds up.

Physical Interpretation and Practical Significance

Understanding these intervals is more than an academic exercise — it offers insights that can be applied in real-life scenarios:

- Traffic safety: Recognizing when a vehicle is decelerating can inform safety systems to alert drivers.
- Vehicle performance analysis: Engineers analyze acceleration intervals to optimize engine performance.
- Driving behavior: Drivers can use such analysis to understand their own driving patterns, such as when they accelerate or brake.

Extending the Analysis: The Role of Critical Points and Inflection Points

While critical points (where v(t)=0) mark potential changes in the direction of motion, inflection points of the position function, where s''(t) changes sign, indicate changes in the acceleration's nature — from increasing to decreasing or vice versa. This layered approach helps in constructing a complete picture of the vehicle's motion profile.

Summary: The Power of Derivatives in Motion Analysis

Using derivatives to analyze a car's motion involves a meticulous process of calculating the first and second derivatives, identifying critical points, and examining the signs of these derivatives across various intervals. This systematic approach allows us to determine:

- When the car is moving forward or backward.
- When it is accelerating or decelerating.
- The moments of change in motion, such as stops, starts, or shifts in speed.

Such analysis exemplifies calculus's practical utility in real-world physics and engineering, transforming abstract functions into meaningful insights about motion. Whether for academic purposes, engineering design, or driving safety, the ability to interpret derivatives and their intervals remains a fundamental skill in understanding dynamic systems.

Final Thoughts

Mastering how to determine intervals of acceleration and deceleration using derivatives enriches our comprehension of motion dynamics. It underscores the importance of mathematical tools in deciphering the complexities of everyday phenomena. As we observe vehicles on the road or analyze mechanical systems, the principles outlined here serve as a foundation for more advanced studies and innovative applications in transportation, robotics, and beyond.

By approaching these problems with clarity and methodical rigor, students and professionals alike can unlock deeper insights into the elegant dance of position, velocity, and acceleration — the core elements that govern the motion of cars and countless other moving objects in our world.

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