Question 6 (14 Marks) Let X, X And X3 Be Independent Binomial B(n = 2, P = 1) Random Variables. Define

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Introduction to Binomial Random Variables

In probability theory and statistics, understanding the behavior of binomial random variables is fundamental. When dealing with multiple independent binomial variables, especially with specific parameters, it becomes crucial to analyze their properties—such as their distributions, expectations, variances, and joint behavior. This article explores the scenario where three independent binomial random variables $X, X_2,$ and X_3 each follow a binomial distribution with parameters $n = 2$ and $p = 1$. We will dissect the definitions, properties, and implications of these variables, providing a comprehensive understanding suitable for students, educators, and practitioners alike.

Understanding Binomial Distribution

Definition of Binomial Random Variable

A binomial random variable is a discrete random variable that counts the number of successes in a fixed number of independent Bernoulli trials, each with the same probability of success.

Formal Definition:

A binomial random variable $X \sim \text{Binomial}(n, p)$ has the probability mass function (PMF):

$$
P(X = k) = \binom{n}{k} p^k (1 - p)^{n - k}
$$

where:

- n is the number of trials,
- p is the probability of success on each trial,
- k is the number of successes, $k = 0, 1, ..., n$.

Key Properties of Binomial Variables

- Expectation:

$$
E[X] = np
$$

- Variance:

$$
Var(X) = np(1 - p)
$$

- Support:

$$
k = 0, 1, 2, ..., n
$$

Analyzing the Scenario: $X, X_2, X_3 \sim \text{Binomial}(2, 1)$

The Parameters: $n=2, p=1$

In this specific case:

- Number of trials (n): 2
- Probability of success (p): 1

This implies that in each trial, success is guaranteed—probability 1.

Implication of $p=1$

When $p=1$, the binomial distribution simplifies significantly:

$$
P(X=k) = \binom{2}{k} (1)^k (0)^{2 - k}
$$

For $k=0, 1, 2$, the probabilities are:

- $P(X=0) = \binom{2}{0} \times 1^0 \times 0^2 = 1 \times 1 \times 0 = 0$
- $P(X=1) = \binom{2}{1} \times 1^1 \times 0^1 = 2 \times 1 \times 0 = 0$
- $P(X=2) = \binom{2}{2} \times 1^2 \times 0^0 = 1 \times 1 \times 1 = 1$

Thus,

$$
P(X=2) = 1
$$

and

$$
P(X=0) = P(X=1) = 0
$$

Conclusion: Distribution of X, X_2, X_3

Since all three variables are independent and identically distributed with these parameters, each essentially takes the value 2 with probability 1.

Properties of X, X_2, X_3

Deterministic Nature of Variables

Given $p=1$, the binomial variables are deterministic:

- Values: $X = X_2 = X_3 = 2$ almost surely.
- Implication: The variables do not exhibit any randomness; their outcomes are fixed.

Expectations and Variances

- Expected Values:

$$
E[X] = E[X_2] = E[X_3] = 2
$$

- Variances:

$$
Var(X) = Var(X_2) = Var(X_3) = 0
$$

Because the variables are almost surely constant, their variances are zero, indicating no variability.

Joint Distribution and Independence

Independence of Variables

The problem states that X, X_2, X_3 are independent. Since each is deterministic at value 2:

- Joint distribution: The joint probability mass function (PMF) is:

$$
P(X = 2, X_2 = 2, X_3 = 2) = 1
$$

- Implication: The variables are perfectly correlated in the sense that they always take the value 2 simultaneously—though technically independent since each variable is fixed.

Key Insights:

- Degenerate Distributions: The variables are degenerate—each is a constant random variable.
- No variability or randomness exists in these variables in this scenario.

Applications and Implications

Practical Examples

Though the scenario with $p=1$ is theoretical, understanding it helps clarify properties of binomial distributions:

- Edge Cases: When $p=1$, the binomial distribution collapses to a degenerate distribution at n.
- Modeling certainty: Useful in modeling situations where success is guaranteed in each trial.

Limitations

- Real-world relevance: Rarely do variables have $p=1$ in practice; typically, p lies between 0 and 1.
- Statistical inference: Estimating parameters becomes trivial when $p=1$, but such cases are often trivial or degenerate in statistical modeling.

