Sketch The Region R In The Xy-plane Bounded By The Lines X = 0, Y = 0 And X+3y=3. Let S Be The Portion

Sketch The Region R In The Xy-plane Bounded By The Lines X = 0, Y = 0 And X+3y=3. Let S Be The Portion of the plane defined by these boundaries. Understanding how to visualize and interpret regions in the XY-plane is fundamental in calculus, especially for setting up integrals, analyzing areas, and understanding the geometric aspects of functions. In this comprehensive guide, we will explore how to accurately sketch the region R, analyze its properties, and understand the significance of the portion S within this region. Whether you're a student preparing for calculus exams or a teacher designing instructional material, this article will provide detailed insights into this particular problem.

Understanding the Boundaries of Region R

Before sketching the region, it is crucial to understand the equations that define its boundaries. The problem specifies three lines:

- X = 0
- Y = 0
- X + 3Y = 3

Each of these lines forms part of the boundary of the region R. Let's analyze each one individually to understand the shape and extent of the region.

The Line X = 0

This is the y-axis, a vertical line where the x-coordinate is always zero. It extends infinitely in both positive and negative y-directions, but since our region is bounded, we'll focus on the relevant portion.

The Line Y = 0

This is the x-axis, a horizontal line where the y-coordinate is zero. Like the y-axis, it extends infinitely in both directions, but within the region, only the portion relevant to the intersection with the other lines is considered.

The Line X + 3Y = 3

This is a straight line with a negative slope, which can be rewritten in slope-intercept form as:

$$ Y = -\frac{1}{3}X + 1 $$

This line intersects the axes at:

- When $X=0$, $Y=1$; so it crosses the y-axis at (0, 1).

- When $Y=0$, $X=3$; so it crosses the x-axis at (3, 0).

Understanding these points is essential for plotting the line accurately.

Steps to Sketch Region R in the XY-plane

To sketch the region R bounded by these lines, follow these systematic steps:

Step 1: Plot the Coordinate Axes

Begin by drawing the x- and y-axes to establish the coordinate plane. Mark units evenly along both axes.

Step 2: Draw the Lines X=0 and Y=0

- Plot the y-axis (x=0), which is the line X=0.
- Plot the x-axis (y=0), which is the line Y=0.

These two lines intersect at the origin (0,0).

Step 3: Draw the Line X + 3Y = 3

- Plot the intercepts:
- At $X=0$, $Y=1$. Plot the point (0,1).
- At $Y=0$, $X=3$. Plot the point (3,0).
- Connect these points with a straight line. This line will slope downward, crossing the y-axis at (0,1) and the x-axis at (3,0).

Step 4: Identify the Intersection Points Forming the Region

The region R is enclosed by the three lines:

- The x-axis (Y=0),
- The y-axis (X=0),
- The line $X + 3Y=3$.

The vertices of the region are at:

1. The origin (0,0), where the x-axis and y-axis intersect.
2. The point where $X + 3Y=3$ intersects the y-axis, at (0,1).
3. The point where $X + 3Y=3$ intersects the x-axis, at (3,0).

Plot these points carefully.

Step 5: Shade the Region R

- The region R is the triangle formed by these three points:

- (0,0)
- (0,1)
- (3,0)

- Shade or outline this triangle to visualize the region.

Understanding the Portion S of the Region R

Once the entire region R is sketched, the next step is to focus on the specific portion S of the region. The problem states: "Let S be the portion"—usually, in calculus contexts, this refers to a particular subregion or a region of interest within R. For example, S could be:

- A subset of R where certain inequalities are satisfied.
- The region under a specific curve within R.
- A region corresponding to a particular range of x or y.

Since the original statement is incomplete, we can consider common scenarios such as:

- S being the region under the line $x + 3y = 3$ between x=0 and x=3.
- S being the region between the coordinate axes and the line $x + 3y=3$.

In many calculus problems, S typically refers to the specific area or region over which an integral is to be evaluated.

Calculating the Area of Region R

Understanding the area of R and the portion S is essential for applications such as calculating integrals, probabilities, or physical quantities.

Using Geometric Methods

Since R is a triangle with vertices at (0,0), (0,1), and (3,0), its area can be found using geometry:

$$
\text{Area} = \frac{1}{2} \times \text{base} \times \text{height}
$$

- Base: Along the x-axis from (0,0) to (3,0), length = 3.
- Height: Along the y-axis from (0,0) to (0,1), length = 1.

Thus,

$$
\text{Area of R} = \frac{1}{2} \times 3 \times 1 = \frac{3}{2} = 1.5
$$

If S is a subset of R, its area can be calculated similarly depending on its boundaries.

Setting Up Integrals Over Region R

In calculus, a common task is to evaluate double integrals over a region like R or S. Here's how to set up the integral:

Choosing the Order of Integration

- Integrating with respect to y first (dy dx):
For a fixed x, y varies between 0 and the line $Y = -\frac{1}{3}X + 1$.

- Integrating with respect to x first (dx dy):
For a fixed y, x varies from 0 to $X=3-3Y$.

