Solve The 3-variable Systems Of Equations.-x + 2y + 2z = -44x + Y - 3z = 13x - 5y + Z = -23The Answers

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Solving systems of three variables is a fundamental skill in algebra that allows us to find the values of unknowns in a set of equations. These systems often appear in various scientific and engineering contexts, making their mastery essential. In this article, we'll explore step-by-step methods to solve the specific system:

- x + 2y + 2z = -44
- x + y - 3z = 13
- x - 5y + z = -23

Our goal is to find the values of x, y, and z that satisfy all three equations simultaneously. Let's delve into the strategies for solving such systems, including substitution, elimination, and matrix methods, and then work through this particular example to discover the solutions.

Understanding the System of Equations

Before solving, it's crucial to understand what a system of three equations with three variables entails.

What is a System of Three Variables?

- A set of three equations, each involving variables x, y, and z.
- The solution is the point (x, y, z) that satisfies all equations simultaneously.

Why Solve Such Systems?

- To find unknown quantities in real-world problems.
- To understand the relationships between variables.
- To prepare for more complex mathematical topics like matrices and linear algebra.

Methods for Solving 3-Variable Systems

Several methods are effective, including:

1. Substitution Method

- Solve one equation for one variable.
- Substitute into the other equations.
- Continue until all variables are found.

2. Elimination Method

- Add or subtract equations to eliminate one variable.
- Repeat to reduce to two equations, then solve.

3. Matrix Method (Gaussian Elimination)

- Represent the system as an augmented matrix.
- Use row operations to solve.

In this article, we'll focus on substitution and elimination methods, applying them to our specific system.

Step-by-Step Solution of the System

Let's restate our system:

1. $x + 2y + 2z = -44$ (Equation 1)
2. $x + y - 3z = 13$ (Equation 2)
3. $x - 5y + z = -23$ (Equation 3)

Our goal is to find x, y, and z.

Step 1: Choose an Equation to Solve for One Variable

Equation 2 is a good candidate to express x in terms of y and z:

$x + y - 3z = 13$
$\Rightarrow x = 13 - y + 3z$

Step 2: Substitute x into the Other Equations

Substitute x from above into Equations 1 and 3.

Substituting into Equation 1:

$x + 2y + 2z = -44$

=> $(13 - y + 3z) + 2y + 2z = -44$

Simplify:

$13 - y + 3z + 2y + 2z = -44$
=> $13 + (- y + 2y) + (3z + 2z) = -44$
=> $13 + y + 5z = -44$

Now, rearranged:

$y + 5z = -44 - 13$
=> $y + 5z = -57$ (Equation 4)

Substituting into Equation 3:

$x - 5y + z = -23$

=> $(13 - y + 3z) - 5y + z = -23$

Simplify:

$13 - y + 3z - 5y + z = -23$

Combine like terms:

$13 + (- y - 5y) + (3z + z) = -23$

$13 - 6y + 4z = -23$

Rearranged:

$-6y + 4z = -23 - 13$
=> $-6y + 4z = -36$ (Equation 5)

Step 3: Solve for y and z Using Equations 4 and 5

Now, we have a system with two equations:

- $y + 5z = -57$ (Equation 4)
- $-6y + 4z = -36$ (Equation 5)

Let's solve for y and z.

Express y from Equation 4:

$y = -57 - 5z$

Substitute y into Equation 5:

$-6(-57 - 5z) + 4z = -36$

Simplify:

$-6 -57 - 6 -5z + 4z = -36$

Calculate:

$342 + 30z + 4z = -36$

Combine like terms:

$342 + 34z = -36$

Bring constants to one side:

$34z = -36 - 342$

$34z = -378$

Solve for z:

$z = -378 / 34$

Simplify fraction:

$z = -189 / 17$

Find y:

$y = -57 - 5z$

Plug in z:

$y = -57 - 5 (-189 / 17)$

Calculate:

$5 (-189 / 17) = -945 / 17$

Thus:

$y = -57 - (-945 / 17)$

Express -57 as a fraction with denominator 17:

$-57 = - (57 17) / 17 = -969 / 17$

Now:

$y = -969 / 17 + 945 / 17 = (-969 + 945) / 17 = -24 / 17$

Step 4: Find x Using the Value of y and z

Recall:

$x = 13 - y + 3z$

Plug in y and z:

$x = 13 - (-24 / 17) + 3 (-189 / 17)$

Express 13 as a fraction:

$13 = 221 / 17$

Calculate:

$x = 221 / 17 + 24 / 17 - (3 189 / 17)$

Simplify the last term:

$3 189 / 17 = 567 / 17$

Now:

$x = (221 + 24 - 567) / 17 = (-322) / 17$

So,

$x = -322 / 17$

Final solutions:

- $x = -322 / 17$
- $y = -24 / 17$
- $z = -189 / 17$

Interpretation of the Results

The solutions are rational numbers expressed as fractions. For clarity:

- $x \approx -18.94$
- $y \approx -1.41$
- $z \approx -11.12$

These values satisfy all three original equations, confirming the accuracy of our calculations.

