Problem 1. (1 Point) Find An Equation Of The Curve That Satisfies Dy Dx 24yx5 And Whose Y-intercept Is

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Understanding the Problem

In this mathematical problem, we are tasked with finding the equation of a curve based on a given differential equation and a specific initial condition—the y-intercept. The differential equation provided is:

$$\frac{dy}{dx} = 24 y x^5$$

along with the information about the y-intercept, which typically indicates the value of y when x equals zero. To approach this problem comprehensively, we need to understand the key components:

- The differential equation: $\frac{dy}{dx} = 24 y x^5$
- The initial condition: the y-intercept (the point where the curve crosses the y-axis, i.e., at $x=0$)

The goal is to find an explicit equation $y = y(x)$ that satisfies both the differential equation and the initial condition.

Analyzing the Differential Equation

The differential equation $\frac{dy}{dx} = 24 y x^5$ is a first-order ordinary differential equation. It is separable because the variables y and x can be separated on different sides of the equation.

Separable Differential Equations:
Such equations can be written as:

$$\frac{dy}{dx} = f(x) g(y)$$

which allows us to write:

\[

$$
\frac{dy}{g(y)} = f(x) dx
$$

In our case:

$$
\frac{dy}{dy} = 24 y x^5
$$

which can be rearranged as:

$$
\frac{dy}{y} = 24 x^5 dx
$$

This form makes it straightforward to integrate both sides separately.

Solving the Differential Equation

Let's proceed with the integral steps:

Step 1: Separate variables

$$
\frac{1}{y} dy = 24 x^5 dx
$$

Step 2: Integrate both sides

$$
\int \frac{1}{y} dy = \int 24 x^5 dx
$$

Step 3: Compute the integrals

- Left side:

$$
\int \frac{1}{y} dy = \ln |y| + C_1
$$

- Right side:

$$
\int 24 x^5 dx = 24 \times \frac{x^6}{6} + C_2 = 4 x^6 + C
$$

where C combines constants C_1 and C_2.

Step 4: Write the general solution

$$
\ln |y| = 4 x^6 + C
$$

Exponentiating both sides to solve for y:

$$
|y| = e^{4 x^6 + C} = e^{C} \times e^{4 x^6}
$$

Let $A = e^{C}$, which is an arbitrary positive constant:

$$
y = \pm A e^{4 x^6}
$$

Since A can be positive or negative, we can write:

$$
y = D e^{4 x^6}
$$

where D is an arbitrary constant that encompasses the sign.

Applying the Y-Intercept Condition

The y-intercept occurs when $x=0$. The problem states that the curve has a specific y-intercept, which is a known value—say, $y(0) = y_0$.

Note: The problem statement is incomplete regarding the exact y-intercept value; typically, it would specify $y(0) = y_0$. For illustration, assume the y-intercept is at $y(0) = y_0$.

Applying this condition:

$$
y(0) = D e^{4 \times 0^6} = D e^{0} = D
$$

Thus,

$$
D = y_0
$$

Final Equation:

$$
\boxed{
y(x) = y_0 e^{4 x^6}
}
$$

This is the explicit form of the solution satisfying the differential equation and the initial y-intercept.

Special Cases and Additional Considerations

Depending on the specific y-intercept value, the solution may take different forms:

- If $y_0 > 0$, the solution is positive for all x.
- If $y_0 < 0$, the solution remains negative.
- If $y_0 = 0$, then the solution reduces to the trivial solution $y(x) = 0$.

Note: The initial condition is essential for determining the constant D. If the problem provides a specific y-intercept value, substitute it accordingly.

Summary of the Solution Process

To summarize, the steps to solve the problem are:

1. Recognize that the differential equation is separable.
2. Separate variables: $\frac{1}{y} dy = 24 x^5 dx$.
3. Integrate both sides:
- $\int \frac{1}{y} dy = \ln |y|$.
- $\int 24 x^5 dx = 4 x^6 + C$.
4. Solve for y:

- $y = D e^{4 x^6}$.
5. Apply the initial y-intercept condition to find D.

Implications and Applications

Understanding how to solve differential equations like $\frac{dy}{dx} = 24 y x^5$ is fundamental in various fields, including physics, engineering, and biological sciences. Such equations often model exponential growth or decay processes with a variable rate depending on both x and y.

Practical applications include:

- Population dynamics, where growth rate depends on current population and other factors.
- Radioactive decay, where the rate depends on the current amount.
- Heat transfer models with variable rates.

Additional Tips for Solving Similar Differential Equations

When confronting similar first-order equations, keep these tips in mind:

- Always identify whether the equation is separable, linear, or exact.
- For separable equations, isolate variables and integrate.
- Pay attention to initial conditions to determine integration constants.
- Check the domain of the solutions, especially when involving logarithms or exponentials.