Example: Double Integral Over R

To compute the area, the integral can be set up as:

$$
\text{Area} = \int_{x=0}^{3} \int_{y=0}^{-\frac{1}{3}x + 1} dy \, dx
$$

or

$$
\text{Area} = \int_{y=0}^{1} \int_{x=0}^{3-3y} dx \, dy
$$

Both integrals yield the same result, confirming the region's area.

Applications and Significance of Sketching Regions Like R

Understanding how to sketch and analyze such regions is fundamental in multiple areas:

- Calculating areas and volumes via double and triple integrals.
- Setting up integrals in multivariable calculus.
- Analyzing the geometric properties of functions over specific domains.
- Solving optimization problems constrained within certain regions.

Additionally, visualizing the region helps in understanding the limits of integration and simplifies the process of solving complex calculus problems.

Conclusion: Mastering the Sketching of Regions in the XY-plane

Sketching the region R bounded by the lines \(x=0\), \(y=0\), and \(x + 3y=3\) is an essential skill in calculus. It involves understanding the boundary equations, plotting key points, drawing the lines accurately, and shading the enclosed area. Recognizing the vertices and the shape of the region enables precise setup of integrals, calculation of areas, and further analysis.

The portion S, depending on the context, can be a specific part of this region used for targeted calculations or analyses. Mastery of these techniques enhances your ability to solve a wide range of problems in mathematics, physics, engineering, and related fields.

Remember, practice is key. Try sketching similar regions bounded by different lines and curves to strengthen your understanding. With consistent effort, you'll develop an intuitive grasp of how to interpret and work with regions in the XY-plane effectively.

Frequently Asked Questions

What are the boundary lines defining region R in the xy-plane?

The region R is bounded by the lines $x = 0$, $y = 0$, and the line $x + 3y = 3$.

How do you find the intercepts of the line $x + 3y = 3$?

The x-intercept is at (3, 0) when $y=0$, and the y-intercept is at (0, 1) when $x=0$.

What is the shape of the region R bounded by these lines?

Region R is a right triangle with vertices at (0, 0), (3, 0), and (0, 1).

How can I sketch the region R in the xy-plane?

Plot the points (0,0), (3,0), and (0,1), then draw the lines $x=0$, $y=0$, and $x + 3y=3$ to form the triangle.

What are the limits of integration if I want to integrate over region R?

For x from 0 to 3, y varies from 0 up to $(3 - x)/3$.

How do I set up the double integral over region R?

The double integral can be set up as:
$\int_0^3 \int_0^{(3 - x)/3} f(x, y)\, dy\, dx$.

What is the significance of the portion S in the problem?

S represents the specific portion or sub-region within R being considered for integration or analysis.

Can you explain how to find the area of region R?

Yes, the area of R is half the area of the triangle with base 3 and height 1, so Area = ½ 3 1 = 1.5 square units.

How does understanding the boundary lines help in setting up integrals over

R?

Knowing the boundary lines allows you to determine the limits of integration for x and y, ensuring accurate calculation over the region.

Is the region R convex, and why is that important?

Yes, R is convex because it is a triangle formed by straight lines, which simplifies integration and geometric calculations.

Additional Resources

Sketch The Region R In The XY-plane Bounded By The Lines X = 0, Y = 0 And X + 3y = 3. Let S Be The Portion of this region.

Understanding the geometry of the region R in the XY-plane defined by these particular boundaries is fundamental for various applications in calculus, physics, and engineering. This comprehensive review aims to explore the characteristics, visualization, and implications of the region R, as well as the associated portion S, through detailed analysis and step-by-step breakdowns.

Introduction to the Region R

The first step in analyzing the region R is to understand its boundaries clearly. The problem specifies that R is bounded by three lines:
- The vertical line $x = 0$ (the y-axis)
- The horizontal line $y = 0$ (the x-axis)
- The line $x + 3y = 3$

Visualizing these boundaries provides the foundation for further analysis.

Understanding the Boundaries

- Line $x = 0$: This is the y-axis, extending infinitely in the positive and negative y-directions.
- Line $y = 0$: This is the x-axis, extending infinitely in both directions along the x-axis.
- Line $x + 3y = 3$: This line can be rewritten as $y = (3 - x)/3$, which is a straight line with a negative slope of $-1/3$ and a y-intercept at $y = 1$ when $x = 0$.

By combining these boundaries, the region R is enclosed in the first quadrant, bounded on the left by the y-axis, below by the x-axis, and above/right by the sloped line $x + 3y = 3$.

Graphical Representation of the Region R

Visualizing R helps in understanding its shape, size, and the relation between the boundaries.

Plotting the Boundaries

- The y-axis ($x=0$): Vertical line at $x=0$.
- The x-axis ($y=0$): Horizontal line at $y=0$.
- The line $x + 3y = 3$: To plot, find intercepts:
- When $x=0$, $y=1$ (point $(0,1)$)
- When $y=0$, $x=3$ (point $(3,0)$)

Plotting these points and drawing the corresponding lines creates a triangle with vertices at $(0,0)$, $(0,1)$, and $(3,0)$.

