Sketch The Curve With The Given Vector Equation By Finding The Following Points. R(t) = (t, 3 - T, 2t)

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Understanding how to analyze and sketch curves defined by vector equations is a fundamental skill in vector calculus and analytic geometry. The given vector function, R(t) = (t, 3 - T, 2t), describes a space curve parametrized by the variable t. To sketch this curve accurately, it is essential to determine specific points on the curve corresponding to different values of t, analyze its behavior, and interpret its geometric properties.

In this comprehensive guide, we'll explore the step-by-step process of finding points on the curve, understanding the nature of the curve through its parametric equations, and ultimately sketching the curve with clarity. This process involves substituting various t-values, plotting points, analyzing the relationships among the coordinates, and understanding the geometric shape the curve forms in three-dimensional space.

Understanding the Vector Equation R(t) = (t, 3 - T, 2t)

Before diving into calculations, it's important to comprehend what the given vector equation represents.

Components of the Vector Equation

The vector function R(t) is expressed as:

- x-component: x(t) = t
- y-component: y(t) = 3 - T
- z-component: z(t) = 2t

This means for any value of t, the point on the curve can be found by substituting t into these component functions.

Parametric Equations Overview

The parametric equations derived from R(t) are:

1. x = t

2. y = 3 - t
3. z = 2t

These equations uniquely define the position of a point in 3D space as t varies over real numbers.

Finding Specific Points on the Curve

To sketch the curve effectively, start by selecting specific values of t and calculating the corresponding points R(t).

Choosing Values of t

Common practice involves choosing a range of t-values to observe the behavior of the curve:

- Negative t-values: t = -2, -1
- Zero: t = 0
- Positive t-values: t = 1, 2, 3

This sampling helps visualize the curve's direction and shape.

Calculating Points for Selected t-values

Let's compute R(t) for these values:

1. **t = -2**

   - x = -2
   - y = 3 - (-2) = 3 + 2 = 5
   - z = 2 (-2) = -4
   - **Point:** (-2, 5, -4)

2. **t = -1**

   - x = -1
   - y = 3 - (-1) = 3 + 1 = 4
   - z = 2 (-1) = -2
   - **Point:** (-1, 4, -2)

3. **t = 0**

   - x = 0
   - y = 3 - 0 = 3

- z = 0
- **Point:** (0, 3, 0)

4. **t = 1**

   - x = 1
   - y = 3 - 1 = 2
   - z = 2 1 = 2
   - **Point:** (1, 2, 2)

5. **t = 2**

   - x = 2
   - y = 3 - 2 = 1
   - z = 4
   - **Point:** (2, 1, 4)

6. **t = 3**

   - x = 3
   - y = 3 - 3 = 0
   - z = 6
   - **Point:** (3, 0, 6)

These points provide a set of locations through which the curve passes, forming the basis for sketching.

Analyzing the Geometric Nature of the Curve

Understanding the geometric properties of the curve helps in sketching and visualizing it accurately.

Relationship Between Components

From the parametric equations, observe:

- x = t
- y = 3 - t
- z = 2t

Express y and z in terms of x:

- Since t = x, then y = 3 - x
- And z = 2x

This allows us to eliminate t and get a direct relationship among x, y, and z.

Deriving the Cartesian Equation

Using the relations:

- $y = 3 - x$
- $z = 2x$

From $z = 2x$, we find $x = z/2$.

Substitute x into y:

- $y = 3 - x = 3 - (z/2)$

This gives the relation:

- $y = 3 - (z/2)$

Or equivalently:

- $2y = 6 - z$

Rearranged as:

- $z + 2y = 6$

This is a plane equation in 3D space.

Implication for the Curve

The parametric equations show that the curve lies on the plane defined by:

- $z + 2y = 6$

and that as t varies, the points trace a line within this plane, with x and z directly related, and y depending on x.

Sketching the Curve

Having identified specific points and understood the geometric relation, proceed to sketch the curve.

Steps to Sketch the Curve

1. Plot the points: Using the coordinates calculated for various t-values, plot these points in 3D space.

2. Connect the points smoothly: Since the parametric equations are linear in t, the points will form a smooth line.

3. Identify the plane: The curve lies on the plane $z + 2y = 6$, which can be visualized as a flat surface in space.

4. Determine the direction: As t increases, x increases, y decreases linearly, and z increases linearly, indicating the direction of the curve.

Visualizing the Curve in 3D

- The curve starts at $t = -2$ with $(-2, 5, -4)$, moves through various points, passing through $(0, 3, 0)$, and ends at $(3, 0, 6)$ for $t = 3$.
- The points suggest a straight line inclined within the plane $z + 2y = 6$.
- The slope and direction can be visualized by connecting these points with a smooth line.

