Set Up A Rational Equation And Then Solve The Following Problems. A Positive Integer Is Twice Another.

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In algebra, setting up and solving rational equations plays a vital role in understanding relationships between variables, especially when dealing with real-world problems. One common type of problem involves two positive integers where one is a multiple of the other. Specifically, when a positive integer is twice another, it offers a practical example to demonstrate the process of translating verbal statements into algebraic equations, particularly rational equations, and then solving them systematically. This article will guide you through the process of setting up rational equations based on such problem scenarios and solving them step-by-step.

Understanding the Problem Scenario

Before diving into the setup of the equations, it’s crucial to understand the problem statement. The key points in our scenario are:

- There are two positive integers.
- One integer is twice the other.

This simple relationship can be expressed algebraically and is often used as a foundation for more complex rational equations.

Step 1: Defining Variables

The first step in solving such problems is to assign variables to the unknown quantities.

Choosing Variables

Suppose:

- Let x be the smaller positive integer.
- Since the other integer is twice the smaller, it can be represented as $2x$.

Because both integers are positive, we know:

- $x > 0$
- $2x > 0$

This variable assignment simplifies the process of translating the word problem into an algebraic equation.

Step 2: Formulating the Rational Equation

Now, consider the specific problem details that involve ratios, fractions, or other rational expressions. For example, a problem might state:

"The sum of the reciprocals of these two integers is $\frac{5}{6}$. Find the integers."

This is a typical rational equation problem because it involves reciprocals (fractions) of variables.

Constructing the Equation

Given the variables x and $2x$, the reciprocals are:

- $\frac{1}{x}$
- $\frac{1}{2x}$

The sum of these reciprocals is given as $\frac{5}{6}$, leading to the equation:

$$
\frac{1}{x} + \frac{1}{2x} = \frac{5}{6}
$$

This is a rational equation involving fractions, which we can solve to find the value of x.

Step 3: Solving the Rational Equation

To solve the equation:

$$
\frac{1}{x} + \frac{1}{2x} = \frac{5}{6}
$$

follow these steps:

Combine the Fractions on the Left Side

Find a common denominator:

$$
\frac{2}{2x} + \frac{1}{2x} = \frac{3}{2x}
$$

So, the equation simplifies to:

$$
\frac{3}{2x} = \frac{5}{6}
$$

Cross-Multiplied to Eliminate Denominators

Cross-multiplied form:

$$
3 \times 6 = 5 \times 2x
$$

which simplifies to:

$$
18 = 10x
$$

Isolate x

Divide both sides by 10:

$$
x = \frac{18}{10} = \frac{9}{5}
$$

$$

Since x must be a positive integer (by problem statement), and $\frac{9}{5}$ is not an integer, this indicates the initial assumption about the problem may need adjustment. Alternatively, the problem could specify different conditions or the reciprocals may be replaced with other functions involving the integers.

Step 4: Adjusting the Problem for Integer Solutions

If the initial problem leads to a non-integer solution, consider changing the problem statement to find the integers themselves, not just their reciprocals. For example:

"The sum of the two integers is 18, and one is twice the other. Find the integers."

In this case:

- Sum: $x + 2x = 3x = 18$
- Solve for x:

$$
3x = 18 \Rightarrow x = 6
$$

- The larger integer:

$$
2x = 2 \times 6 = 12
$$

Thus, the integers are 6 and 12.

Step 5: General Approach to Similar Problems

The process outlined above can be generalized to other problems involving rational equations and integers. Here is a step-by-step guide:

1. **Identify the variables:** Assign variables to unknown quantities based on

the problem statement.

2. **Translate words into algebraic expressions:** Express relationships explicitly, especially those involving ratios, reciprocals, or fractions.

3. **Formulate the rational equation:** Write an equation involving the variables and rational expressions as per the problem's conditions.

4. **Solve the rational equation:** Simplify, cross-multiply, and isolate variables to find solutions.

5. **Check solutions:** Verify that solutions satisfy all conditions, especially positivity and integrality where necessary.

Additional Examples and Practice Problems

To deepen your understanding, here are some practice problems involving rational equations where a positive integer is twice another:

Example 1

"The sum of the reciprocals of two positive integers is $\frac{3}{4}$. One integer is twice the other. Find the integers."

