# Question 3: Derive The Expression Of Input Impedance As Seen By The Primary Side Of The Linked Coil As

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## Introduction

In electrical engineering, particularly in the analysis of coupled inductors or transformers, understanding the input impedance as seen from the primary side is fundamental. It influences how signals are transferred, how power is delivered, and how the system interacts with external circuits. The primary side impedance, when viewed through the lens of the coupled coil, depends on various parameters such as the inductances of the coils, the mutual inductance, and the load connected to the secondary coil.

This article will provide an in-depth derivation of the expression for the input impedance as seen from the primary side of a linked coil (transformer). We will systematically analyze the coupled circuit, introduce relevant parameters, and step through the mathematical derivation, culminating in the general impedance formula. Such understanding is essential for designing efficient transformers, analyzing coupled inductors, and optimizing circuit performance.

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## Basics of Mutual Induction and Coupled Coils

Before diving into the derivation, let’s review some fundamental concepts:

### Self-Inductance

- The self-inductance $L$ of a coil is the property that opposes changes in current flowing through it.
- It is measured in henrys (H) and relates the coil's magnetic field to the current.

### Mutual Inductance

- When two coils are placed close to each other, a change in current in one coil induces an emf in the other.
- This phenomenon is characterized by the mutual inductance $M$, also measured in henrys.
- The mutual inductance depends on the coil geometry, the core material, and the relative positioning of the coils.

### Transformer Analogy

- A typical transformer consists of a primary coil (input side) and a secondary coil (output side).
- The coils are magnetically linked, allowing energy transfer via magnetic flux.

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## Modeling The Coupled Coils

Consider a transformer with the following parameters:

- Primary coil with inductance $L_1$
- Secondary coil with inductance $L_2$
- Mutual inductance $M$
- Load connected across secondary with impedance $Z_L$

The circuit can be modeled as two inductors coupled through mutual inductance, with the secondary connected to a load.

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## Mathematical Representation of the Circuit

The voltages and currents in the primary and secondary coils are related as:

\[

\begin{cases}
V_1 = j \omega L_1 I_1 + j \omega M I_2 \\
V_2 = j \omega M I_1 + j \omega L_2 I_2
\end{cases}
\]

where:
- \(V_1\) and \(V_2\) are the voltages across primary and secondary, respectively.
- \(I_1\) and \(I_2\) are the currents in the primary and secondary.
- \(j\) is the imaginary unit, and \(\omega\) is the angular frequency.

Assuming sinusoidal steady-state operation, the impedance relationships can be written as:

\[
V_1 = Z_{in} I_1
\]

Our goal is to determine \(Z_{in}\), the input impedance as seen from the primary.

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# Deriving The Input Impedance

Let's proceed step-by-step to derive the expression:

## Step 1: Express Secondary Current \(I_2\)

The secondary is connected to a load \(Z_L\), so:

\[
V_2 = Z_L I_2
\]

Using the second circuit equation:

\[
V_2 = j \omega M I_1 + j \omega L_2 I_2
\]

Substitute \(V_2 = Z_L I_2\):

$$
Z_L I_2 = j \omega M I_1 + j \omega L_2 I_2
$$

Rearranged to solve for $I_2$:

$$
(Z_L - j \omega L_2) I_2 = j \omega M I_1
$$

$$
I_2 = \frac{j \omega M I_1}{Z_L - j \omega L_2}
$$

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## Step 2: Express Primary Voltage $V_1$ in Terms of $I_1$

From the first equation:

$$
V_1 = j \omega L_1 I_1 + j \omega M I_2
$$

Substitute the expression for $I_2$:

$$
V_1 = j \omega L_1 I_1 + j \omega M \times \frac{j \omega M I_1}{Z_L - j \omega L_2}
$$

