

QA4-Torsion Theory A Hollow Shaft Of 100 Mm External Diameter And 50 Mm Internal Diameter Transmits 60

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Understanding torsion in mechanical components is fundamental to ensuring the safety, efficiency, and longevity of rotating machinery. One such component often analyzed is the hollow shaft, especially when subjected to torsional loads. In this article, we explore the principles underlying torsion theory, focusing on a specific case: a hollow shaft with an external diameter of 100 mm and an internal diameter of 50 mm, transmitting a torque of 60 Nm. We will delve into the fundamental concepts, calculations, and practical considerations associated with such shafts, providing a comprehensive understanding suitable for engineers, students, and professionals in mechanical design.

Introduction to Torsion and Hollow Shafts

Torsion refers to the twisting of an object due to an applied torque. When a shaft experiences torsion, shear stresses develop within its material, which can lead to deformation or failure if the stresses exceed material limits. Hollow shafts are especially popular in engineering applications because they offer high strength-to-weight ratios, ease of manufacturing, and versatility.

A hollow shaft's geometry significantly influences its ability to withstand torsional loads. The key parameters include:

- External diameter (D)
- Internal diameter (d)
- Wall thickness ($t = (D - d)/2$)
- Material properties, especially shear modulus (G)

In our case, the shaft has an external diameter of 100 mm and an internal diameter of 50 mm, making it a relatively thick-walled tube suitable for transmitting moderate torques.

Fundamental Concepts in Torsion Theory

To analyze torsional stresses and deformations, several theoretical models are used:

1. Torsion Theory (Maximum Shear Stress Theory)

- Assumes failure occurs when the maximum shear stress reaches the shear strength of the material.
- Useful for ductile materials like steel.

2. Torsion Formula for Shafts

The basic relation between torque (T), shear stress (τ), and the shaft's polar section modulus (J):

$$T = \tau \times J$$

where:

- T = applied torque
- τ = shear stress at the outer surface
- J = polar moment of inertia

3. Polar Moment of Inertia (J) for Hollow Shaft

The polar moment of inertia for a hollow shaft is calculated as:

$$J = \frac{\pi}{32} (D^4 - d^4)$$

This value reflects the shaft's capacity to resist torsion.

Calculations for the Hollow Shaft

Let's proceed with the detailed calculations based on the given parameters.

1. Given Data

- External diameter, $(D = 100\text{ mm})$
- Internal diameter, $(d = 50\text{ mm})$
- Applied torque, $(T = 60\text{ Nm})$

2. Convert Units to Consistent Measurements

Since diameters are in millimeters, convert torque to N·mm for consistency:

$$T = 60\text{ Nm} = 60 \times 10^3\text{ N}\cdot\text{mm} = 60,000\text{ N}\cdot\text{mm}$$

3. Calculate Polar Moment of Inertia (J)

$$J = \frac{\pi}{32} (D^4 - d^4)$$

Plugging in the values:

$$J = \frac{\pi}{32} \left[(100)^4 - (50)^4 \right]$$

Calculate each term:

$$100^4 = (100)^4 = 100^2 \times 100^2 = (10,000) \times (10,000) = 100,000,000$$

$$50^4 = (50)^4 = 50^2 \times 50^2 = (2,500) \times (2,500) = 6,250,000$$

Now, compute (J) :

$$J = \frac{\pi}{32} (100,000,000 - 6,250,000) = \frac{\pi}{32} \times 93,750,000$$

Calculate:

$$\frac{J}{93,750,000} \approx \frac{3.1416}{32} \times 93,750,000 \approx 0.09817 \times 93,750,000 \approx 9,201,262 \text{ mm}^4$$

4. Determine Shear Stress (τ) at the Outer Surface

Using the torsion formula:

$$\tau = \frac{T}{J}$$

$$\tau = \frac{60,000 \text{ N}\cdot\text{mm}}{9,201,262 \text{ mm}^4} \approx 0.00652 \text{ N/mm}^2$$

which is approximately 6.52 kPa.

Note: This shear stress value is very low, indicating the shaft is well within safe limits for typical steel materials.

Material Selection and Safety Considerations

Choosing appropriate materials for the shaft is critical. Steel alloys like mild steel, alloy steel, or stainless steel are common choices because of their high shear strength.

Key material properties:

- Yield shear strength (τ_{yield}) typically ranges from 250 MPa to 600 MPa for various steels.
- Shear modulus (G) typically around 80 GPa for steel.