Solution:

- Let x be the smaller integer.
- Larger integer: $2x$.
- Sum of reciprocals:

$$
\frac{1}{x} + \frac{1}{2x} = \frac{3}{4}
$$

- Combine the fractions:

$$
\frac{2}{2x} + \frac{1}{2x} = \frac{3}{4}
$$
$$
\frac{3}{2x} = \frac{3}{4}
$$

- Cross-multiplied:

$$
3 \times 4 = 3 \times 2x
$$
$$
12 = 6x
$$
$$
x = 2
$$

- Larger integer:

$$
2x = 4
$$

Answer: The integers are 2 and 4.

Example 2

"The product of two positive integers, where one is twice the other, is 48. Find the integers."

Solution:

- Let x be the smaller integer.
- Larger integer: $2x$.
- Product:

$$
x \times 2x = 48
$$
$$
2x^2 = 48
$$
$$
x^2 = 24
$$

Since x must be positive, $x = \sqrt{24}$, which is not an integer. Since the problem specifies integers, this indicates no integer solutions exist under these conditions.

Conclusion

Setting up and solving rational equations involving positive integers where one is twice the other requires careful translation of the problem statement into algebraic expressions. By defining variables appropriately, translating verbal statements into equations, and solving systematically, you can find the solutions efficiently. Remember to verify that solutions meet all the problem's conditions, including positivity and integrality. Practice with various problems enhances your understanding of rational equations and strengthens your algebraic problem-solving skills.

As you become more comfortable with these techniques, you'll be equipped to tackle a wide array of mathematical problems involving ratios, fractions, and algebraic relationships.

Keywords: Rational Equations, Algebra, Positive Integers, Ratios, Fractions, Word Problems, Algebraic Equations, Problem Solving, Mathematics, Integer Solutions

Frequently Asked Questions

How do you set up a rational equation when given that a positive integer is twice another?

First, define the smaller integer as a variable (e.g., x). Then, express the larger integer as 2x. Use these expressions to set up the rational equation based on the problem's conditions, such as ratios or sums involving these integers.

What is the main step to solve a rational equation involving two integers where one is twice the other?

The main step is to substitute the variable expression for the smaller integer into the equation, then clear denominators by multiplying through by the least common denominator to solve for the variable.

Can you provide an example of setting up a rational equation with a positive integer and its double?

Yes. For example, if the sum of two integers is 36, and one is twice the other, let the smaller be x. Then, the larger is 2x. The equation is $x + 2x = 36$, which simplifies to $3x = 36$.

How do you verify your solution when solving a rational equation involving integers related by a factor of two?

After solving for the variable, substitute the value back into the definitions of the integers to ensure they satisfy the original conditions and check for any extraneous solutions caused by clearing denominators.

What are common mistakes to avoid when solving rational equations with this type of problem?

Common mistakes include forgetting to find a common denominator before clearing fractions, neglecting to check for extraneous solutions after solving, and confusing the variable definitions for the integers involved.

Additional Resources

Set Up A Rational Equation And Then Solve The Following Problems. A Positive Integer Is Twice Another.

Understanding how to set up and solve rational equations is a fundamental skill in algebra that helps students analyze real-world problems involving relationships between quantities. One common type of problem involves two positive integers where one is twice the other, creating an interesting opportunity to develop equations that accurately model the scenario. This article provides a comprehensive overview of how to formulate rational equations based on such conditions, along with detailed steps for solving them. We will explore the core concepts, demonstrate the problem-solving process, and analyze the implications of the solutions, all within a structured, educational framework.

Introduction to Rational Equations and Their Significance

Rational equations are equations that involve rational expressions—fractions where the numerator and/or denominator are polynomials. These equations are essential in modeling situations where ratios or proportions are involved, such as rates, concentrations, or comparative quantities.

Why are rational equations important?

- They help in real-world problem-solving where relationships involve division.

- They allow for modeling complex relationships that are not linear.
- They serve as foundational tools for advanced topics in algebra, calculus, and applied mathematics.

In the context of the problem at hand, setting up a rational equation involves translating verbal descriptions into algebraic expressions involving ratios of integers. Once formulated, these equations can be manipulated and solved to find the values of the unknowns.

Understanding the Scenario: A Positive Integer Is Twice Another

The core of the problem involves two positive integers, with one being exactly twice the other. Let's define the variables:

- Let x be the smaller positive integer.
- Since the other integer is twice the smaller, let 2x be the larger positive integer.

The problem might involve additional constraints or conditions, such as sums, differences, or ratios involving these integers, which need to be translated into equations.

Key points:

- Both x and 2x are positive integers.
- The relationship is straightforward: larger integer = 2 smaller integer.
- The goal is to set up a rational equation that models the problem accurately.

