Please Help Here Is The Graph Of Y = X2 + 2x - 3The Roots Of The Function ==orThe Turning Point Of The

Please Help Here Is The Graph Of Y = X^2 + 2x - 3The Roots Of The Function ==orThe Turning Point Of The

Understanding the behavior of quadratic functions is a fundamental aspect of algebra and calculus, as these functions model a wide array of real-world phenomena—from projectile motion to economic models. The specific quadratic function in question, $y = x^2 + 2x - 3$, offers an excellent case study for exploring key features such as roots (zeros), vertex (turning point), and the graph's general shape. This in-depth article aims to clarify these concepts, provide step-by-step methods for analyzing the function, and interpret its graph comprehensively.

Analyzing the Quadratic Function $y = x^2 + 2x - 3$

This quadratic function is a parabola opening upwards because the coefficient of x^2 is positive. To understand its graph thoroughly, we need to determine several key aspects:

- The roots (x-intercepts)
- The vertex (turning point)
- The axis of symmetry
- The y-intercept
- The domain and range

Each of these features provides insight into the shape and position of the parabola on the coordinate plane.

Finding the Roots of the Function

Understanding Roots of a Quadratic

The roots or zeros of a quadratic function are the x-values at which the function intersects the x-axis, i.e., where $y=0$. For the quadratic $y = x^2 + 2x - 3$, roots can be found using factoring, completing the square, or the quadratic formula.

Using the Quadratic Formula

The quadratic formula is a universal method for solving quadratic equations:

$$
x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}
$$

Given the quadratic $y = x^2 + 2x - 3$, the coefficients are:

- $a = 1$
- $b = 2$
- $c = -3$

Plugging into the formula:

$$
x = \frac{-2 \pm \sqrt{(2)^2 - 4(1)(-3)}}{2(1)} = \frac{-2 \pm \sqrt{4 + 12}}{2} = \frac{-2 \pm \sqrt{16}}{2}
$$

$$
x = \frac{-2 \pm 4}{2}
$$

Calculating the two roots:

- $x = \frac{-2 + 4}{2} = \frac{2}{2} = 1$
- $x = \frac{-2 - 4}{2} = \frac{-6}{2} = -3$

Roots: $x = 1$ and $x = -3$

Factoring the Quadratic

Alternatively, the quadratic can be factored:

$$
x^2 + 2x - 3 = (x + 3)(x - 1)
$$

Setting each factor to zero:

- $x + 3 = 0 \Rightarrow x = -3$
- $x - 1 = 0 \Rightarrow x = 1$

This confirms the roots found via the quadratic formula.

Summary of Roots

- The parabola intersects the x-axis at $(-3, 0)$ and $(1, 0)$.
- These points are the solutions to the equation $y=0$.

Finding the Turning Point (Vertex)

Understanding the Vertex of a Parabola

The vertex is the highest or lowest point on the parabola, depending on whether it opens downward or upward. Since the coefficient of x^2 is positive, the parabola opens upward, making the vertex the minimum point.

Using the Vertex Formula

The x-coordinate of the vertex for a quadratic $y = ax^2 + bx + c$ is:

$$
x_{v} = -\frac{b}{2a}
$$

Plugging in our values:

$$
x_{v} = -\frac{2}{2 \times 1} = -\frac{2}{2} = -1
$$

To find the y-coordinate:

$$
y_{v} = a x_{v}^2 + b x_{v} + c
$$

Calculating:

$$
y_{v} = 1 \times (-1)^2 + 2 \times (-1) - 3 = 1 - 2 - 3 = -4
$$

Turning point (vertex): $(-1, -4)$

Interpreting the Vertex

- The vertex is the minimum point of the parabola.
- Its x-coordinate is at $x = -1$.
- The y-coordinate at the vertex is $y = -4$.
- The parabola is symmetric about the vertical line $x = -1$.

Additional Key Features of the Graph

Axis of Symmetry

The axis of symmetry is the vertical line passing through the vertex:

$$
x = -1
$$

This line divides the parabola into two mirror-image halves.

