Prove Each Statement Using A Proof By Exhaustion. (a) For Every Integer N Such That 0 N < 3, (n 1)2

Prove Each Statement Using A Proof By Exhaustion. (a) For Every Integer N Such That 0 N < 3, $(N + 1)^2 \geq N^2$

Introduction to Proof By Exhaustion

Proof by exhaustion, also known as proof by cases, is a logical reasoning method used to establish the truth of a statement by dividing the problem into a finite number of cases and proving each case individually. This method is particularly effective when the set of possible cases is small and manageable, as it allows for comprehensive verification of all potential scenarios.

In the context of the statement "For every integer N such that $0 \leq N < 3$, $(N + 1)^2 \geq N^2$," the domain of N is limited to a small set of integers: N = 0, 1, and 2. Using proof by exhaustion, we will verify the inequality for each of these specific values, thereby establishing the validity of the statement for the entire domain.

Understanding the Statement

Statement Breakdown

- Domain: All integers N such that $0 \leq N < 3$
- Claim: $(N + 1)^2 \geq N^2$

Implications

Since N is an integer in the range 0 to 2 inclusive, we only need to verify the inequality for N = 0, N = 1, and N = 2. Confirming the statement for these three values will prove its truth for the entire domain.

Case 1: N = 0

Substitute N = 0 into the inequality

Compute both sides:

1. Left side: $(N + 1)^2 = (0 + 1)^2 = 1^2 = 1$
2. Right side: $N^2 = 0^2 = 0$

Compare the Results

- Is $1 \geq 0$? Yes, it is.
- Therefore, for $N = 0$, the inequality holds true.

Case 2: N = 1

Substitute N = 1 into the inequality

Calculate both sides:

1. Left side: $(N + 1)^2 = (1 + 1)^2 = 2^2 = 4$
2. Right side: $N^2 = 1^2 = 1$

Compare the Results

- Is $4 \geq 1$? Yes, it is.
- Thus, for $N = 1$, the inequality is verified.

Case 3: N = 2

Substitute N = 2 into the inequality

Compute both sides:

1. Left side: $(N + 1)^2 = (2 + 1)^2 = 3^2 = 9$
2. Right side: $N^2 = 2^2 = 4$

Compare the Results

- Is $9 \geq 4$? Yes, it is.
- Therefore, for $N = 2$, the inequality holds.

Conclusion

Having verified the inequality for all values of N in the set $\{0, 1, 2\}$ using proof by exhaustion, we can confidently conclude that the statement "For every integer N such that $0 \leq N < 3$, $(N + 1)^2 \geq N^2$" is true for all such N.

This case-based approach effectively demonstrates the validity of the inequality within the specified range, exemplifying the utility of proof by exhaustion for finite and small domains.

Additional Insights and Generalizations

Why Proof by Exhaustion is Suitable Here

- The domain of N is finite and small, making case analysis straightforward and manageable.
- Each case can be explicitly verified without ambiguity or complexity.

Potential Extensions

- For larger domains, proof by exhaustion may become impractical, and algebraic or inductive methods are preferable.
- The inequality $(N + 1)^2 \geq N^2$ can also be proved using algebraic manipulation, but proof by exhaustion here provides clear and concrete validation for the small set considered.

Summary

In this exploration, we have demonstrated how to apply proof by exhaustion to verify the inequality $(N + 1)^2 \geq N^2$ for all integers N in the specified domain $0 \leq N < 3$. By systematically checking each case, we confirmed the statement's correctness for $N = 0$, 1, and 2. This method underscores the effectiveness of proof by exhaustion when dealing with finite, manageable sets of cases, providing a transparent and rigorous validation of the statement.

Frequently Asked Questions

What is the main goal when using proof by exhaustion for the statement 'For every integer N such that 0 ≤

N < 3, (N + 1)^2'?

The goal is to verify the statement holds true for all integers within the specified range, specifically N = 0, 1, and 2, by checking each case individually.

How do you begin a proof by exhaustion for the statement about (N + 1)^2 for N = 0, 1, 2?

Start by evaluating the expression for each value of N in the set {0, 1, 2} and verify the statement holds true for each case.

What is the value of (N + 1)^2 when N = 0?

When N = 0, (0 + 1)^2 = 1^2 = 1.

What is the value of (N + 1)^2 when N = 1?

When N = 1, (1 + 1)^2 = 2^2 = 4.

What is the value of (N + 1)^2 when N = 2?

When N = 2, (2 + 1)^2 = 3^2 = 9.

How does verifying each case confirm the statement for all N in the range?

Since the set of values is finite and each case has been checked individually, confirming the statement in each case ensures it holds for all N within 0 ≤ N < 3.

Why is proof by exhaustion suitable for this particular statement?

Because the range of N is finite and small (0, 1, 2), making it practical to verify each case individually without the need for more complex proof methods.

What is the conclusion after performing proof by exhaustion on this statement?

The statement is proved to be true for all integers N with 0 ≤ N < 3, since it holds for each specific case tested.

Additional Resources

Proof by Exhaustion: A Deep Dive into Verifying (n + 1)^2 for Integers N Where 0 ≤ N < 3

Introduction

Mathematics often challenges us to establish truths that hold across entire sets of numbers. One of the oldest and most straightforward methods for such proofs is proof by exhaustion—a technique that involves systematically checking every possible case within a finite set to confirm a statement's validity. While it might seem simplistic at first glance, this method is both rigorous and elegant, especially when dealing with finite or small domains.

In this article, we explore how to prove the statement:

> For every integer N such that $0 \leq N < 3$, the expression $(N + 1)^2$ holds true, through the method of proof by exhaustion.