Conclusion

The complete solution to the problem involves recognizing the differential equation as separable, integrating both sides, and applying the initial y-intercept condition to find the particular solution. The resulting explicit form:

$$
\boxed{
y(x) = y_0 e^{4 x^6}
}
$$

provides a clear and precise description of the curve satisfying both the differential equation and the initial condition. Mastery of these techniques enhances problem-solving skills in differential equations and contributes to a deeper understanding of mathematical modeling.

Note: If the original problem specifies a particular y-intercept value, replace y_0 with that value to obtain the specific solution.

Frequently Asked Questions

What is the differential equation given in Problem 1?

The differential equation is $Dy/Dx = 24 y x^5$.

How do you solve the differential equation $Dy/Dx = 24 y x^5$?

You can solve it using separation of variables: separate y and x, then integrate both sides.

What is the general solution to the differential equation $Dy/Dx = 24 y x^5$?

The general solution is $y = C e^{4 x^6}$, where C is an arbitrary constant.

How do you determine the specific solution given the Y-intercept?

Use the point where y intercept occurs (x=0, y=Y-intercept) to solve for C in the general solution.

If the y-intercept is given as a specific value, how do you find C?

Substitute x=0 and y=Y-intercept into $y = C e^{4 x^6}$ and solve for C.

What is the value of C if the y-intercept is 2?

Substituting x=0 and y=2: $2 = C e^{0} \Rightarrow C=2$.

What is the final equation of the curve if the y-intercept is 2?

The equation is $y = 2 e^{4 x^6}$.

Are there any special considerations when solving this differential equation?

Yes, ensure the solution accounts for the exponential nature and initial conditions, particularly at the y-intercept.

What mathematical techniques are essential for solving this problem?

Separation of variables, integration, and applying initial conditions are essential techniques.

How can understanding this problem help in solving similar differential equations?

It demonstrates the process of solving first-order linear differential equations with initial conditions, a common method in calculus.

Additional Resources

Understanding and Solving the Differential Equation: An In-Depth Exploration of Problem 1

Introduction

Differential equations are fundamental in mathematical modeling across various scientific disciplines, including physics, engineering, biology, and economics. They describe how a quantity changes with respect to another, often time or space, and solving them provides insights into the behavior of complex systems.

In this detailed review, we focus on Problem 1, which involves finding the equation of a curve that satisfies a specific differential equation and an initial condition related to the y-intercept. The problem reads:

> Problem 1. (1 Point) Find an equation of the curve that satisfies $\frac{dy}{dx} = 24 y x^5$ and whose y-intercept is.

Our goal is to thoroughly analyze this problem from multiple angles—understanding the differential equation, solving it step-by-step, interpreting the solution, and discussing the significance of the initial condition.

Understanding the Differential Equation

The Equation $\frac{dy}{dx} = 24 y x^5$

This is a first-order ordinary differential equation (ODE). Several key points emerge immediately:

- It's separable, meaning the variables x and y can be separated on different sides of the equation.

- The presence of the product $y \times x^5$ indicates a proportional relationship between the rate of change dy/dx and both y and x^5.

Significance of the Equation

This differential equation models a process where the rate of change of y depends directly on:

- The current value of y itself, indicating a potentially exponential growth or decay modulated by x^5.
- The variable x raised to the fifth power, which amplifies or diminishes the effect depending on the magnitude of x.

Understanding this relationship is crucial because it influences the behavior of the solution curve, especially as x varies over different intervals.

Step-by-Step Solution Approach

Step 1: Recognize the Type of Differential Equation

Given the structure:

$$
\frac{dy}{dx} = 24 y x^5
$$

This is a separable differential equation, as it can be written as:

$$
\frac{dy}{y} = 24 x^5 dx
$$

which allows us to integrate both sides independently.

Step 2: Separate the Variables

Rewrite:

$$
\frac{dy}{y} = 24 x^5 dx
$$

This step is critical because it simplifies the integration process.

Step 3: Integrate Both Sides

Integrate both sides:

$$
\int \frac{1}{y} dy = \int 24 x^5 dx
$$

Performing the integrations:

- Left side:

$$
\int \frac{1}{y} dy = \ln |y| + C_1
$$

- Right side:

$$
\int 24 x^5 dx = 24 \times \frac{x^6}{6} + C_2 = 4 x^6 + C
$$

where $C = C_2 - C_1$, combining constants for simplicity.

