Prove Or Disprove That The Point (1,2 \sqrt{2}) Lies On A Circle Centered At The Origin And Containing

Prove Or Disprove That The Point (1, 2 \sqrt{2}) Lies On A Circle Centered At The Origin And Containing

When dealing with geometric problems involving circles, a fundamental question often arises: does a specific point lie on a given circle? In this case, we are asked to determine whether the point $(1, 2\sqrt{2})$ is on a circle that is centered at the origin (0,0) and contains this point. To address this question, we need to analyze the properties of the circle and the coordinates of the point itself, applying fundamental principles of geometry and algebra.

Understanding the Problem

Before diving into calculations, it is essential to clarify what the problem entails and what information is available.

The Key Elements

- **Point in question:** $(1, 2\sqrt{2})$
- **Circle's center:** at the origin (0,0)
- **Containment:** The circle must contain the point

The primary goal is to determine whether the point $(1, 2\sqrt{2})$ satisfies the equation of some circle centered at the origin, which means verifying if it lies on a circle with a certain radius.

What Is Needed To Prove?

- To prove that the point lies on such a circle, we need to find a radius r such that the point satisfies the equation of the circle:

$$
x^2 + y^2 = r^2
$$

- To disprove the claim, we need to show that no such radius exists, meaning the point does not satisfy the equation for any positive radius.

Mathematical Formulation

The equation of a circle centered at the origin with radius r is:

$$
x^2 + y^2 = r^2
$$

Given the point (1, $2\sqrt{2}$), substituting its coordinates into the circle's equation yields:

$$
(1)^2 + (2\sqrt{2})^2
$$

Calculating each term:

- $1^2 = 1$
- $(2\sqrt{2})^2 = 2^2 \times (\sqrt{2})^2 = 4 \times 2 = 8$

Adding these:

$$
1 + 8 = 9
$$

This indicates that if the point lies on a circle centered at the origin, the radius squared must be 9, implying that the radius r is:

$$
r = \sqrt{9} = 3
$$

Therefore, the circle that contains the point $(1, 2\sqrt{2})$ and is centered at the origin has the equation:

\[
x^2 + y^2 = 9
\]

Verification: Does the Point Lie On the Circle?

The calculation above suggests that the point $(1, 2\sqrt{2})$ satisfies the circle's equation:

\[
1^2 + (2\sqrt{2})^2 = 9
\]

which simplifies to:

\[
1 + 8 = 9
\]

Since this equality holds true, the point indeed lies on the circle of radius 3 centered at the origin.

Conclusion: The point $(1, 2\sqrt{2})$ lies on the circle centered at (0,0) with radius 3.

Implications and Additional Considerations

While the above calculation confirms the point's position relative to a specific circle of radius 3, the original problem statement involves understanding whether this point lies on any circle centered at the origin that contains it.

Are there other possible circles?

- The equation of a circle centered at the origin is unique for a given radius.
- Since the point satisfies \(x^2 + y^2 = 9 \), it lies on the circle with radius 3.

Thus, the point $(1, 2\sqrt{2})$ lies on exactly one circle centered at the origin with radius 3.

What about other circles?

- Any circle centered at the origin with a radius different from 3 will not contain this point.
- For example, a circle with radius 2 (or any other value not equal to 3) will not include the point because the sum of the squares of its coordinates does not equal the square of that radius.

Summary and Final Verdict

Based on the algebraic calculation, we have:

- The sum of the squares of the point's coordinates: 9
- The radius squared of the circle centered at the origin that contains this point: 9

Since the point satisfies the circle equation:

$$
x^2 + y^2 = r^2
$$

with $r^2 = 9$, the point $(1, 2\sqrt{2})$ lies on the circle centered at the origin with radius 3.

Therefore, we can conclusively state that:

The point $(1, 2\sqrt{2})$ does indeed lie on a circle centered at the origin, specifically on the circle with radius 3.

Additional Insights and Related Problems

Understanding the position of a point relative to a circle is a fundamental skill in coordinate geometry. Here are some related topics for further exploration:

1. **Distance Formula:** How to determine the distance between two points in the plane and its relation to circle equations.

2. **Equation of a Circle:** Deriving the circle's equation from given points or geometric conditions.

3. **Multiple Circles:** How the same point can lie on different circles, depending on their radii.

4. **Geometric Constructions:** Using compass and straightedge to construct circles passing through specific points.

Conclusion

In conclusion, the initial question about whether the point $(1, 2\sqrt{2})$ lies on a circle centered at the origin and containing it can be answered affirmatively. The algebraic verification confirms that the point satisfies the circle's equation for radius 3, thus confirming its position on that circle. This problem exemplifies how algebra and geometry intersect to provide clear and definitive answers to questions about spatial relationships in the coordinate plane.

