

# Simple Random Sample Of 25 Observations Is Derived From A Normally Distributed Population With A Known

**Simple Random Sample Of 25 Observations Is Derived From A Normally Distributed Population With A Known** - An In-Depth Exploration

---

## Introduction to Sampling and Its Significance in Statistics

Sampling is a fundamental concept in statistics, enabling researchers and analysts to draw meaningful inferences about entire populations based on smaller subsets of data. When dealing with large populations, collecting data from every individual is often impractical, costly, or time-consuming. Therefore, sampling methods, especially simple random sampling, become essential tools for statistical analysis.

A simple random sample (SRS) involves selecting a subset of observations such that each member of the population has an equal probability of being chosen. This method helps ensure that the sample accurately reflects the population, minimizing biases and supporting valid statistical conclusions.

---

## Understanding the Scenario: A Sample of 25 Observations from a Normally Distributed Population

In this context, we focus on a sample size of 25 observations drawn via simple random sampling from a population that follows a normal distribution with a known population mean ( $\mu$ ) and population standard deviation ( $\sigma$ ). This setting is common in many practical applications, including quality control, clinical trials, and manufacturing processes.

The key features of this scenario include:

- Sample Size ( $n$ ): 25 observations
- Population Distribution: Normal distribution
- Known Parameters: Population mean ( $\mu$ ) and standard deviation ( $\sigma$ )

Knowing the population parameters significantly influences the choice of statistical methods and the interpretation of results.

---

# Why Focus on a Normally Distributed Population?

Normal distribution plays a central role in statistical inference because of its desirable properties and the Central Limit Theorem (CLT). When the population is normally distributed, several benefits arise:

- Exact Distribution of Sample Mean: The sample mean follows a normal distribution regardless of sample size, provided the population is normal.
- Simplified Hypothesis Testing: Confidence intervals and tests are more straightforward when parameters are known.
- Accurate Probabilities: Precise calculation of probabilities associated with sample statistics.

This known normality allows statisticians to apply specific analytical techniques, such as Z-tests, with confidence.

---

## Sampling Distribution of the Sample Mean

Understanding the sampling distribution is crucial for inference.

### Characteristics of the Sampling Distribution

When drawing a simple random sample of size  $n$  from a normal population:

- The sampling distribution of the sample mean ( $\bar{x}$ ) is normally distributed with mean  $\mu$  and standard deviation  $\frac{\sigma}{\sqrt{n}}$ .

Mathematically:

$$\bar{x} \sim N\left(\mu, \frac{\sigma}{\sqrt{n}}\right)$$

Given that  $\sigma$  is known, this distribution allows for precise probability calculations and hypothesis testing.

### Implications for Sample Size

A sample size of 25 is often considered sufficient for the CLT to apply, especially when the population is normal. Since the population distribution is known to be normal, even smaller samples can provide reliable estimates.

---

# Estimating Population Parameters from the Sample

When working with a sample, statisticians often aim to estimate the population mean and assess the variability.

## Sample Mean ( $\bar{x}$ )

The sample mean serves as an unbiased estimator of the population mean:

$$\bar{x} = \frac{1}{n} \sum_{i=1}^n x_i$$

where  $(x_i)$  are individual observations.

## Sample Standard Deviation ( $s$ ) and Known $(\sigma)$

In this scenario,  $(\sigma)$  is known, simplifying many analyses. If  $(\sigma)$  were unknown, the sample standard deviation  $(s)$  would be used instead.

---

## Constructing Confidence Intervals for the Population Mean

A key objective in statistical inference is estimating the population mean  $(\mu)$  with a certain level of confidence.

### Confidence Interval Formula (Known $(\sigma)$ )

Given the normality and known  $(\sigma)$ , the confidence interval for  $(\mu)$  is constructed using the Z-distribution:

$$\text{CI: } \bar{x} \pm Z_{\alpha/2} \times \frac{\sigma}{\sqrt{n}}$$

Where:

-  $(Z_{\alpha/2})$  is the critical value from the standard normal distribution corresponding to the desired confidence level (e.g., 1.96 for 95%).