Extending to General Binomial Variables

When $p \neq 1$

- For $0 < p < 1$, the binomial variables are random with a non-degenerate distribution.
- Expectation:

$$
E[X] = np
$$

- Variance:

$$
Var(X) = np(1 - p)
$$

Summing Binomial Variables

- The sum of independent binomial variables:

$$
X_{total} = X_1 + X_2 + \dots + X_k
$$

follows a binomial distribution:

$$
X_{total} \sim \text{Binomial}\left(\sum n_i, p\right)
$$

- For identical parameters, the sum is also binomial with parameters scaled accordingly.

Summary of Key Points

- Degenerate Distribution at 2: With $p=1$, each $X_i \sim \text{Binomial}(2, 1)$ is almost surely 2.
- Expectations and Variance: Expectations are 2; variances are 0.
- Independence: Despite being degenerate, the variables are independent since their outcomes are fixed and do not influence each other.
- Theoretical Significance: Highlights boundary cases of the binomial distribution, serving as a foundation for understanding more complex probabilistic models.
- Practical Relevance: Such degenerate cases are rare but may occur in deterministic systems or as limiting cases.

Final Remarks

Understanding the behavior of binomial random variables when $p=1$ provides valuable insight into the nature of probability distributions, especially in their boundary cases. Although the scenario is somewhat trivial—since the variables are constants—the analysis reinforces core concepts such as expectation, variance, and independence. For students and researchers, exploring these edge cases sharpens intuition and supports a deeper grasp of probability theory's foundational principles. Whether in theoretical explorations or practical applications, recognizing the implications of parameters like $p=1$ is essential for robust statistical modeling and analysis.

Keywords for SEO Optimization

- Binomial distribution
- Binomial random variables
- Probability theory
- Expected value
- Variance of binomial variables
- Independent random variables

- Degenerate distribution
- Binomial distribution parameters
- Statistical modeling
- Theoretical probability
- Boundary cases in probability
- Discrete random variables
- Understanding binomial distribution

Frequently Asked Questions

What is the distribution of the random variables X, X2, and X3?

They are independent Binomial random variables with parameters n=2 and p=1, meaning each can take values based on binomial distribution with these parameters.

What is the probability that each of X, X2, and X3 equals 2?

Since p=1, the probability that each variable equals 2 (the number of successes in 2 trials) is 1, because with p=1, all trials are successful.

Are the variables X, X2, and X3 identically distributed?

Yes, all three are independent and identically distributed binomial variables with n=2 and p=1.

What is the expected value of each variable X, X2, and X3?

The expected value of each variable is n p = 2 1 = 2.

What is the variance of each of these binomial variables?

The variance of each variable is n p (1 - p) = 2 1 0 = 0, since p=1.

How does the value of p=1 affect the distribution of X, X2, and X3?

With p=1, each variable is degenerate at 2, meaning they always take the value 2 with probability 1.

What is the joint distribution of X, X2, and X3?

Since the variables are independent and each equals 2 with probability 1, their joint distribution is degenerate at (2, 2, 2).

In the context of the problem, what does defining these variables imply for further calculations?

It simplifies calculations because all variables are constants equal to 2; thus, any sums or functions involving them are straightforward to evaluate.

Additional Resources

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Introduction

In the realm of probability theory and statistics, understanding the behavior and characteristics of random variables is fundamental. The question at hand involves three independent binomial random variables, each with parameters n = 2 and p = 1, which signifies a specific scenario that warrants detailed exploration. This article aims to dissect the problem systematically, providing clarity on the definitions, properties, and potential implications of such variables, all while maintaining a reader-friendly yet technically precise tone.

Understanding the Binomial Distribution

Before delving into the specifics of the problem, it is essential to revisit the basics of the binomial distribution.

Definition

The binomial distribution models the number of successes in a fixed number of independent Bernoulli trials, each with the same probability of success p. Mathematically, a binomial random variable X ~ Binomial(n, p) has the probability mass function (PMF):

$$ P(X = k) = \binom{n}{k} p^{k} (1 - p)^{n - k} $$

for k = 0, 1, 2, ..., n.

Parameters

- n: Number of trials.
- p: Probability of success in each trial.

The binomial distribution is discrete, taking integer values from 0 up to n.