Note that:

$$
j \omega M \times j \omega M = (j)^2 (\omega)^2 M^2 = - (\omega)^2 M^2
$$

Therefore:

$$
V_1 = j \omega L_1 I_1 - \frac{(\omega)^2 M^2 I_1}{Z_L - j \omega L_2}
$$

Factor out $I_1$:

$$
V_1 = I_1 \left[ j \omega L_1 - \frac{\omega^2 M^2}{Z_L - j \omega L_2} \right]
$$

So, the input impedance $Z_{in}$ as seen from the primary is:

$$
Z_{in} = \frac{V_1}{I_1} = j \omega L_1 - \frac{\omega^2 M^2}{Z_L - j \omega L_2}
$$

This is the general expression for the input impedance of a coupled coil with load $Z_L$ connected to the secondary.

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# Final Expression of Input Impedance

The derived formula:

$$
\boxed{
Z_{in} = j \omega L_1 - \frac{\omega^2 M^2}{Z_L - j \omega L_2}
}
$$

encapsulates how the primary impedance is influenced by the secondary load, the mutual inductance, and the inductances of both coils.

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# Special Cases and Interpretations

Understanding the behavior of the impedance expression under specific conditions is vital:

## 1. Open-Circuited Secondary ($Z_L \to \infty$)

- When the secondary is open, no current flows ($I_2 \to 0$), so:

$$
Z_{in} = j \omega L_1
$$

- The primary sees only its self-inductance.

## 2. Short-Circuited Secondary ($Z_L = 0$)

- When secondary is shorted:

$$
Z_{in} = j \omega L_1 - \frac{\omega^2 M^2}{- j \omega L_2} = j \omega L_1 + \frac{\omega^2 M^2}{j \omega L_2}
$$

Simplify:

$$
Z_{in} = j \omega L_1 + \frac{\omega M^2}{L_2}
$$

- The primary impedance is increased by the mutual coupling.

## 3. Matched Load ($Z_L = j \omega L_2$)

- When load impedance matches the secondary inductance's impedance:

$$
Z_{in} = j \omega L_1 - \frac{\omega^2 M^2}{j \omega L_2 - j \omega L_2} \rightarrow \text{Indeterminate}
$$

- Special care is required here, but generally, this indicates resonance or specific coupling conditions.

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## Implications for Transformer Design and Analysis

The derived expression guides engineers in predicting how the primary impedance varies with load and coupling:

- Designing for Efficiency: Knowing $Z_{in}$ helps match the source impedance for maximum power transfer.
- Tuning Resonance: Adjusting inductances or coupling affects the resonant conditions.
- Leakage and Coupling Coefficient: The mutual inductance $M$ relates to the coupling coefficient $k$:

$$
M = k \sqrt{L_1 L_2}
$$

which influences the impedance transformation ratio.

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## Conclusion

In this comprehensive analysis, we derived the expression for the input impedance as seen from the primary side of a linked coil or transformer:

$$
\boxed{
Z_{in} = j \omega L_1 - \frac{\omega^2 M^2}{Z_L - j \omega L_2}
}
$$

This formula encapsulates the effects of mutual inductance, secondary load, and coil inductances, providing crucial insights for transformer design, circuit analysis, and electromagnetic compatibility considerations. Understanding how to manipulate and interpret this impedance aids engineers in optimizing performance, achieving desired transfer characteristics, and ensuring system stability.

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## Frequently Asked Questions

## How is the input impedance seen from the primary side of a linked coil derived in a transformer?

The input impedance is derived by referring the secondary impedance to the primary side using the square of the turns ratio. It involves expressing the secondary impedance in terms of primary parameters and applying the impedance transformation formula $Z_{in} = (N1/N2)^2 Z_{secondary}$.

## What is the significance of the turns ratio in calculating the primary input impedance of a linked coil?

The turns ratio (N1/N2) determines how the secondary impedance appears on the primary side. It allows us to transform the secondary impedance to an equivalent primary impedance, which is essential for accurately analyzing the circuit from the primary perspective.

## Can you provide the general formula for the input impedance as seen from the primary side of a transformer with a linked coil?