In our case, the calculated shear stress (~6.52 kPa) is negligible compared to typical material strengths, indicating the shaft is safe under the applied torque.

Safety factors are used in engineering to account for uncertainties:

- Common safety factors range from 1.5 to 3.
- For example, with a safety factor of 2, the permissible shear stress becomes:

$$\tau_{\text{permit}} = \frac{\tau_{\text{yield}}}{2} \approx \frac{250 \text{ MPa}}{2} = 125 \text{ MPa}$$

Since the actual shear stress is only about 0.00652 MPa, the shaft's capacity exceeds requirements by a significant margin.

Design Considerations for Hollow Shafts

When designing hollow shafts for torsional loads, consider:

1. **Material Strength:** Selecting materials with high shear strength and good fatigue properties.
2. **Geometry Optimization:** Balancing wall thickness for strength and weight savings.
3. **Manufacturing Constraints:** Ensuring that the internal diameter and external diameter are feasible for manufacturing processes.
4. **Stress Concentrations:** Avoiding sharp corners or notches that can cause localized stresses.
5. **Fatigue Life:** Ensuring the shaft can withstand repeated torsional loads without failure.

Design Tip: Increasing the internal diameter reduces weight but may compromise strength; hence, an optimal balance must be achieved.

Applications of Hollow Shafts Transmitting Torsion

Hollow shafts are widely used in various mechanical systems:

- Gearboxes and transmission systems
- Automotive drive shafts

- Rotor shafts in turbines and generators
- Aircraft and aerospace components
- Industrial machinery and robotics

Their advantages include reduced weight, improved stiffness, and ease of mounting internal components such as bearings or gears.

Practical Example: Real-World Application

Suppose an engineer needs to select a shaft for a conveyor system transmitting a torque of 60 Nm. The design constraints specify a hollow shaft with a 100 mm external diameter and a 50 mm internal diameter.

Design steps:

1. Calculate the shear stress using the torsion formula.
2. Verify that the shear stress is within permissible limits for the selected material.
3. Incorporate safety factors to ensure reliability.
4. Confirm that the shaft's dimensions are manufacturable.
5. Consider the dynamic effects, such as torsional vibrations.

In this case, calculations show the shear stress is minimal, and the shaft is more than capable of transmitting the designated torque safely.

Conclusion

Analyzing torsion in hollow shafts is essential for ensuring mechanical integrity in rotating systems. The specific case of a hollow shaft with 100 mm external diameter and 50 mm internal diameter transmitting 60 Nm exemplifies how fundamental torsion theory guides design decisions. By calculating the polar moment of inertia, shear stresses, and considering material properties and safety factors, engineers can optimize shaft design for strength, weight, and durability.

This analysis underscores the importance of understanding torsion principles and applying them accurately in real-world scenarios. Proper design not only ensures safety and performance

Frequently Asked Questions

What is QA4-Torsion Theory and how is it applied to hollow shafts?

QA4-Torsion Theory is a failure criterion used to analyze the torsional strength of materials, especially in hollow shafts. It helps determine the maximum shear stress a hollow shaft can withstand under torsion before failure occurs.

How do you calculate the torsional shear stress in a hollow shaft with an external diameter of 100 mm and an internal diameter of 50 mm?

The shear stress τ is calculated using $\tau = Tr/J$, where T is the torque, r is the radius at the point of interest, and J is the polar moment of inertia. For a hollow shaft, $J = (\pi/32) (D_o^4 - D_i^4)$.

Given a hollow shaft with external diameter 100 mm, internal diameter 50 mm, and transmitting 60 Nm torque, what is the maximum shear stress?

First, compute J : $J = (\pi/32) (100^4 - 50^4) \text{ mm}^4$. Then, maximum shear stress $\tau = Tr/J$, where $r = D_o/2 = 50 \text{ mm}$. Plugging in the values yields the shear stress in MPa.

What are the advantages of using QA4-Torsion Theory for hollow shafts in power transmission?

QA4-Torsion Theory provides a reliable assessment of shear stress limits, helping to prevent failure, optimize material usage, and improve the safety and efficiency of hollow shafts under torsional loads.

How does the internal diameter affect the torsional capacity of a hollow shaft?

Increasing the internal diameter reduces the polar moment of inertia J , which decreases the shaft's torsional capacity. Conversely, decreasing the internal diameter increases J and enhances torsional strength.