In essence, the point $(1, 2\sqrt{2})$ does indeed lie on the circle centered at the origin with radius 3, and this can be proved through straightforward algebraic substitution and validation.

Frequently Asked Questions

Does the point $(1, 2\sqrt{2})$ lie on a circle centered at the origin?

To determine if the point lies on such a circle, we need the radius. If the circle is centered at the origin with radius r, then the point must satisfy $x^2 + y^2 = r^2$. For $(1, 2\sqrt{2})$, $x^2 + y^2 = 1^2 + (2\sqrt{2})^2 = 1 + (2\sqrt{2})^2 = 1 + 4\ 2 = 1 + 8 = 9$. Therefore, the point lies on the circle of radius 3 centered at the origin.

What is the equation of the circle centered at the origin passing through $(1, 2\sqrt{2})$?

The radius r is $\sqrt{(1^2 + (2\sqrt{2})^2)} = \sqrt{(1 + 8)} = \sqrt{9} = 3$. Therefore, the circle's equation is $x^2 + y^2 = 9$.

Can $(1, 2\sqrt{2})$ be proven to lie on the circle $x^2 + y^2 = 9$?

Yes. Substituting $x = 1$ and $y = 2\sqrt{2}$ into the circle's equation: $1^2 + (2\sqrt{2})^2 = 1 + 8 = 9$, which satisfies the equation. Hence, the point lies on the circle.

Is the point $(1, 2\sqrt{2})$ outside, inside, or on the circle $x^2 + y^2 = 9$?

The point $(1, 2\sqrt{2})$ lies exactly on the circle $x^2 + y^2 = 9$, as its coordinates satisfy the equation.

If the circle is centered at the origin with radius less than 3, does $(1, 2\sqrt{2})$ lie on it?

No. Since the point's distance from the origin is 3, any circle with radius less than 3 will not contain the point, which lies on the circle $x^2 + y^2 = 9$.

If the circle is centered at the origin with radius greater than 3, does $(1, 2\sqrt{2})$ lie on it?

Yes. The point $(1, 2\sqrt{2})$ has a distance of 3 from the origin, so it lies on all circles centered at the origin with radius 3 or more. For radius greater than 3, the point still lies inside or on the circle.

How can we generalize whether a point lies on a circle centered at the origin?

A point (x, y) lies on a circle centered at the origin with radius r if and only if $x^2 + y^2 = r^2$. To check, substitute the point's coordinates into this equation.

What is the significance of the point $(1, 2\sqrt{2})$ in relation to circles centered at the origin?

The point $(1, 2\sqrt{2})$ lies on the circle $x^2 + y^2 = 9$, which has radius 3. This highlights that the point is exactly at a distance of 3 from the origin, making it a point on that specific circle.

Can we conclude that $(1, 2\sqrt{2})$ lies on all circles centered at the origin?

No. It only lies on the circle with radius 3. For any other radius, the point will not be on the circle unless the radius matches its distance from the origin.

Additional Resources

Prove Or Disprove That The Point $(1, 2\sqrt{2})$ Lies On A Circle Centered At The Origin And Containing

In the realm of coordinate geometry, a fundamental question often arises: given a specific point, can it be situated on a particular geometric figure such as a circle? This question becomes especially intriguing when the circle in question is centered at the origin, and its radius is unknown or unspecified. Today, we explore the case of the point $(1, 2\sqrt{2})$ and examine whether it lies on a circle centered at the origin that contains this point. The problem demands not just a straightforward answer but a thorough analysis rooted in mathematical principles, providing clarity and insight into the geometric relationships involved.

Understanding the Problem: The Geometric Context

Before delving into proofs or disproofs, it is essential to clarify what the problem entails. The question can be paraphrased as: "Is the point $(1, 2\sqrt{2})$ located on some circle centered at the origin (0,0) that also contains this point?" Implicitly, this involves:

- The circle being centered at the origin, which simplifies the problem since its equation will have a specific form.
- The circle containing the point $(1, 2\sqrt{2})$, meaning the point must satisfy the equation of the circle.
- The question of whether such a circle exists, given the constraints.

The core of the problem reduces to determining whether the point $(1, 2\sqrt{2})$ satisfies the general equation of a circle centered at the origin. If it does, then the point lies on that circle; if not, then it does not. The phrase "and containing" suggests the circle must include this point, which is straightforward: the point must satisfy the circle's equation.

Mathematical Foundations: Equation of a Circle Centered at the Origin

To analyze whether $(1, 2\sqrt{2})$ lies on such a circle, we need to understand the general equation of a circle centered at the origin.