## Example Calculation

Suppose:

- Sample mean ( $\bar{x}$ ) = 50
- Population standard deviation ( $\sigma$ ) = 10
- Sample size ( $n$ ) = 25
- Confidence level = 95% (so,  $Z_{0.025} \approx 1.96$ )

Then, the confidence interval is:

$$50 \pm 1.96 \times \frac{10}{\sqrt{25}} = 50 \pm 1.96 \times 2 = 50 \pm 3.92$$

Resulting in a 95% confidence interval: (46.08, 53.92)

This interval suggests that we are 95% confident the true population mean ( $\mu$ ) lies within this range.

---

## Hypothesis Testing with a Known Population Distribution

In addition to estimation, hypothesis testing helps determine whether a specific claim about the population parameter holds.

### Testing the Population Mean

Suppose we want to test:

- Null Hypothesis ( $H_0$ ):  $\mu = \mu_0$
- Alternative Hypothesis ( $H_A$ ):  $\mu \neq \mu_0$

Using the sample data:

$$Z = \frac{\bar{x} - \mu_0}{\sigma / \sqrt{n}}$$

- If  $|Z|$  exceeds the critical value  $Z_{\alpha/2}$ , we reject  $H_0$ .

## Example

Assuming:

- $\bar{x} = 52$
- $\mu_0 = 50$
- $\sigma = 10$
- $n = 25$
- Significance level  $\alpha = 0.05$

Calculate:

$$Z = \frac{52 - 50}{10 / \sqrt{25}} = \frac{2}{2} = 1$$

Since  $Z=1$  is less than 1.96, we fail to reject  $H_0$  at the 5% significance level, indicating insufficient evidence to assert that the population mean differs from 50.

---

## Advantages of Knowing the Population Parameters

Having prior knowledge of the population mean and standard deviation simplifies statistical analysis:

- Precise Inference: No need to estimate  $\sigma$  from the sample.
- Exact Confidence Intervals: Use of Z-distribution instead of t-distribution.
- Simplified Hypothesis Testing: Direct application of Z-tests without additional adjustments.

This knowledge is typical in controlled experimental settings or when historical data provides reliable population parameters.

---

## Limitations and Assumptions

Despite the advantages, several assumptions underpin the analysis:

- Normality of the Population: The population distribution must be normal. In this case, it is given.
- Random Sampling: The sample must be randomly selected to avoid bias.
- Known  $\sigma$ : The population standard deviation is known with certainty.
- Independence: Observations are independent of each other.

Violations of these assumptions can lead to inaccurate conclusions, emphasizing the need for careful sampling and data validation.

---

## Practical Applications of Sampling from a Normal Population

This statistical framework is applied across various fields:

- Manufacturing: Monitoring product quality where process data follows a normal distribution.
- Clinical Trials: Estimating average treatment effects with known variability.
- Environmental Studies: Measuring pollutant levels that tend to be normally distributed.
- Economics and Finance: Analyzing returns or other financial data modeled as normal.

In each case, understanding how to derive and interpret a simple random sample from a known normal population enhances decision-making and policy formulation.

---

## Conclusion

In summary, a simple random sample of 25 observations drawn from a normally distributed population with known parameters provides a powerful foundation for statistical inference. This approach allows for precise estimation of the population mean, construction of confidence intervals, and hypothesis testing with clarity and confidence. Recognizing the importance of the normality assumption and the availability of known parameters enables statisticians and analysts to apply straightforward analytical techniques, leading to accurate and meaningful conclusions. Whether in quality control, research, or policy analysis, understanding the principles outlined here equips practitioners with the tools necessary for robust statistical analysis rooted in probability theory and statistical inference.

---

Keywords: simple random sample, normal distribution, population parameters, confidence interval, hypothesis testing, statistical inference, sample mean, known  $\sigma$ , sampling distribution

## Frequently Asked Questions

### What is a simple random sample in the context of a normally distributed population?

A simple random sample is a subset of observations where each individual has an equal chance of being selected, and every possible sample of a given size has the same probability of occurrence, drawn from a normally distributed population.

## **Why is it important that the population be normally distributed when drawing a simple random sample of 25 observations?**

Because many statistical inference techniques, such as t-tests and confidence intervals, rely on the assumption of normality, especially for small sample sizes like 25, to ensure accurate results.

## **How does knowing the population standard deviation affect the analysis of the sample?**

Knowing the population standard deviation allows for the use of z-tests instead of t-tests, simplifying the analysis and providing more precise probability calculations.