Additional Resources

QA4-Torsion Theory: An In-Depth Analysis of a Hollow Shaft with 100 mm External Diameter and 50 mm Internal Diameter Transmitting 60 Nm

Introduction to Torsion Theory and Its Significance

Torsion theory, also known as strength of materials theory for torsion, is fundamental in understanding how shafts behave under twisting loads. It helps engineers determine whether a shaft can withstand a given torque without failure, ensuring safety, reliability, and efficiency in mechanical systems.

Among the various types of shafts, hollow shafts are prevalent because they offer a favorable strength-to-weight ratio, ease of manufacturing, and specific applications such as in power transmission and aerospace components. This review focuses on a particular hollow shaft with an external diameter of 100 mm and an internal diameter of 50 mm, tasked with transmitting a torque of 60 Nm, analyzed through the lens of the QA4-Torsion Theory.

Fundamental Concepts in Torsion Theory for Shafts

Key Parameters:

- External Diameter (D): 100 mm
- Internal Diameter (d): 50 mm
- Torque (T): 60 Nm
- Material properties: Typically, the ultimate tensile strength (S_{ut}), yield strength (S_y), and shear strength (S_{shear}) are considered, depending on the failure criterion.

Assumptions:

- The shaft is solid or hollow, uniform, and homogeneous.
- The load is purely torsional, with no bending or axial forces.
- The material obeys Hooke's law within elastic limits.
- The shaft is subjected to steady torsion.

Geometric Properties of the Hollow Shaft

Cross-Sectional Area:

The hollow shaft's cross-section is a ring, which influences its torsional properties significantly.

Polar Moment of Inertia (J):

The torsional strength and deformation are directly related to the polar moment of inertia, given by:

$$J = \frac{\pi}{32} (D^4 - d^4)$$

Calculating for our shaft:

$$\begin{aligned} & \backslash[\\ D &= 100\backslash, \backslash\text{mm} = 0.1\backslash, \backslash\text{m} \\ & \backslash] \\ & \backslash[\\ d &= 50\backslash, \backslash\text{mm} = 0.05\backslash, \backslash\text{m} \\ & \backslash] \end{aligned}$$

$$\begin{aligned} & \backslash[\\ J &= \frac{\pi}{32} \left((0.1)^4 - (0.05)^4 \right) \\ & \backslash] \end{aligned}$$

$$\begin{aligned} & \backslash[\\ J &= \frac{\pi}{32} \left(0.0001 - 0.00000625 \right) = \frac{\pi}{32} \times 0.00009375 \\ & \backslash] \end{aligned}$$

$$\begin{aligned} & \backslash[\\ J &\approx 0.0000092\backslash, \backslash\text{m}^4 \\ & \backslash] \end{aligned}$$

This value is vital for calculating shear stresses and deformation under torsion.

Shear Stress Distribution in the Hollow Shaft

Maximum Shear Stress ($\tau_{\max}$)

The shear stress at the outer surface and inner surface depends on the torque transmitted and the polar section:

$$\begin{aligned} & \backslash[\\ \tau &= \frac{T \times r}{J} \\ & \backslash] \end{aligned}$$

where r is the radius at the point of interest.

Maximum shear stress occurs at the outer radius:

$$\begin{aligned} & \backslash[\\ r_{\text{outer}} &= \frac{D}{2} = 50\backslash, \backslash\text{mm} = 0.05\backslash, \backslash\text{m} \\ & \backslash] \end{aligned}$$

$$\begin{aligned} & \backslash[\\ \tau_{\max} &= \frac{T \times r_{\text{outer}}}{J} \\ & \backslash] \end{aligned}$$

Convert torque to SI units:

$$\begin{aligned} & \backslash[\\ T &= 60\backslash, \backslash\text{Nm} \end{aligned}$$

\]

Calculating:

$$\tau_{\max} = \frac{60 \times 0.05}{0.0000092} \approx \frac{3}{0.0000092} \approx 326,087 \text{ Pa} \approx 0.326 \text{ MPa}$$

Similarly, the shear stress at the internal surface (inner radius) is:

$$r_{\text{inner}} = 25 \text{ mm} = 0.025 \text{ m}$$

$$\tau_{\text{inner}} = \frac{T \times r_{\text{inner}}}{J} \approx \frac{60 \times 0.025}{0.0000092} \approx 163,043 \text{ Pa} \approx 0.163 \text{ MPa}$$

The shear stress distribution is linear between these two points, with maximum at the outer surface.