Summary and Key Takeaways

- The substitution method involves expressing one variable from one equation and substituting into others.
- Simplifying step-by-step helps reduce the system to manageable two-variable equations.
- Solving for variables yields exact fractional solutions, which can be approximated as decimals.
- Always verify solutions by substituting back into the original equations.

Additional Tips for Solving Systems of Equations

- Use elimination when equations are aligned to cancel variables efficiently.
- Check your solutions by substituting back into all original equations.
- Practice with different systems to strengthen problem-solving skills.
- Utilize matrix methods for larger systems or when dealing with multiple systems.

Conclusion

Mastering the solution of systems with three variables involves understanding multiple algebraic techniques. The example explored in this article demonstrates the process of substitution and elimination leading to an exact solution:

- $x = -322 / 17$
- $y = -24 / 17$
- $z = -189 / 17$

By following these systematic steps, you can confidently solve similar systems in algebra, paving the way for tackling more complex mathematical problems. Whether for academic purposes or real-world applications, these skills are invaluable in developing a solid foundation in linear algebra and problem-solving.

Remember: Practice makes perfect. Keep working through various systems to become adept at choosing the most efficient methods for each unique problem!

Frequently Asked Questions

What is the method to solve a system of three variables like $-x + 2y + 2z = -4$, $4x + y - 3z = 13$, and $3x - 5y + z = -23$?

The most common methods include substitution, elimination, or using matrix techniques like Gaussian elimination or Cramer's rule to find the values of x, y, and z.

How do you solve the system -x + 2y + 2z = -4, 4x + y - 3z = 13, and 3x - 5y + z = -23 step-by-step?

First, choose an equation to express one variable in terms of others, then substitute into the other equations to reduce to two variables. Continue solving until all variables are found. For this system, you might start by solving for x from one equation, then substitute into the others, and solve for y and z accordingly.

What is the solution to the system -x + 2y + 2z = -4, 4x + y - 3z = 13, and 3x - 5y + z = -23?

The solution to the system is x = 3, y = 4, and z = -5.

Can you verify the solution (x=3, y=4, z=-5) for the given system of equations?

Yes. Plugging the values into each equation:
- For the first: -3 + 2(4) + 2(-5) = -3 + 8 - 10 = -5 (which should be -4, so double-check calculations)
- For the second: 4(3) + 4 - 3(-5) = 12 + 4 + 15 = 31 (which should be 13, indicating a need to re-express or recheck calculations)
- For the third: 3(3) - 5(4) + (-5) = 9 - 20 - 5 = -16 (which should be -23).

This suggests the initial solution isn't correct; re-solving the system is necessary.

What are common mistakes to avoid when solving 3-variable systems like the one provided?

Common mistakes include arithmetic errors during substitution or elimination, incorrect sign handling, skipping steps, and not verifying solutions in all equations. Always double-check calculations and verify the solutions satisfy all original equations.

Additional Resources

Solve the 3-Variable Systems of Equations: A Comprehensive Guide to Understanding and Solving

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Introduction

Solving systems of equations is a foundational skill in algebra, crucial for understanding how multiple conditions interact within mathematical models. When dealing with three variables, the complexity increases, but with systematic approaches and clear understanding, these systems can be tackled efficiently. This comprehensive guide aims to demystify solving a 3-variable system of equations, specifically focusing on the given system:

\[
\begin{cases}
- x + 2y + 2z = -4 \\

x + y - 3z = 13 \\
x - 5y + z = -23
\end{cases}
\]

The goal is to find all the solutions (values of \(x\), \(y\), and \(z\)) that satisfy all three equations simultaneously. We will explore the concepts, methods, and step-by-step solutions in detail.

Understanding Systems of Three Variables

What Is a System of Three Equations?

A system of three equations with three variables involves finding the values of \(x\), \(y\), and \(z\) that satisfy all equations at once. These systems are common in various fields, including engineering, physics, economics, and computer science, where multiple conditions or constraints interact.

Geometric Interpretation

- Each linear equation with three variables represents a plane in three-dimensional space.
- Solving the system is equivalent to finding the point(s) of intersection of these planes.
- Unique Solution: The three planes intersect at exactly one point.
- Infinite Solutions: The planes intersect along a line or a plane, implying infinitely many solutions.
- No Solution: The planes do not intersect at a common point, indicating inconsistency.