Graphical Tools and Techniques

To accurately sketch the curve, you can use:

- Graphing software: GeoGebra 3D Graphing, Desmos (3D mode), or WolframAlpha.
- Manual sketching: Draw the coordinate axes, plot points, and connect them with a straight line, emphasizing the plane's orientation.

Additional Concepts and Tips for Sketching

Understanding the Direction of the Curve

- As t increases from negative to positive, x increases, y decreases, and z increases.
- The direction vector of the line is derived from the coefficients of t:

$$
\vec{d} = (1, -1, 2)
$$

- This vector indicates the direction in which the curve extends in space.

Parameter Range and Limits

- While t can be any real number, focusing on specific ranges helps in visualization.
- For example, t in [-2, 3] gives a clear segment of the curve.

Understanding the Curve's Behavior at Infinity

- As t approaches infinity, x and z tend to infinity, y tends to negative infinity.
- The curve extends infinitely along its direction, but for practical sketching, focus on a finite t-range.

Common Mistakes to Avoid

- Confusing the order of points; always verify calculations.
- Forgetting the relationship between components when eliminating variables.
- Overlooking the plane on which the curve lies.

Summary and Final Tips

- The key to sketching a curve defined by a vector equation is to find multiple points by substituting different t-values.
- Expressing the parametric equations in terms of Cartesian coordinates helps in understanding the geometric shape.
- Recognizing the plane on which the curve lies simplifies the visualization process.
- Using graphical tools enhances accuracy and understanding.

Final tip: Practice sketching such curves with different vector equations to develop intuition about their geometry and behavior in space.

Conclusion

Sketching the curve given by the vector equation R(t) = (t, 3 - T, 2t) involves systematically finding points for various t-values, understanding the relationships between the components, and visualizing the curve within its geometric context. By following these steps—calculating points, deriving Cartesian relations, and understanding the curve's direction—you can accurately depict the space curve and deepen your

Frequently Asked Questions

What is the vector equation given for sketching the curve?

The vector equation is R(t) = (t, 3 - t, 2t).

How do you find the point on the curve when t = 0?

Substitute t = 0 into R(t): R(0) = (0, 3 - 0, 0) = (0, 3, 0).

What is the point on the curve when t = 1?

Substitute t = 1 into R(t): R(1) = (1, 3 - 1, 21) = (1, 2, 2).

How do you find the point on the curve when t = -1?

Substitute t = -1 into R(t): R(-1) = (-1, 3 - (-1), 2(-1)) = (-1, 4, -2).

What is the significance of plotting these points when sketching the curve?

Plotting these points helps visualize the shape and orientation of the curve in three-dimensional space.

How can you determine the general trend or direction of the curve from these points?

By observing how the points change as t varies, noting that x = t, y = 3 - t, and z = 2t, indicating a linear relationship guiding the curve's direction.

Are there specific values of t that simplify the plotting process?

Yes, choosing values like t = -1, 0, 1, and 2 makes calculations straightforward and helps in accurately sketching the curve.

Additional Resources

Comprehensive Guide to Sketching the Curve Defined by the Vector Equation R(t) = (t, 3 - t, 2t)

Understanding how to visualize and sketch curves described by vector equations is a fundamental skill in vector calculus and analytical geometry. The given vector function \(R(t) = (t, 3 - t, 2t) \) offers a rich opportunity to explore the behavior of a parametric curve in three-dimensional space. In this detailed review, we will delve deeply into the process of analyzing, plotting, and interpreting this curve, emphasizing the importance of calculating specific points, understanding the underlying geometry, and employing both algebraic and visual techniques.

Understanding the Vector Equation $R(t) = (t, 3 - t, 2t)$

Before plotting the curve, it is crucial to comprehend what the vector equation represents.

The Components of the Vector Equation

The vector function $R(t)$ maps a real number t (the parameter) to a point in three-dimensional space with coordinates:

- $x(t) = t$
- $y(t) = 3 - t$
- $z(t) = 2t$

This means that as t varies over the real numbers, the point $R(t)$ traces out a specific path or curve in 3D space.

Significance of Each Component

- $x(t) = t$: The x-coordinate increases linearly with t.
- $y(t) = 3 - t$: The y-coordinate decreases linearly as t increases.
- $z(t) = 2t$: The z-coordinate increases linearly at twice the rate of t.

Analyzing the Curve: Key Steps

To sketch the curve effectively, we need to analyze it thoroughly. This involves:

- Calculating specific points for various values of t
- Understanding the relationship among the coordinates
- Identifying the nature of the curve (line, plane, space curve)
- Exploring the domain and range of the parameter t

Let's explore each aspect in detail.