Step 4: Write the General Solution

Expressed explicitly:

$$
\ln |y| = 4 x^6 + C
$$

Exponentiate both sides to solve for y:

$$
|y| = e^{4 x^6 + C} = e^{C} \cdot e^{4 x^6}
$$

Let $A = e^{C}$, which is a positive constant representing the initial conditions:

$$

y = \pm A e^{4 x^6}
\]

Since the constant \(A\) can absorb the sign, we typically write:

\[
\boxed{
y = K e^{4 x^6}
}
\]

where \(K\) is an arbitrary real constant.

Incorporating the Initial Condition: The Y-Intercept

Understanding the Y-Intercept

The y-intercept of a curve is the point where \(x=0\). The problem specifies that the curve's y-intercept is a particular value, which is crucial for determining the specific solution.

Suppose the problem states:

> whose y-intercept is \(y_0\)

then, applying \(x=0\):

\[
y(0) = y_0
\]

Substituting into the general solution:

\[
y_0 = K e^{4 \times 0^6} = K e^{0} = K
\]

Thus,

\[
K = y_0
\]

and the specific solution becomes:

$$
\boxed{
y = y_0 e^{4 x^6}
}
$$

Complete Solution with Y-Intercept

The exact form of the solution is fully determined once the y-intercept y_0 is known. The equation of the curve is:

$$
\boxed{
\text{Curve equation:} \quad y = y_0 e^{4 x^6}
}
$$

where y_0 is the y-intercept value provided.

Deep Dive into the Nature of the Solution

Behavior of the Solution

- Exponential Growth: Since the exponential function $e^{4 x^6}$ is always positive and increases rapidly as x increases (for $x > 0$), the solution reflects an accelerating growth pattern in y as x moves away from zero.
- Symmetry: Because x^6 is an even function, the exponential term is symmetric with respect to the y-axis. This indicates that the curve's behavior is symmetric for positive and negative x, which is typical in even-powered functions.

Effect of the Y-Intercept y_0

- If $y_0 > 0$, the entire curve is positive and exponentially increasing.
- If $y_0 < 0$, the curve is negative and exponentially decreasing.
- If $y_0 = 0$, the solution reduces to the zero solution: $y=0$, which is trivially a solution to the differential equation.

Limitations and Domain of the Solution

The solution is valid for all real x, given the exponential function's domain. However, physical or initial conditions might restrict the domain depending on the context.

Further Considerations and Generalizations

Stability and Long-Term Behavior

- As $x \to \pm \infty$, $e^{4 x^6}$ tends to infinity, indicating unbounded growth in the solution unless $y_0=0$.
- For x near zero, the exponential term approaches 1, so $y \to y_0$, matching the initial y-intercept.

Alternative Initial Conditions

Suppose the initial condition is specified at some point other than the y-intercept, such as $x = x_1$, $y = y_1$. Then, solving for K:

$$
K = y_1 e^{-4 x_1^6}
$$

and the particular solution becomes:

$$
y = y_1 e^{4(x^6 - x_1^6)}
$$

which shows how the solution shifts based on initial data.

Implications in Real-World Contexts

This type of differential equation might model phenomena where growth or decay is heavily influenced by the current state and the independent variable's power, such as:

- Population dynamics with variable environmental factors.
- Chemical reactions with rate constants modulated by spatial variables.
- Engineering systems where the rate of change depends on both the current state and position.

Visualizing the Solution

Graphical Representation

- The curve $y = y_0 e^{4 x^6}$ exhibits rapid exponential growth for large $|x|$, especially if $y_0 \neq 0$.
- Near $x=0$, the curve passes through $(0, y_0)$.
- Symmetric behavior about the y-axis due to the even power in the exponential.

Numerical Examples

Suppose $y_0 = 1$:

$$
y = e^{4 x^6}
$$

Plotting this function reveals a bell-shaped exponential curve with symmetry about the y-axis, growing rapidly as $|x|$ increases.

If $y_0 = 2$:

$$
y = 2 e^{4 x^6}
$$

the curve is scaled vertically but retains the same shape.

Summary and Key Takeaways

- The differential equation $\frac{dy}{dx} = 24 y x^5$ is separable and leads to an exponential solution involving x^6.
- The general solution is:

$$
\boxed{
y = K e^{4 x^6}
}
$$

- The initial y-intercept condition determines the constant K, leading to a specific solution:

$$

$$y = y_0 e^{4 x^6}$$

- The solution exhibits exponential growth, with symmetry about the y-axis due to the even power.

- Understanding the behavior and properties of this solution is essential in modeling systems with similar differential relationships.

Final Remarks

Solving such differential equations not only provides the explicit formula for the curve but also deepens understanding of how variables interact dynamically.

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