This seemingly simple statement provides a perfect opportunity to demonstrate the power of exhaustive verification, helping you understand how to approach similar proofs in more complex contexts.

Understanding the Statement

Before diving into the proof, it's essential to clarify what the statement asserts.

- Domain: All integers N where $0 \leq N < 3$. This means N can only take on the values 0, 1, and 2.
- Expression to evaluate: $(N + 1)^2$.

The goal is to confirm that the statement, perhaps implying a particular property of $(N + 1)^2$ (like its value or divisibility), holds true for all N within this set.

Clarifying the Goal of the Proof

The statement as given is somewhat open-ended. Typically, proofs by exhaustion are used to verify specific properties, such as:

- The exact value of $(N + 1)^2$ for each N.
- That $(N + 1)^2$ satisfies a particular property, say being even, divisible by some number, or fitting a pattern.

For this article, let's assume that the intended statement is:

> For all integers N such that $0 \leq N < 3$, the value of $(N + 1)^2$ is as follows:

N	$N + 1$	$(N + 1)^2$
0	1	1
1	2	4
2	3	9

Alternatively, if the goal was to prove a property such as "for all N in this set, $(N + 1)^2$ is odd," then the proof would verify this property for each case.

For the purpose of this article, we'll focus on demonstrating the exact values of $(N + 1)^2$ for each N in the set $\{0, 1, 2\}$, confirming the calculations via proof by exhaustion.

Why Use Proof by Exhaustion?

In mathematics, proof by exhaustion is particularly effective when:

- The set of cases is finite and small.
- Each case can be checked individually without ambiguity.
- The goal is to verify specific properties that are easier to confirm case-by-case.

Given our limited domain, proof by exhaustion becomes the most straightforward and reliable method to verify the statement, ensuring no case is overlooked.

Step-by-Step Verification

Let's proceed to verify each case where N takes on the values 0, 1, and 2.

Case 1: When $N = 0$

Step 1: Calculate $N + 1$.

$$
0 + 1 = 1
$$

Step 2: Compute $(N + 1)^2$.

$$
(1)^2 = 1
$$

Conclusion:

- The value of $(0 + 1)^2$ is 1.
- This confirms that when $N = 0$, the statement holds true, as $(0 + 1)^2 = 1$.

Case 2: When $N = 1$

Step 1: Calculate $N + 1$.

$$
1 + 1 = 2
$$

Step 2: Compute $(N + 1)^2$.

\[

$$
(2)^2 = 4
$$

Conclusion:

- The value of $(1 + 1)^2$ is 4.
- The statement holds for $N = 1$.

Case 3: When $N = 2$

Step 1: Calculate $N + 1$.

$$
2 + 1 = 3
$$

Step 2: Compute $(N + 1)^2$.

$$
(3)^2 = 9
$$

Conclusion:

- The value of $(2 + 1)^2$ is 9.
- The statement holds for $N = 2$.

Summarizing the Results

N	$N + 1$	$(N + 1)^2$
0	1	1
1	2	4
2	3	9

Since each case has been explicitly verified, we conclude that the property holds for all N such that $0 \leq N < 3$.

Extending the Concept: Why Is This Important?

While the proof here is straightforward due to the small set of cases, the methodology exemplifies the core principle of proof by exhaustion—checking all possible cases to establish the truth of a statement.

This technique is particularly powerful in the following contexts:

- Finite domains: where the set of possibilities is limited.
- Base cases in induction: verifying initial steps.
- Verifying properties of small sets of solutions: such as in combinatorics or number theory.

In more complex scenarios, proof by exhaustion might involve hundreds or thousands of cases, but the process remains fundamentally the same:

enumerate, verify, and conclude.

Additional Insights: Generalizing the Approach

Suppose you want to prove more complex properties of $(N + 1)^2$ for the set $\{0, 1, 2\}$. Here are some potential properties you could verify via proof by exhaustion:

- Parity: Is $(N + 1)^2$ odd or even?
- For $N = 0$: 1 (odd)
- For $N = 1$: 4 (even)
- For $N = 2$: 9 (odd)

- Divisibility: Is $(N + 1)^2$ divisible by 3?
- When $N = 2$: 9 is divisible by 3.
- Others are not divisible by 3.

- Pattern Recognition: Observing the sequence $1, 4, 9$, which are perfect squares, and analyzing their properties.

By verifying each case explicitly, you can establish these properties confidently.

Limitations of Proof by Exhaustion

While proof by exhaustion is straightforward and rigorous for small sets, it becomes impractical as the size of the domain increases. For large or infinite sets, alternative methods such as induction or algebraic proofs are more appropriate.

In our example, the set is finite and small, making proof by exhaustion ideal.

Final Remarks

The process demonstrated here underscores the importance of thoroughness in mathematical proofs. Whether dealing with simple numbers or complex structures, verifying every case ensures no detail is overlooked, leading to a solid and irrefutable conclusion.

In summary:

- Proof by exhaustion involves systematically checking each possible case within a finite set.
- For the set $\{0, 1, 2\}$, verifying $(N + 1)^2$ involves straightforward calculations.
- The method guarantees the statement's validity across the entire domain.

This approach is foundational in mathematics, emphasizing that sometimes, the simplest method—checking all cases—is the most effective for small, finite problems. Whether you're verifying properties of numbers or examining complex systems, proof by exhaustion remains a vital tool in the mathematician's toolkit.

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In conclusion, verifying the statement "For every integer \( N \) such that
\( 0 \leq N < 3 \), the value of \( (N + 1)^2 \) is as calculated" through
proof by exhaustion not only confirms its truth but also exemplifies a
fundamental proof technique that continues to underpin rigorous mathematical
reasoning.
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