Step-by-Step Approach to Setting Up the Rational Equation

To effectively set up a rational equation based on the problem, follow these steps:

1. Read and Understand the Problem

- Identify what is given: the relationship between the two integers.

- Determine what is asked: whether to find the integers themselves or to find a related quantity.

2. Define Variables Clearly

- Assign variables to the unknown quantities.
- In this case, as above, x for the smaller integer and $2x$ for the larger.

3. Translate the Verbal Conditions into Algebraic Expressions

- Use the variables to formulate the relationships.
- For example, if the problem states "the sum of the two integers is 30," then:
$x + 2x = 30$.

- If the problem involves ratios, differences, or other relationships, express them accordingly.

4. Formulate the Rational Equation

- Rational equations typically involve fractions.
- For instance, if the problem states "the ratio of the larger to the smaller is 3," then:
$\frac{2x}{x} = 3$.

- Or if the problem involves a scenario where the sum of the reciprocals is given, such as:
$\frac{1}{x} + \frac{1}{2x} = \text{some value}$.

5. Simplify and Prepare for Solution

- Simplify the algebraic expressions.
- Clear denominators when necessary to facilitate solving.

Example Problem: Formulating and Solving

Let’s illustrate the process with a specific example.

Problem Statement:

A positive integer is twice another positive integer. The sum of their reciprocals is $\frac{7}{12}$. Find the integers.

Step 1: Define Variables

- Let x be the smaller positive integer.
- The larger integer is 2x.

Step 2: Write the Relationship

- The sum of their reciprocals:
$\frac{1}{x} + \frac{1}{2x} = \frac{7}{12}$.

Step 3: Set Up the Rational Equation

- Combine the left side:
$\frac{1}{x} + \frac{1}{2x} = \frac{2}{2x} + \frac{1}{2x} = \frac{3}{2x}$.

- So, the equation becomes:
$\frac{3}{2x} = \frac{7}{12}$.

Step 4: Solve the Equation

- Cross-multiplied:
$3 \times 12 = 7 \times 2x$,

$36 = 14x$.

- Solve for x:
$x = \frac{36}{14} = \frac{18}{7}$.

Step 5: Analyze the Solution

- Since $x = \frac{18}{7}$ is not an integer, this suggests that under the current conditions, there is no solution with positive integers satisfying the sum of reciprocals as $\frac{7}{12}$.

Conclusion:
This indicates that either the problem constraints need adjustment or the specific values do not correspond to integers.

Alternative Problems and Variations

The above example demonstrates the process of setting up and solving rational equations involving integers. Other variations include:

- Sum of the integers: Find integers where their sum or difference equals a certain value.
- Product relationships: Involving the product of the integers.
- Ratios involving other quantities: For example, "the ratio of the larger integer to the sum of both is $\frac{2}{3}$."

Each variation requires carefully translating the verbal statement into an algebraic rational equation, simplifying, and solving.

Key Strategies for Solving Rational Equations

When solving rational equations, consider the following strategies:

- Clear denominators: Multiply both sides of the equation by the least common denominator (LCD) to eliminate fractions.
- Check for extraneous solutions: Rational equations can introduce solutions that are not valid due to restrictions like division by zero.
- Factor and simplify: Use factoring techniques to simplify complex expressions.
- Verify solutions: Plug solutions back into the original equation to confirm their validity.

Implications and Real-World Applications

Understanding how to set up and solve rational equations has practical implications in various fields:

- Finance: Calculating interest rates, ratios, or amortization schedules.
- Physics: Modeling rates, such as speed or density.
- Biology: Analyzing ratios in population studies.
- Engineering: Designing systems involving ratios and proportional relationships.

In educational contexts, mastering the process enhances problem-solving skills, logical reasoning, and mathematical literacy.

Conclusion

Setting up a rational equation based on a problem involving positive integers, especially when one is twice the other, is an essential algebraic skill that combines conceptual understanding with procedural techniques. The process involves translating verbal descriptions into algebraic expressions, simplifying, and solving systematically. The example provided illustrates this methodology, emphasizing the importance of careful variable definition, algebraic manipulation, and verification of solutions.

By mastering these steps, students and practitioners can confidently approach a wide range of problems involving ratios, proportions, and rational expressions, deepening their understanding of algebraic relationships and enhancing their analytical abilities. Whether applied in academic settings or real-world scenarios, these skills form the backbone of quantitative reasoning and problem-solving excellence.

References:

- Algebra and Trigonometry Textbooks
- Educational Websites on Rational Equations
- Practice Worksheets and Problem Sets

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