Shape and Area of the Region

The region R is a right triangle with these vertices:
- Vertex A: $(0,0)$ (origin)
- Vertex B: $(0,1)$ (on the y-axis)
- Vertex C: $(3,0)$ (on the x-axis)

The hypotenuse is the segment between B and C, lying along the line $x + 3y = 3$.

Area Calculation:
The base can be taken as the segment along the x-axis from $(0,0)$ to $(3,0)$: length 3.
The height is the segment along the y-axis from $(0,0)$ to $(0,1)$: length 1.

Thus, the area of R is:
$$ \text{Area} = \frac{1}{2} \times \text{base} \times \text{height} = \frac{1}{2} \times 3 \times 1 = 1.5 $$

Mathematical Description of the Region

To analyze R mathematically, it is essential to describe it as a set of points (x,y) satisfying certain inequalities.

Set of Inequalities

Given the boundaries:
- $x \ge 0$ (right of y-axis)
- $y \ge 0$ (above x-axis)
- $x + 3y \le 3$ (below the line $x + 3y = 3$)

Therefore, the region R can be expressed as:
$$ R = \{ (x,y) \mid x \ge 0, y \ge 0, x + 3y \le 3 \} $$

This description is vital for setting up integrals or for computational purposes.

Bounding Functions

- The lower bounds are simply $x=0$ and $y=0$.
- The upper bound for y, for given x, can be derived from the line:

$$ y \le \frac{3 - x}{3} $$

This allows integrating with respect to y first, then x, or vice versa, depending on the problem context.

Sketching the Region R

Creating an accurate sketch of R involves plotting the axes and the line, then shading the enclosed triangular area.

Step-by-step Sketching Process

1. Draw the coordinate axes (x and y).
2. Mark the intercepts of the line $x + 3y = 3$: (0,1) and (3,0).
3. Draw the line connecting these points.
4. Shade the area bounded between the axes and this line in the first quadrant.

This visual aid helps in understanding the region's shape and is crucial for visual learners.

Applications and Further Analysis of Region R

Understanding the region R is not just an academic exercise; it has practical implications in multiple fields.

Calculating Area and Integrals

- The area of R has been computed as 1.5, which can act as a baseline for probability density functions or physical measurements.
- Double integrals over R can be set up using the inequalities for x and y, enabling calculations of mass, volume, or other quantities where density functions are involved.

Setting Up Double Integrals

Given the description:
$$ \text{Area} = \iint_R dA $$

One way to express this integral is:
$$ \int_{x=0}^{3} \int_{y=0}^{\frac{3 - x}{3}} dy\, dx $$

Alternatively, for integration with respect to x:
$$ \int_{y=0}^{1} \int_{x=0}^{3 - 3y} dx\, dy $$

These setups are essential for calculating moments, masses, or probabilities over the region.

Advantages and Limitations of the Region R

Understanding the features of this region helps in appreciating its utility and potential limitations.

Pros / Features

- Simplicity: The region is a straightforward triangle in the first quadrant, making calculations manageable.
- Clear Boundaries: Line equations provide explicit bounds for integration.
- Visualization Ease: Easy to sketch and interpret geometrically.
- Applicability: Useful in problems involving linear constraints, optimization, and probability.

Cons / Limitations

- Limited Complexity: The region's simplicity may not suit problems requiring more intricate boundaries.
- Restrictive Domain: Confined to the first quadrant; extensions to other quadrants require adjustments.
- Linearity Constraint: The boundaries are linear; non-linear boundaries require different approaches.

Extensions and Related Topics

Beyond the initial sketch and analysis, there are several avenues for further exploration.

Transforming the Region

- Applying coordinate transformations can simplify integration or analyze the region under different perspectives.
- For example, substitution methods like $u = x + 3y$ can linearize boundaries or facilitate change of variables.

Integrating Over the Region

- Calculating moments, center of mass, or centroid:

$$\bar{x} = \frac{1}{\text{Area}} \iint_R x\, dA$$

$$\bar{y} = \frac{1}{\text{Area}} \iint_R y\, dA$$

- Evaluating functions over R: For example, integrating $f(x,y) = xy$ over R.

Real-World Applications

- Physics: Determining regions of influence or feasible regions in mechanics.
- Economics: Modeling constraints in resource allocation.
- Engineering: Analyzing stress regions within bounded materials.

Conclusion

The region R in the XY-plane, bounded by x=0, y=0, and x + 3y=3, exemplifies fundamental concepts of coordinate geometry and calculus. Its triangular shape, clear boundaries, and straightforward mathematical description make it an ideal case study for understanding planar regions, setting up integrals, and visualizing geometric constraints. This comprehensive analysis underscores its educational value and practical relevance, providing a solid foundation for more complex applications involving regions bounded by lines or curves. With tools like graphing, inequalities, and integral setup, students and professionals alike can leverage the properties of R for diverse mathematical and real-world problem-solving tasks.

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