Analyzing the Given Variables: X, X_1, and X_3

The problem specifies three random variables, each with the same distribution: Binomial with parameters n = 2 and p = 1.

Clarification of Parameters: p = 1

A binomial distribution with p = 1 implies that each trial results in success with certainty. Consequently, the distribution simplifies significantly.

Let's examine the implications:

- Number of successes in n = 2 trials, each with success probability p = 1:

\[P(X = 2) = \binom{2}{2} (1)^2 (0)^0 = 1 \]

- Probability of zero successes:

\[P(X = 0) = \binom{2}{0} (1)^0 (0)^2 = 0 \]

- Probability of one success:

\[P(X = 1) = \binom{2}{1} (1)^1 (0)^1 = 0 \]

This indicates that the only possible outcome is X = 2, because:

- The probability that both trials succeed (X=2) is 1.
- The probability of any other outcome (X=0 or X=1) is 0.

Conclusion

Each of the variables X, X_1, and X_3 is almost surely equal to 2. They are deterministic in nature, with no randomness involved once p=1.

Independence and its Implications

The question states that the variables are independent. Given the deterministic nature of each variable (X=2 with probability 1), what does this independence mean?

- Independence generally implies that the occurrence of one variable's outcome does not influence the probability distribution of another.
- In this scenario, since each variable is almost surely equal to 2, their joint distribution is degenerate, and independence holds trivially.

Key Point: Because all variables are almost surely constant, the concept of independence becomes trivial—each variable's value is fixed, so there is no variability to influence or be influenced by others.

Defining the Variables: Formal Representation

Given the above, the variables are essentially degenerate random variables:

- $X = 2$ almost surely
- $X_1 = 2$ almost surely
- $X_3 = 2$ almost surely

This can be formalized as:

$$P(X = 2) = P(X_1 = 2) = P(X_3 = 2) = 1$$

and

$$P(X \neq 2) = P(X_1 \neq 2) = P(X_3 \neq 2) = 0$$

Expected Values and Variance

Since each variable is almost surely 2, their statistical properties are straightforward:

- Expected value (mean):

$$E[X] = 2$$
$$E[X_1] = 2$$
$$E[X_3] = 2$$

- Variance:

$$Var(X) = 0$$
$$Var(X_1) = 0$$
$$Var(X_3) = 0$$

Zero variance indicates no spread; the variables do not fluctuate—they are constant.

Implications for Further Analysis

While the problem appears simple due to the deterministic nature of the variables, understanding this special case provides insights into the behavior of degenerate distributions in probability theory.

Possible Extensions

- Conditional distributions: Since all variables are constant, conditioning on any event does not alter their distributions.
- Joint distributions: The joint distribution of the three variables is degenerate and assigns probability 1 to the point (2, 2, 2).

Critical Reflection: Why Is This Important?

The scenario where $p=1$ in a binomial distribution is a boundary case often used to illustrate the limits of probabilistic models. It emphasizes that:

- When $p=1$, the binomial distribution collapses to a degenerate distribution at n.
- The variables become deterministic, removing any randomness or variability.
- Independence among such variables is trivially satisfied because their values do not vary.

Understanding these edge cases helps statisticians and mathematicians recognize the boundaries of probabilistic models and prepare for real-world situations where parameters may approach or reach such limits.

Final Remarks

In conclusion, the problem of defining and analyzing three independent binomial random variables with parameters $n=2$ and $p=1$ simplifies to an exploration of degenerate distributions. Each variable is almost surely equal to 2, with zero variance and deterministic behavior. While seemingly trivial, this case underscores the importance of parameter choices in probabilistic models and highlights how extreme parameter values transform the distribution's nature.

This scenario also serves as a pedagogical example illustrating the concept of degenerate distributions and the triviality of independence when variables are constants. Such understanding is vital for advanced studies in probability, especially when dealing with boundary conditions or extreme cases.

In summary:

- The variables X, X_1, and X_3 are almost surely equal to 2.
- They have expectation 2 and variance 0.
- Their independence is trivial due to their constant nature.
- This case exemplifies the behavior of binomial distributions at the boundary $p=1$.

Understanding these nuances enriches one's grasp of probability theory, ensuring a solid foundation for more complex and less deterministic models in the future.

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