Y-Intercept

The y-intercept occurs when $x=0$:

$$
y = (0)^2 + 2 \times 0 - 3 = -3
$$

- The y-intercept is at $(0, -3)$.

Domain and Range

- Domain: All real numbers ($-\infty, \infty$), since quadratic functions are defined for all real x.
- Range: Since the parabola opens upward with a minimum at $y=-4$:

\[
\boxed{

\text{Range}: y \geq -4
}
\]

Graphical Representation and Sketching

Plotting Key Points

To sketch the graph:

1. Plot the roots: $(-3, 0)$ and $(1, 0)$.
2. Plot the vertex: $(-1, -4)$.
3. Plot the y-intercept: $(0, -3)$.
4. Draw the axis of symmetry at $x = -1$.

Drawing the Parabola

Connect these points smoothly, ensuring symmetry about $x = -1$:

- The parabola dips to its minimum at $(-1, -4)$.
- It crosses the x-axis at -3 and 1.
- It crosses the y-axis at -3.

Real-World Applications and Significance

Quadratic functions like $y = x^2 + 2x - 3$ model numerous real-world phenomena:

- Projectile motion: The path of an object under gravity can be represented by quadratic equations.
- Optimization problems: Finding maximum profit or minimum cost.
- Economics: Modeling cost functions or revenue functions.

Understanding roots and the vertex helps in solving practical problems such as determining maximum profit points or the time when an object reaches its lowest point.

Summary and Key Takeaways

- The roots of $y = x^2 + 2x - 3$ are at $x = -3$ and $x = 1$.
- The parabola opens upward, with its vertex at $(-1, -4)$, which is the minimum point.
- The axis of symmetry is the vertical line $x = -1$.
- The y-intercept is at $(0, -3)$, and the domain is all real numbers.
- The range of the function is $y \geq -4$.

Understanding these features allows for accurate graphing and interpretation of quadratic functions, providing a foundation for more advanced topics in algebra and calculus.

Conclusion

The quadratic function $y = x^2 + 2x - 3$ exemplifies classic properties of parabolas. By calculating its roots, vertex, axis of symmetry, and intercepts, we gain a comprehensive understanding of its graph. Recognizing these features not only aids in graphing but also enhances problem-solving skills across various mathematical and real-world contexts. Whether you are analyzing projectile trajectories or optimizing functions in business, mastering the analysis of quadratics is essential.

Frequently Asked Questions

What is the graph of the function y = x^2 + 2x - 3?

The graph of y = x^2 + 2x - 3 is a parabola opening upwards, with its vertex representing the turning point, and it intersects the x-axis at its roots.

How do you find the roots of y = x^2 + 2x - 3?

You can find the roots by factoring the quadratic or using the quadratic formula; in this case, factoring gives (x + 3)(x - 1) = 0, so roots are x = -3 and x = 1.

What is the significance of the roots in the graph of y =

$x^2 + 2x - 3$?

The roots are the points where the parabola intersects the x-axis, indicating the x-values at which y equals zero.

How do you find the turning point (vertex) of the quadratic $y = x^2 + 2x - 3$?

The vertex of a parabola $y = ax^2 + bx + c$ can be found using $x = -b/(2a)$. For this function, the vertex is at $x = -2/(21) = -1$.

What is the y-coordinate of the vertex of $y = x^2 + 2x - 3$?

Substitute $x = -1$ into the equation: $y = (-1)^2 + 2(-1) - 3 = 1 - 2 - 3 = -4$. Therefore, the vertex is at $(-1, -4)$.

Why is understanding the roots and vertex important in analyzing the quadratic graph?

They help identify key features like intercepts and the maximum or minimum point, which are essential for graph interpretation and solving related problems.

Can the function $y = x^2 + 2x - 3$ have more than two roots?

No, quadratic functions can have at most two real roots, which are the x-intercepts of the parabola.

How does the graph of $y = x^2 + 2x - 3$ change if the coefficients are altered?

Changing coefficients affects the parabola's shape and position: increasing the quadratic coefficient makes it narrower, changing the linear coefficient shifts the vertex horizontally, and altering the constant shifts the graph vertically.

