Prove That For All Whole Values Of N The Value Of The Expression (n-3)(n+2)-(n-3)(n+8) Is Divisible By

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Understanding the divisibility of algebraic expressions is a fundamental aspect of number theory and algebra. In this article, we will rigorously demonstrate that for all whole numbers n, the expression $(n-3)(n+2) - (n-3)(n+8)$ is divisible by a specific number. The process involves expanding the expressions, simplifying, and analyzing the resulting algebraic form to identify the divisor.

1. Introduction to the Problem

The problem asks us to prove that the value of the expression

$$
(n-3)(n+2) - (n-3)(n+8)
$$

is divisible by a certain number for all whole values of n. Whole numbers typically include zero and all positive integers ($0, 1, 2, 3, \ldots$).

Our goal is to:
- Expand both products
- Simplify the resulting expression
- Find the divisor that divides the entire expression for all integers n.

2. Expanding the Expression

To analyze the divisibility, the first step is to expand each product:

2.1 Expand $(n-3)(n+2)$

Using the distributive property:

$$
(n-3)(n+2) = n \times n + n \times 2 - 3 \times n - 3 \times 2
$$

$$
= n^2 + 2n - 3n - 6 = n^2 - n - 6
$$

2.2 Expand $(n-3)(n+8)$

Similarly:
$$
(n-3)(n+8) = n \times n + n \times 8 - 3 \times n - 3 \times 8
$$
$$
= n^2 + 8n - 3n - 24 = n^2 + 5n - 24
$$

3. Simplifying the Expression

Now, substitute the expanded forms into the original expression:
$$
(n-3)(n+2) - (n-3)(n+8) = (n^2 - n - 6) - (n^2 + 5n - 24)
$$

Distribute the subtraction:
$$
= n^2 - n - 6 - n^2 - 5n + 24
$$

Combine like terms:
$$
n^2 - n^2 = 0
$$
$$
- n - 5n = -6n
$$
$$
-6 + 24 = 18
$$

So, the simplified form of the expression is:
$$
-6n + 18
$$

Or, equivalently:
$$
6(3 - n)
$$

4. Analyzing Divisibility

The simplified expression is:
$$
6(3 - n)
$$

This indicates that for every whole number n, the original expression equals $6 \times (3 - n)$.

4.1 Divisibility by 6

Since the expression is explicitly a multiple of 6, it is divisible by 6 for all whole numbers n. This is because $6(3 - n)$ is an integer multiple of 6 for any integer n.

4.2 General Divisibility

The key question is: "Is the expression divisible by any larger number?" To determine this, we analyze the factors involved.

- The expression's value: $6(3 - n)$
- The divisor: 6

Because $(3 - n)$ can be any integer (since n is any whole number), the product $6(3 - n)$ can be any multiple of 6.

However, it will not necessarily be divisible by any larger number unless that number divides 6, i.e., 1, 2, 3, or 6.

5. Formal Proof of Divisibility

Let's formalize the proof that the expression is divisible by 6:

5.1 For all whole n, $E(n) = (n-3)(n+2) - (n-3)(n+8)$ is divisible by 6.

- From the previous calculations, $E(n) = 6(3 - n)$.
- Since 6 divides any multiple of 6, $E(n)$ is divisible by 6 for all n.

Therefore,

$$
\boxed{\text{The expression is divisible by } 6 \text{ for all whole values of } n.}
$$

6. Conclusion

In conclusion, the algebraic manipulation reveals that the given expression simplifies to $6(3 - n)$. Because this is always a multiple of 6, the expression is divisible by 6 for every whole value of n.

Final statement:

\[
\boxed{
\text{For all whole numbers } n, \quad (n-3)(n+2) - (n-3)(n+8) \text{ is divisible by } 6.

}
\]

This proof not only confirms the divisibility but also demonstrates the process of expanding, simplifying, and analyzing algebraic expressions for divisibility properties—a fundamental skill in mathematics.

Frequently Asked Questions

What is the expression to be tested for divisibility in the problem?

The expression is (n - 3)(n + 2) - (n - 3)(n + 8).

How can the given expression be simplified before testing for divisibility?

Factor out (n - 3) to get (n - 3) [(n + 2) - (n + 8)] which simplifies further to (n - 3)(-6).

What is the simplified form of the expression, and how does it relate to divisibility?

The simplified form is -6(n - 3). Since this is a multiple of 6, the entire expression is divisible by 6 for all integer values of n.

For which values of n is the expression divisible by 6?

For all integers n, the expression simplifies to -6(n - 3), which is always divisible by 6 regardless of n.

What is the key step in proving that the expression is divisible by a certain number?

The key step is factoring the expression to identify a common multiple (here, -6), which proves divisibility for all whole numbers n.

Does the divisibility hold for all whole values of n, including negative numbers?

Yes, since the simplified form is -6(n - 3), it is divisible by 6 for all integer values of n, including negative numbers.

What is the general conclusion about the divisibility of the

given expression?

The expression (n - 3)(n + 2) - (n - 3)(n + 8) is divisible by 6 for all whole number values of n.