Yes. The general formula is $Z_{in} = (N1/N2)^2 Z_{secondary}$, where $Z_{secondary}$ is the impedance connected across the secondary coil, and N1 and N2 are the number of turns on the primary and secondary coils, respectively.

## How does leakage impedance affect the derivation of input impedance on the primary side of a linked coil?

Leakage impedance adds to the ideal transformed impedance, increasing the total input impedance seen from the primary side. It must be included in the secondary impedance before transforming it to the primary side to accurately derive the total input impedance.

## What assumptions are typically made when deriving the input impedance of a linked coil as seen from the primary side?

Common assumptions include neglecting core losses, assuming ideal coupling (no leakage flux), and considering the transformer to be linear and lossless. These simplify the derivation to mainly involve the turns ratio and secondary impedance transformation.

# Additional Resources

Question 3: Derive The Expression Of Input Impedance As Seen By The Primary Side Of The Linked Coil As

Understanding the input impedance as seen from the primary side of a linked coil (or transformer) is fundamental in the analysis and design of electrical and electronic systems. It influences how a circuit interacts with the source, affects power transfer efficiency, and determines the matching conditions for optimal operation. This comprehensive review explores the derivation of the input impedance, breaking down the concepts, mathematical formulations, and practical considerations involved in analyzing such coupled inductive systems.

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# Introduction to Coupled Inductive Circuits

Coupled inductors or coils are widely used in transformers, inductive filters, and wireless power transfer systems. When two coils are magnetically linked, the behavior of one coil influences the other through mutual inductance. The primary coil is typically connected to a source, while the secondary coil is connected to a load or other circuit elements.

The core idea underlying the analysis of such systems is that the primary side "sees" an impedance that depends on the secondary circuit and the nature of the coupling. Deriving this impedance involves understanding the relationships between voltages, currents, inductances, and mutual coupling.

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# Basic Concepts and Definitions

Before delving into the derivation, it is crucial to clarify some fundamental concepts:

- Self-Inductance ($L_1$, $L_2$): The property of each coil that opposes changes in current within itself.
- Mutual Inductance (M): The measure of the magnetic coupling between the two coils, representing how effectively the magnetic flux produced by one coil links with the other.
- Coefficient of Coupling (k): Describes the fraction of flux linkage between coils, where $0 \leq k \leq 1$. The relation between mutual inductance and self-inductances is $M = k \sqrt{L_1 L_2}$.
- Impedance (Z): The total opposition to current, combining resistance and reactance.
- Input Impedance ($Z_{in}$): The impedance "seen" by the source connected to the primary coil.

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# Mathematical Model of Linked Coils

Consider two coils, the primary (with inductance $L_1$) and secondary (with inductance $L_2$), coupled via magnetic flux characterized by mutual inductance $M$. The primary is connected to a source voltage $V_1$, and the secondary has an impedance $Z_2$.

The voltage-current relationships for the primary and secondary are:

$$
V_1 = L_1 \frac{d i_1}{dt} + M \frac{d i_2}{dt}
$$

$$
V_2 = L_2 \frac{d i_2}{dt} + M \frac{d i_1}{dt}
$$

where:

- $i_1$ and $i_2$ are the currents in the primary and secondary coils, respectively.
- $V_2$ is the voltage across the secondary, which relates to its impedance $Z_2$ as $V_2 = Z_2 i_2$.

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# Derivation of Input Impedance

The goal is to express the primary input impedance $Z_{in}$ as seen from the source terminals connected to the primary coil. This involves eliminating $i_2$ and expressing $V_1$ in terms of $i_1$.