Torsional Shear Stress and Material Strength Criteria

Understanding whether the shaft can safely transmit 60 Nm depends on the failure criterion applied, chiefly:

- Maximum Shear Stress Theory (Tresca Criterion): Assumes failure occurs when shear stress reaches the shear strength.
- Distortion Energy Theory (von Mises): Considers combined stresses; more conservative for ductile materials.
- Maximum Principal Stress Theory: Less relevant in pure torsion but used in combined loading scenarios.

Given the shear stress calculations, the key is to compare $\tau_{\max}$ with the material's shear strength S_s .

Material Selection and Safety Considerations

Material Properties:

- Common materials for shafts: Steel (e.g., mild steel, alloy steel), Aluminum alloys.
- Typical shear strengths:
 - Mild Steel: $S_s \approx 250 \text{ MPa}$
 - Alloy Steel: $S_s \approx 400 \text{ MPa}$

Factor of Safety (FoS):

Designs usually incorporate FoS (e.g., 1.5 to 3), depending on application criticality.

For example, with mild steel:

$$\begin{aligned} & \text{Allowable shear stress} = \frac{S_s}{\text{FoS}} \approx \\ & \frac{250}{2} = 125 \text{ MPa} \end{aligned}$$

Our calculated shear stress (~0.326 MPa) is well below this limit, indicating the shaft is safe under the applied torque.

Torsion Theory Application: QA4-Torsion Principles

QA4-Torsion Theory emphasizes the comprehensive analysis of torsional stresses, deformation, and failure modes for hollow shafts. Its core principles include:

- Precise calculation of shear stresses using geometric properties.
- Evaluation of material's shear strength for safety assurance.
- Consideration of stress concentration factors if present.
- Assessment of deformation (angle of twist) to ensure functional integrity.

Applying QA4-Torsion involves:

1. Calculating Shear Stresses:

As shown above, shear stresses are comfortably within permissible limits.

2. Determining Angle of Twist (θ):

$$\theta = \frac{T \times L}{G \times J}$$

where:

- L is the length of the shaft.
- G is the shear modulus (~80 GPa for steel).

For example, assuming $L = 1 \text{ m}$:

$$\theta = \frac{60 \times 1}{80 \times 10^9 \times 0.0000092} \approx 8.15 \times 10^{-6} \text{ radians}$$

This indicates negligible deformation, aligning with design expectations.

3. Stress Concentration Analysis:

- Check for keyways, holes, or surface flaws that could amplify stresses.
- Proper surface finish and fillet radii are recommended.

4. Failure Mode Analysis:

- Confirm shear stresses do not exceed material shear strength.
- Perform fatigue analysis if the shaft is subjected to cyclic loads.

Practical Design Implications and Optimization

Material Choice:

- Steel alloys are preferred for high strength and ductility.
- Aluminum or composites if weight reduction is critical, with corresponding shear strength considerations.

Shaft Dimensions:

- The given dimensions (100 mm outer, 50 mm inner) provide a robust margin for the 60 Nm torque.
- Adjustments can be made for higher torque applications or to optimize weight.

Manufacturing Constraints:

- Machining hollow shafts requires special tooling.
- Heat treatment may be necessary to enhance strength.

Cost and Weight Efficiency:

- Hollow shafts are lighter than solid shafts of similar diameter.
- Adequate design ensures cost-effective manufacturing and operational safety.

Advanced Topics in Torsion Theory for Hollow Shafts

Stress Concentration Factors:

In real-world applications, holes, keyways, or surface imperfections increase local stresses. QA4-Torsion recommends:

- Use of stress concentration factors (K_t).
- Reinforced design features.
- Finite element analysis (FEA) for complex geometries.

Fatigue Life Estimation:

Repeated torsional loading can cause fatigue failure. Using S-N curves and Miner's rule, engineers can estimate the shaft's lifespan.

Dynamic Load Considerations:

- Sudden torque spikes can induce higher stresses.
- Damping and shock absorption may be incorporated.

Surface Treatments:

- Hardening, polishing, or coating to reduce surface flaws and improve fatigue life.

Conclusion and Final Recommendations

The detailed analysis of the QA4-Torsion Theory applied to a hollow shaft with 100 mm external and 50 mm internal diameters transmitting 60 Nm demonstrates:

- The shear stresses generated are well within the shear strength limits of common materials like mild steel.
- The torsional deformation is minimal, ensuring functional integrity.
- The design

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