Additional Resources

Prove That For All Whole Values Of N The Value Of The Expression (n-3)(n+2)-(n-3)(n+8) Is Divisible By

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Introduction: Understanding the Significance of Divisibility in Mathematics

In the realm of mathematics, divisibility plays a crucial role in understanding the fundamental relationships between numbers. Whether in number theory, algebra, or even applied fields like cryptography, establishing whether a particular expression is divisible by a certain number underpins many theoretical and practical applications.

Today, we delve into a specific algebraic expression:

(n - 3)(n + 2) - (n - 3)(n + 8)

Our goal is to prove that, for all whole (non-negative integer) values of n, this expression is divisible by a certain number. Such proofs not only reinforce algebraic manipulation skills but also deepen our understanding of how algebraic expressions behave across the set of whole numbers.

Statement of the Problem

The problem asks us to demonstrate that:

> For all whole numbers n, the value of the expression
> (n - 3)(n + 2) - (n - 3)(n + 8)
> is divisible by a specific integer.

While the problem does not explicitly specify the divisor, the structure of the question suggests that the divisor is an integer that consistently divides the expression for all whole values of n. As we proceed, we will identify this divisor and rigorously prove the divisibility.

Step 1: Simplify the Expression

Algebraic Expansion

The first logical step is to expand both products in the expression:

1. Expand (n - 3)(n + 2):

$(n - 3)(n + 2) = n n + n 2 - 3 n - 3 2$
$= n^2 + 2n - 3n - 6$
$= n^2 - n - 6$

2. Expand $(n - 3)(n + 8)$:

$(n - 3)(n + 8) = n n + n 8 - 3 n - 3 8$
$= n^2 + 8n - 3n - 24$
$= n^2 + 5n - 24$

Write the Difference

Now, subtract the second from the first:

$(n^2 - n - 6) - (n^2 + 5n - 24) = n^2 - n - 6 - n^2 - 5n + 24$

Simplify by combining like terms:

- n^2 and $-n^2$ cancel out
- $-n$ and $-5n$ combine to $-6n$
- -6 and $+24$ combine to $+18$

Thus, the simplified form is:

$-6n + 18$

Or, equivalently,

$18 - 6n$

Step 2: Analyze the Divisibility

Recognizing the Pattern

The simplified expression, $18 - 6n$, is linear in n. To demonstrate that this expression is divisible by a certain number for all whole n, we need to find that divisor.

Observe that:

- 6 divides $6n$ for all integers n.
- 18 is divisible by 6.

Therefore, $18 - 6n$ is always divisible by 6, regardless of the value of n.

Formal Conclusion

Since $18 - 6n = 6(3 - n)$, it is explicitly factored as a multiple of 6 for all whole n. This confirms that the original expression:

(n - 3)(n + 2) - (n - 3)(n + 8)

is divisible by 6 for all whole values of n.

Step 3: Verifying with Examples

To reinforce our conclusion, let’s test a few values of n:

n	Expression $(n - 3)(n + 2) - (n - 3)(n + 8)$	Simplified Form $6(3 - n)$	Divisible by 6?
0	(0 - 3)(0 + 2) - (0 - 3)(0 + 8)	6(3 - 0) = 18	Yes
1	(1 - 3)(1 + 2) - (1 - 3)(1 + 8)	6(3 - 1) = 12	Yes
2	(2 - 3)(2 + 2) - (2 - 3)(2 + 8)	6(3 - 2) = 6	Yes
3	(3 - 3)(3 + 2) - (3 - 3)(3 + 8)	6(3 - 3) = 0	Yes
4	(4 - 3)(4 + 2) - (4 - 3)(4 + 8)	6(3 - 4) = -6	Yes

All these examples confirm that the expression is divisible by 6.

Step 4: General Proof and Conclusion

Formal Proof

1. Starting from the simplified expression:

$(n - 3)(n + 2) - (n - 3)(n + 8) = 6(3 - n)$

2. Since 6 is a constant divisor, and $3 - n$ is an integer for all whole n, their product is always divisible by 6.

3. Therefore, for all whole values of n, the original expression:

$$
(n - 3)(n + 2) - (n - 3)(n + 8)
$$

is divisible by 6.

Final Statement

Proven: For all whole numbers n, the value of the expression $(n - 3)(n + 2) - (n - 3)(n + 8)$ is divisible by 6.

Additional Insights and Broader Implications

Why is this important?

This proof exemplifies how algebraic simplification can reveal inherent divisibility properties of expressions that might initially seem complex. Recognizing factors and common terms allows mathematicians and students alike to understand the structure underlying algebraic relationships.

Broader applications

Divisibility proofs like this are foundational in number theory, especially in areas such as modular arithmetic, cryptography, and coding theory. They help establish properties that can be generalized or used in constructing algorithms, proofs, or solving more complex problems.

Summary

In this article, we rigorously demonstrated that the algebraic expression $(n - 3)(n + 2) - (n - 3)(n + 8)$ simplifies neatly to $6(3 - n)$, which is divisible by 6 for all whole number values of n. This highlights the power of algebraic manipulation in uncovering fundamental divisibility properties that hold across an entire set of numbers. Whether in pure mathematics or applied fields, such proofs reinforce the importance of pattern recognition, algebraic skill, and logical reasoning in mathematical problem-solving.

In conclusion, the expression in question is divisible by 6 for all whole values of n, confirming a universal property rooted in algebraic structure, and exemplifying the elegance of mathematical reasoning.

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