What real-world applications can be modeled using the quadratic function $y = x^2 + 2x - 3$?

Quadratic functions model various real-world phenomena such as projectile motion, profit maximization in economics, and areas in geometry, among others.

Additional Resources

Comprehensive Analysis of the Quadratic Function $y = x^2 + 2x - 3$: Roots and Turning Point

Understanding quadratic functions is fundamental in algebra, as they appear frequently across various fields such as physics, engineering, economics, and beyond. The specific function $y = x^2 + 2x - 3$ provides an excellent example to explore the core concepts of quadratic equations, including their roots and turning points. This discussion aims to delve into every aspect of this function, providing clarity through detailed explanations, graphical insights, and mathematical derivations.

Introduction to the Quadratic Function $y = x^2 + 2x - 3$

Quadratic functions are polynomial functions of degree two, generally expressed in the form $y = ax^2 + bx + c$, where a, b, and c are constants, with $a \neq 0$. The function $y = x^2 + 2x - 3$ is a classic parabola opening upward, owing to the positive coefficient of x^2.

Key characteristics:
- The parabola is symmetric with respect to its axis of symmetry.
- It can have two real roots, one real root (a repeated root), or no real roots, depending on the discriminant.
- It possesses a vertex, which is the 'turning point' of the parabola, representing either a maximum or minimum.

Graphical Representation of $y = x^2 + 2x - 3$

Visualizing the graph helps in understanding the function's behavior:

- The parabola opens upward because the coefficient of x^2 (which is 1) is positive.
- The vertex is the minimum point of the parabola.
- The x-intercepts are the roots where the parabola crosses the x-axis.
- The y-intercept can be found by evaluating y at $x=0$.

Graph features:

1. Vertex: The lowest point on the parabola.
2. Axis of symmetry: Vertical line passing through the vertex, dividing the parabola into mirror images.
3. Roots (x-intercepts): The points where $y=0$.
4. Y-intercept: The point where the graph crosses the y-axis.

Finding the Roots of $y = x^2 + 2x - 3$

The roots of a quadratic function are the solutions to the equation $y=0$:

$$ x^2 + 2x - 3 = 0 $$

Methods to find roots:

- Factoring
- Completing the square
- Quadratic formula

1. Factoring:

Looking for two numbers that multiply to -3 and add to 2:

- Factors of -3: (1, -3) and (-1, 3)
- Sum of (1, -3) = -2
- Sum of (-1, 3) = 2

Since (-1) and 3 sum to 2, the factors are:

$$ (x - 1)(x + 3) = 0 $$

Set each factor equal to zero:

$$ x - 1 = 0 \Rightarrow x=1 $$
$$ x + 3= 0 \Rightarrow x= -3 $$

Roots: $x = 1$ and $x = -3$

2. Quadratic formula:

$$ x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} $$

Substituting $a=1$, $b=2$, $c=-3$:

$$ x = \frac{-2 \pm \sqrt{(2)^2 - 4(1)(-3)}}{2(1)} $$
$$ x= \frac{-2 \pm \sqrt{4 + 12}}{2} $$
$$ x= \frac{-2 \pm \sqrt{16}}{2} $$
$$ x= \frac{-2 \pm 4}{2} $$

Calculating both solutions:

- $x= \frac{-2 + 4}{2} = \frac{2}{2} = 1$
- $x= \frac{-2 - 4}{2} = \frac{-6}{2} = -3$

These match the factoring method results, confirming the roots are x=1 and x=-3.

The Turning Point: The Vertex of the Parabola

The vertex represents the point where the parabola changes direction — the minimum for an upward-opening parabola such as this.

Calculating the vertex:

The x-coordinate of the vertex (x_0) is given by:

$$ x_0 = -\frac{b}{2a} $$

For our function:

$$ x_0 = -\frac{2}{2 \times 1} = -\frac{2}{2} = -1 $$

Now, find the y-coordinate (y_0) by substituting x_0 back into the original equation:

$$ y(-1) = (-1)^2 + 2(-1) - 3 = 1 - 2 - 3 = -4 $$

Vertex Coordinates: (-1, -4)

Interpretation:

- The vertex is at (-1, -4).
- This point is the minimum of the parabola.
- The vertex's y-value indicates the lowest point on the graph.