The Standard Equation of a Circle Centered at the Origin

A circle centered at the origin (0,0) with radius r is described by the equation:

$$x^2 + y^2 = r^2$$

where:

- (x, y) are the coordinates of any point on the circle.
- r is the radius of the circle, a positive real number.

Any point (x, y) satisfying this equation lies on the circle; those that do not satisfy it are outside or inside the circle, depending on whether $x^2 + y^2$ is greater than or less than r^2.

Determining if a Point Lies on the Circle

Given a point (x_0, y_0), the question of whether it lies on the circle reduces to checking whether:

$$x_0^2 + y_0^2 = r^2$$

for some positive real r. If such an r exists, then (x_0, y_0) is on the circle with radius r.

In our case, the point in question is:

$$(1, 2\sqrt{2})$$

and the candidate circle has the equation:

$$x^2 + y^2 = r^2$$

Substituting the point's coordinates into the circle's equation gives:

$$1^2 + (2\sqrt{2})^2 = r^2$$

Calculating the sum:

$$1 + (2\sqrt{2})^2$$

Recall that:

$$(\text{a})^2 = a^2$$

and

$$(2\sqrt{2})^2 = 2^2 \times (\sqrt{2})^2 = 4 \times 2 = 8$$

Hence:

$$1 + 8 = 9$$

Therefore, for the point $(1, 2\sqrt{2})$ to lie on the circle, the radius squared must be:

$$r^2 = 9$$

which implies:

$$r = \pm 3$$

Since radius length is positive, $r = 3$.

Conclusion: The point $(1, 2\sqrt{2})$ lies on the circle centered at the origin with radius 3.

Does The Circle Contain The Point? A Confirmed Answer

Based on the calculations above, the circle centered at the origin with radius $r = 3$ contains the point $(1, 2\sqrt{2})$. This is a specific case, but the original question is broader: does there exist such a circle? The answer is yes, because:

- The circle with radius $r = 3$ centered at the origin passes through $(1, 2\sqrt{2})$.
- Any circle centered at the origin with radius $r \geq 3$ will contain the point, as the point's distance from the origin is $\sqrt{1^2 + (2\sqrt{2})^2} = \sqrt{9} = 3$.

Thus, the point lies on the circle with radius exactly 3, and:

- For $r > 3$, the point still lies on or inside the circle.
- For $r = 3$, it lies exactly on the circle.
- For $r < 3$, the point does not lie on the circle.

Hence, the point $(1, 2\sqrt{2})$ does indeed lie on a circle centered at the origin, specifically the one with radius 3.

Implications and Broader Context

The above conclusion highlights a fundamental principle in coordinate geometry: the distance of a point from the circle's center determines whether the point lies on, inside, or outside the circle.

Geometric Interpretation

- The distance from the origin to $(1, 2\sqrt{2})$ is:

$$\sqrt{1^2 + (2\sqrt{2})^2} = \sqrt{1 + 8} = \sqrt{9} = 3$$

- The circle centered at the origin with radius 3 precisely contains the point.

- Any circle centered at the origin with radius greater than 3 will also contain the point, while smaller radii will exclude it.

Extension to Other Scenarios

This analysis can be generalized to questions about points and circles:

- Given a point, determine the radius of a circle centered at the origin that passes through it: simply compute the distance.

- Given a radius, check if the point lies on the circle: compare the point's distance from the origin to the radius.

- Multiple points: find a circle centered at the origin that contains multiple points by ensuring the distances of those points from the origin are less than or equal to the chosen radius.

Summary and Final Remarks

The question, "Prove or disprove that the point $(1, 2\sqrt{2})$ lies on a circle centered at the origin," is answerable through straightforward coordinate geometry. The key steps involve computing the distance of the point from the origin and comparing it with the radius of the circle.

Final verdict: The point $(1, 2\sqrt{2})$ does lie on the circle centered at the origin with radius 3.

This conclusion is rooted in the fundamental distance formula:

$$\text{Distance} = \sqrt{x^2 + y^2}$$

which, in this case, confirms the point's position relative to the circle.

In essence, the problem underscores the elegance of Euclidean geometry: a simple calculation reveals the inclusion or exclusion of a point on a circle, illustrating the harmony between algebra and geometry.

Note: If the question intended to ask whether any such circle, with an unspecified radius, containing the point, exists—then the answer is affirmative, since choosing $r = 3$ yields such a circle. Conversely, if the question was whether a particular circle not specified contains the point, then the answer hinges on the radius, which we have determined to be 3 for the circle passing through the point.

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