Starting from the secondary equation:

$$
V_2 = Z_2 i_2
$$

Rearranged as:

$$
i_2 = \frac{V_2}{Z_2}
$$

But $V_2$ can also be expressed via the primary current:

$$
V_2 = M \frac{d i_1}{dt} + L_2 \frac{d i_2}{dt}
$$

In the phasor domain (assuming sinusoidal steady state with $e^{j \omega t}$ dependence), derivatives translate into multiplication by $j \omega$:

$$
V_1 = j \omega L_1 i_1 + j \omega M i_2
$$

$$
V_2 = j \omega M i_1 + j \omega L_2 i_2
$$

Since $V_2 = Z_2 i_2$, we get:

$$
j \omega M i_1 + j \omega L_2 i_2 = Z_2 i_2
$$

Rearranged as:

$$
j \omega M i_1 = (Z_2 - j \omega L_2) i_2
$$

Thus,

$$
i_2 = \frac{j \omega M}{Z_2 - j \omega L_2} i_1
$$

Substituting $i_2$ into the primary voltage equation:

$$
V_1 = j \omega L_1 i_1 + j \omega M \times \frac{j \omega M}{Z_2 - j \omega L_2} i_1
$$

Simplify:

$$
V_1 = j \omega L_1 i_1 + \frac{(j \omega M)^2}{Z_2 - j \omega L_2} i_1
$$

Note that:

$$
(j \omega M)^2 = - (\omega M)^2
$$

Therefore:

$$
V_1 = \left[ j \omega L_1 - \frac{ (\omega M)^2 }{ Z_2 - j \omega L_2 } \right] i_1
$$

From the definition of input impedance:

$$
Z_{in} = \frac{V_1}{i_1} = j \omega L_1 - \frac{ (\omega M)^2 }{ Z_2 - j \omega L_2 }
$$

This is the fundamental expression for the input impedance seen from the primary side of a coupled coil system.

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## Special Cases and Simplifications

The general expression can be simplified under certain conditions:

- Ideal coupling ($k = 1$): When the coupling coefficient is unity, mutual inductance reaches its maximum $M = \sqrt{L_1 L_2}$.
- Secondary short-circuited ($Z_2 = 0$): When the secondary is shorted, the impedance simplifies to:

$$
Z_{in} = j \omega L_1 - \frac{ (\omega M)^2 }{ - j \omega L_2 } = j \omega L_1 + \frac{ (\omega M)^2 }{ j \omega L_2 }
$$

which further simplifies to:

$$
Z_{in} = j \omega L_1 + j \frac{ (\omega M)^2 }{ \omega L_2 } = j \omega L_1 + j \omega \frac{ M^2 }{ L_2 }
$$

- Infinite load impedance (open secondary): When the secondary is open, $Z_2 \to \infty$, and the expression reduces to:

$$
Z_{in} = j \omega L_1
$$

indicating the primary behaves as an inductor.

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## Features and Practical Implications

Understanding the derived expression's features helps in application and design:

Pros:
- Accurate modeling: Provides precise insight into how secondary loads influence primary impedance.
- Design flexibility: Enables the design of impedance matching networks for optimal power transfer.
- Versatility: Applicable across RF transformers, inductive sensors, and wireless power systems.

Cons:
- Complex calculations: The expression involves complex algebra, especially with reactive loads.
- Ideal assumptions: Real systems have parasitic effects, non-ideal coupling, and losses not accounted for in simplified models.
- Frequency dependence: The impedance varies with frequency, demanding frequency-specific analysis.

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## Conclusion and Summary

The derivation of the input impedance as seen from the primary side of a linked coil (transformer) hinges on understanding mutual inductance, load conditions on the secondary, and the frequency of operation. The key formula:

$$
Z_{in} = j \omega L_1 - \frac{ (\omega M)^2 }{ Z_2 - j \omega L_2 }
$$

captures the complex interplay between self-inductance, mutual inductance, and secondary load impedance. It reveals that the primary impedance is not static but dynamic, varying with load conditions, coupling efficiency, and frequency.

This comprehensive understanding equips engineers and students with the analytical tools necessary to design, analyze, and optimize coupled inductive systems for a broad range of applications, from power transformers to wireless communication devices. Mastery of these derivations and their implications is critical in advancing modern electrical engineering solutions.

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