Properties Derived from the Vertex and Roots

1. Axis of Symmetry:

The parabola is symmetric about the vertical line passing through the vertex:

$$ x = x_0 = -1 $$

2. Roots and Vertex Relationship:

The roots are at x=1 and x=-3. The axis of symmetry is exactly midway between these roots:

$$\frac{-3 + 1}{2} = -1$$

which aligns with the x-coordinate of the vertex, confirming the symmetry.

3. Discriminant (Δ):

The discriminant indicates the nature of roots:

$$\Delta = b^2 - 4ac$$

Calculating:

$$\Delta = 2^2 - 4 \times 1 \times (-3) = 4 + 12 = 16$$

Since $\Delta > 0$, the roots are real and distinct, which we already confirmed.

Graphical Analysis and Sketching

Creating an accurate graph involves plotting key points:

- Vertex: (-1, -4)
- Roots: (-3, 0) and (1, 0)
- Y-intercept: when x=0, y= -3
- Symmetry about x=-1

Step-by-step sketching:

1. Plot the vertex at (-1, -4).
2. Mark the x-intercepts at (-3, 0) and (1, 0).
3. Plot the y-intercept at (0, -3).
4. Draw the parabola opening upward, passing through these points, ensuring symmetry about x=-1.

This visual provides insight into the function's behavior, including how it intersects axes and its minimum point.

Applications and Significance of the Function

Quadratic functions like $y = x^2 + 2x - 3$ are fundamental in modeling real-world phenomena:

- Physics: Trajectory of projectiles with quadratic equations.

- Economics: Cost and revenue functions, profit maximization.
- Engineering: Structural analysis, parabolic reflectors.
- Mathematics Education: Foundation for understanding polynomial behavior and calculus concepts.

Understanding roots and turning points helps in optimizing solutions, predicting behaviors, and solving practical problems.

Summary of Key Findings

- The roots of $y = x^2 + 2x - 3$ are $x=1$ and $x=-3$.
- The parabola opens upward, with its vertex at $(-1, -4)$.
- The axis of symmetry is the vertical line $x = -1$.
- The discriminant is 16, confirming two real, distinct roots.
- The y-intercept is at $(0, -3)$.
- The graph is symmetric about the line $x = -1$.

Conclusion: The Significance of Roots and Turning Points in Quadratic Functions

Analyzing the roots and vertex of quadratic functions provides essential insights into their behavior:

- Roots determine where the parabola intersects the x-axis, critical for solving equations.
- The vertex indicates the maximum or minimum value, crucial for optimization problems.
- The axis of symmetry simplifies graphing and analysis.
- The discriminant helps classify the nature of solutions without graphing.

In the specific case of $y = x^2 + 2x - 3$, the roots at $x=1$ and $x=-3$, along with the vertex at $(-1, -4)$, offer a comprehensive understanding of its shape and behavior. Recognizing these features enhances problem-solving skills and deepens appreciation for the elegance of quadratic functions.

Final thoughts: Whether for academic pursuits or practical applications, mastering the analysis of quadratic functions like $y = x^2 + 2x - 3$ is fundamental. It empowers learners and professionals alike to interpret, manipulate, and utilize these mathematical models effectively.

Please Help Here Is The Graph Of Y X2 2x 3the Roots Of The Function Orthe Turning Point Of The

Please Help Here Is The Graph Of Y X2 2x 3the Roots Of The Function Orthe Turning Point Of The

Related Articles

- Perform The Following Operations Expressing The Answer With The Correct Number Of Significant Figures.
- On January 1 Brooks Incorporated Borrows $90,000 From A Bank And Signs A 5% Installment Note Requiring
- Mr. Chang Bought 264 Pencils To Divide Evenly Among His 22 Students. How Many Pencils Did Each Student

Back to Home