## **What is the significance of the sample size being 25 in this context?**

A sample size of 25 is often considered sufficient for the Central Limit Theorem to apply approximately, but since the population is normal and standard deviation is known, this size ensures reliable inference using z-scores.

## **How can the results from this sample be used to estimate the population mean?**

Using the sample mean and the known population standard deviation, one can construct a z-interval to estimate the population mean with a specified confidence level.

## **What assumptions are made when deriving conclusions from this simple random sample?**

The key assumptions are that the sample is truly random, the population is normally distributed, and the population standard deviation is known.

## **Can the findings from this sample be generalized to the entire population? Why or why not?**

Yes, because the sample is random and representative, and the population distribution is normal, allowing for valid generalizations within the context of statistical inference.

## **What statistical methods are appropriate for analyzing this sample data?**

Since the population standard deviation is known and the population is normal, z-tests for means and z-intervals are appropriate for analysis.

## If the population standard deviation were unknown, how would the analysis change?

The analysis would then rely on the t-distribution instead of the normal distribution, using the sample standard deviation to estimate variability and perform t-tests or confidence intervals.

## Additional Resources

Simple Random Sample Of 25 Observations Is Derived From A Normally Distributed Population With A Known: An In-Depth Examination

---

### Introduction

In the realm of statistical analysis, the concept of sampling is fundamental. It forms the backbone of inferential statistics, allowing researchers to make conclusions about populations based on data collected from a subset of observations. Among various sampling techniques, the simple random sample (SRS) stands out due to its straightforward implementation and desirable statistical properties. When an SRS of size 25 is obtained from a population that is normally distributed with a known mean and variance, it opens up a plethora of analytical opportunities. This article delves into the nuances of this scenario, exploring its theoretical foundations, practical implications, and the significance in statistical inference.

---

### Understanding the Basics of Simple Random Sampling

#### What is Simple Random Sampling?

Simple Random Sampling is a sampling method where each member of a population has an equal chance of being selected, and every possible sample of a specified size has an equal probability of occurrence. This technique ensures that the sample is representative of the population, minimizing bias and allowing for the application of classical statistical methods.

#### Why Choose SRS?

- Unbiasedness: Each member has an equal chance, reducing selection bias.
- Ease of Analysis: The randomness simplifies probability calculations.
- Foundation for Inferential Statistics: Many statistical tests assume SRS for valid results.

#### Limitations of SRS

While powerful, SRS can be impractical for very large populations due to logistical constraints. Additionally, it assumes complete randomness, which may be compromised in real-world data collection.

---

## The Significance of a Normally Distributed Population

### Characteristics of Normal Distribution

The normal distribution, often called the Gaussian distribution, is characterized by its symmetric bell-shaped curve. Its key features include:

- Mean ( $\mu$ ): The central location of the distribution.
- Standard deviation ( $\sigma$ ): Measures the spread or variability.
- Empirical rule: Approximately 68% of data within one  $\sigma$  of the mean, 95% within two, and 99.7% within three.

### Why Is Normality Important?

Many statistical inference techniques, such as t-tests and confidence intervals, rely on the assumption of normality, especially when sample sizes are small. When the population is known to be normally distributed, it simplifies analysis and enhances the accuracy of inferential procedures.

---

### The Scenario: Sample Size of 25 from a Known Normal Population

#### The Setting

Suppose a researcher draws a simple random sample of size 25 from a population that is normally distributed, with a known mean ( $\mu$ ) and known standard deviation ( $\sigma$ ). This scenario is common in quality control, manufacturing, and scientific research where prior knowledge about the population parameters exists.

#### Implications of Known Parameters

Having known population parameters provides several advantages:

- Direct calculation of probabilities: Using the standard normal distribution.
- Precise confidence intervals: Without the need to estimate parameters.
- Simplified hypothesis testing: Using Z-tests instead of t-tests.

---

### Analytical Framework for the Sample

#### Distribution of the Sample Mean

When sampling from a population with a normal distribution and known parameters, the sample mean ( $\bar{X}$ ) is itself normally distributed:

$$\bar{X} \sim N\left(\mu, \frac{\sigma^2}{n}\right)$$

where:

- $\mu$  is the population mean,
- $\sigma^2$  is the population variance,
- $n$  is the sample size (here, 25).