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1. Calculating Points for Specific Values of t

Choosing strategic values of t helps in visualizing the path of the curve.

Sample calculations:

t	$x = t$	$y = 3 - t$	$z = 2t$	Point $R(t)$
-2	-2	5	-4	$(-2, 5, -4)$
-1	-1	4	-2	$(-1, 4, -2)$
0	0	3	0	$(0, 3, 0)$
1	1	2	2	$(1, 2, 2)$
2	2	1	4	$(2, 1, 4)$
3	3	0	6	$(3, 0, 6)$

Plotting these points reveals the general shape and orientation of the curve. For example:

- At $t = 0$, the point is at $(0, 3, 0)$.
- When t increases, x and z increase linearly, while y decreases.
- The points suggest a continuous, smooth path in space.

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2. Understanding the Relationship Among Coordinates

Examining the parametric equations, we can uncover relationships between x, y, and z.

Deriving relations:

- From $x(t) = t$, we have $t = x$.
- Substituting into $y(t) = 3 - t$, yields:
$$
y = 3 - x
$$
- Similarly, $z(t) = 2t = 2x$.

Implication:

- The y-coordinate is linearly related to x, with $y = 3 - x$.
- The z-coordinate is directly proportional to x, with $z = 2x$.

Combined relation:

- Eliminating t, the parametric curve satisfies:

$$
\boxed{
y = 3 - x,\quad z = 2x
}
$$

This describes a line in 3D space with a specific parametric form.

Identifying the Geometric Nature of the Curve

Based on the relations among x, y, and z, we can determine the geometric nature of the curve.

The Curve as a Line in Space

Since $y = 3 - x$ and $z = 2x$, the curve lies on the line:

$$
\text{Line } L: \quad y = 3 - x,\quad z = 2x
$$

or equivalently, parameterized as:

$$
\begin{cases}
x = t \\
y = 3 - t \\
z = 2t
\end{cases}
$$

which matches the original vector function.

Equation of the Line

Expressed in symmetric form:

$$
\text{From } y: y = 3 - x \Rightarrow x = 3 - y
$$
$$
z = 2x
$$

Or, in parametric form:

$$
x = t, \quad y = 3 - t, \quad z = 2t
$$

This confirms that the curve is a straight line in 3D space, passing through points corresponding to various t.

Visualizing the Line in Space

To sketch the line effectively, identify key points and the direction vector.

Direction Vector

The vector indicating the direction of the line is:

$$
\vec{d} = \frac{dR}{dt} = (1, -1, 2)
$$

This vector points along the line, indicating how the position changes with respect to t.

Points on the Line

Using specific values of t, as previously calculated:

- For $t = 0$: $(0, 3, 0)$
- For $t = 1$: $(1, 2, 2)$
- For $t = -1$: $(-1, 4, -2)$

Plotting these points and drawing the line through them provides a clear visualization.

Sketching the Curve: Step-by-Step Approach

Now that we've established the curve is a line, the sketching process becomes systematic.

Step 1: Plot Key Points

- Mark the points for selected t values:
- $(-2, 5, -4)$
- $(-1, 4, -2)$
- $(0, 3, 0)$
- $(1, 2, 2)$
- $(2, 1, 4)$
- $(3, 0, 6)$

Step 2: Draw the Line

- Connect the points smoothly, noting the linear progression.
- Indicate the direction with arrows, showing increasing t.

Step 3: Understand the Orientation

- The line passes through the point $(0, 3, 0)$ when $t=0$.

- The line extends infinitely in both directions, but for sketching purposes, focus on a segment that captures the curve’s trend.

Step 4: Annotate the Graph

- Label key points and the direction vector.
- Indicate the relation $y = 3 - x$ and $z = 2x$.

Additional Insights and Advanced Considerations

While the curve is a line, further exploration can deepen understanding.

1. Parameter Domain:

- Typically, t can range over all real numbers, meaning the line extends infinitely in both directions.
- For practical sketching, select a finite interval, e.g., $t \in [-2, 3]$.

2. Line Equation in Symmetric Form:

- From the parametric form:

$$
\frac{x - 0}{1} = \frac{y - 3}{-1} = \frac{z - 0}{2}
$$

- This symmetric form describes the line explicitly and aids in visualization.

3. Relationship to Planes:

- The line lies at the intersection of the planes:

$$
\begin{cases}
y + x = 3 \\
z = 2x
\end{cases}
$$

- Recognizing these can help in 3D plotting software or physical models.

Practical Tips for Sketching the Curve in 3D