This property is pivotal because it allows direct probabilistic statements about the sample mean.

### Standard Error of the Mean

The standard error (SE) quantifies the variability of the sample mean:

$$SE = \frac{\sigma}{\sqrt{n}}$$

For a sample size of 25:

$$SE = \frac{\sigma}{5}$$

Knowing  $\sigma$  enables precise calculation of the likelihood of observing particular sample means, which is essential for constructing confidence intervals and conducting hypothesis tests.

---

## Statistical Inference with a Known Normal Population

### Confidence Intervals for the Population Mean

When the population standard deviation  $\sigma$  is known, the confidence interval for the mean simplifies to:

$$\bar{X} \pm Z_{\alpha/2} \times \frac{\sigma}{\sqrt{n}}$$

where:

- $Z_{\alpha/2}$  is the critical value from the standard normal distribution corresponding to the desired confidence level (e.g., 1.96 for 95%).

Example: For a 95% confidence interval with  $\sigma$  known:

$$\text{CI} = \bar{X} \pm 1.96 \times \frac{\sigma}{5}$$

This interval provides a range that, with specified confidence, contains the true population mean.

## Hypothesis Testing

Suppose we want to test:

- Null hypothesis  $(H_0: \mu = \mu_0)$ ,
- Alternative hypothesis  $(H_a: \mu \neq \mu_0)$ .

Given the known  $\sigma$ , the test statistic is:

$$Z = \frac{\bar{X} - \mu_0}{\sigma / \sqrt{n}}$$

This Z-statistic follows the standard normal distribution under  $(H_0)$ . Critical values determine whether to reject  $(H_0)$ .

---

## Practical Applications and Considerations

### Quality Control and Manufacturing

In industries where product characteristics are normally distributed (e.g., thickness, weight), sampling 25 items provides a reliable estimate of the process mean. Known  $\sigma$  allows quick decision-making based on Z-tests and confidence intervals.

### Scientific Research

In experiments where prior calibration provides known population parameters, sampling 25 observations simplifies analysis, making the process efficient and statistically robust.

### Limitations and Assumptions

- Normality: The population must truly be normal; deviations can affect results.
- Known  $\sigma$ : In practice,  $\sigma$  is often unknown, requiring t-distribution-based methods.
- Randomness: The sample must be truly random to ensure validity.

---

## Extending the Analysis: When $\sigma$ Is Unknown

If  $\sigma$  is unknown, the sampling distribution of  $(\bar{X})$  follows a t-distribution with  $(n-1)$  degrees of freedom. The confidence intervals and hypothesis tests then become:

$$\bar{X} \pm t_{\alpha/2, n-1} \times \frac{s}{\sqrt{n}}$$

where  $(s)$  is the sample standard deviation. This adjustment accounts for the additional uncertainty introduced by estimating  $\sigma$ .

---

## Conclusion

The scenario of drawing a simple random sample of 25 observations from a normally distributed population with known parameters encapsulates a fundamental aspect of statistical inference. The normality assumption ensures that the sample mean's distribution is well-characterized, facilitating precise estimation and testing. Recognizing when the population parameters are known allows statisticians to employ Z-based methods, which are more straightforward and powerful than their t-based counterparts.

This foundational setup underpins numerous real-world applications across industries and sciences, from quality assurance to experimental research. Understanding the theoretical underpinnings, practical implications, and limitations of such sampling schemes is essential for accurate data analysis and sound decision-making.

In essence, the combination of simple random sampling, normal distribution assumptions, and known parameters creates a robust framework for making reliable inferences about the underlying population—highlighting the elegance and utility of classical statistical methods.

## **Simple Random Sample Of 25 Observations Is Derived From A Normally Distributed Population With A Known**

Simple Random Sample Of 25 Observations Is Derived From A Normally Distributed Population With A Known

## **Related Articles**

- [Tristan Has \\$1.40 Worth Of Nickels And Dimes. He Has Twice As Many Nickels As Dimes. Write A System Of](#)
- [The Figure Below Shows A Shaded Region And A Non-shaded Region. Angles In The Figure That Appear To Be](#)
- [Typically, The Ice Sheet Begins To Show Evidence Of Thawing In Late Summer, Following Several Weeks Of](#)

[Back to Home](#)