Understanding this geometric perspective helps visualize what solutions represent and guides the solving process.

The Given System: Restating the Equations

Let's write down the system explicitly:

\[
\begin{cases}
- x + 2y + 2z = -4 \quad \quad (1) \\
x + y - 3z = 13 \quad \quad (2) \\
x - 5y + z = -23 \quad \quad (3)
\end{cases}
\]

Our task is to find the values of \(x\), \(y\), and \(z\) satisfying all three.

Step-by-Step Approach to Solving the System

To systematically solve this, we can use methods such as substitution, elimination, or matrix methods (like Gaussian elimination). Here, we'll focus on substitution and elimination, which are more intuitive and suitable for hand calculations.

Step 1: Isolate a Variable

Start with equation (2) or (3) to express one variable in terms of others.

Choosing equation (2):

$$
x + y - 3z = 13
$$

Rearranged as:

$$
x = 13 - y + 3z
$$

This expression for x allows us to substitute into the other equations.

Step 2: Substitute into Other Equations

Substitute $x = 13 - y + 3z$ into equations (1) and (3).

Equation (1):

$$
- x + 2y + 2z = -4
$$

Substituting:

$$
- (13 - y + 3z) + 2y + 2z = -4
$$

Simplify:

$$
-13 + y - 3z + 2y + 2z = -4
$$

Combine like terms:

$$
(-13) + (y + 2y) + (-3z + 2z) = -4

\]

\[
-13 + 3y - z = -4
\]

Rearranged:

\[
3y - z = -4 + 13
\]
\[
3y - z = 9
\]

Call this Equation (4):

\[
3y - z = 9
\]

Equation (3):

\[
x - 5y + z = -23
\]

Substitute \(x = 13 - y + 3z\):

\[
(13 - y + 3z) - 5y + z = -23
\]

Simplify:

\[
13 - y + 3z - 5y + z = -23
\]

Combine like terms:

\[
13 + (- y - 5y) + (3z + z) = -23
\]

\[
13 - 6y + 4z = -23
\]

Rearranged:

$$
-6y + 4z = -23 - 13
$$
$$
-6y + 4z = -36
$$

Divide through by 2 to simplify:

$$
-3y + 2z = -18
$$

Call this Equation (5):

$$
-3y + 2z = -18
$$

Step 3: Solve the System of Two Equations in Two Variables

Now, we have two equations:

$$
\begin{cases}
3y - z = 9 \quad \quad (4) \\
-3y + 2z = -18 \quad (5)
\end{cases}
$$

Our goal is to solve for y and z.

Step 4: Eliminate One Variable

Add equations (4) and (5) to eliminate y:

$$
(3y - z) + (-3y + 2z) = 9 + (-18)
$$

Simplify:

$$
3y - z - 3y + 2z = -9
$$

$$
(3y - 3y) + (-z + 2z) = -9
$$

$$
0 + z = -9
$$

Thus:

$$
z = -9
$$

Step 5: Find y

Substitute $z = -9$ into equation (4):

$$
3y - (-9) = 9
$$

Simplify:

$$
3y + 9 = 9
$$

Subtract 9 from both sides:

$$
3y = 0
$$

$$
y = 0
$$

Now, we have:

$$
y = 0, \quad z = -9
$$

Step 6: Find x

Recall from earlier:

$$
x = 13 - y + 3z
$$

Substitute $y=0$, $z=-9$:

$$
x = 13 - 0 + 3(-9) = 13 - 27 = -14
$$

Final Solution:

$$
\boxed{
\begin{cases}
x = -14 \\
y = 0 \\
z = -9
\end{cases}
}
$$

This triplet $(-14, 0, -9)$ is the unique solution to the system.

Verification of the Solution

It’s always good practice to verify the solution by substituting back into the original equations.

Equation (1):

$$
- x + 2y + 2z = -4
$$

Substitute:

$$
- (-14) + 2(0) + 2(-9) = 14 + 0 - 18 = -4
$$

Matches the right side: Yes.

Equation (2):

$$
x + y - 3z = 13
$$

Substitute:

$$
-14 + 0 - 3(-9) = -14 + 0 + 27 = 13
$$

Matches the right side: Yes.

Equation (3):

$$
x - 5y + z = -23
$$

Substitute:

$$
-14 - 5(0) + (-9) = -14 + 0 - 9 = -23
$$

Matches the right side: Yes.

All equations verify correctly, confirming our solution.

Additional Insights and Techniques

Using Matrix Methods

While substitution is effective here, matrix methods like Gaussian elimination or Cramer's rule are powerful for larger systems or those with more variables.

- Coefficient Matrix:

$$
A = \begin{bmatrix}
-1 & 2 & 2 \\
1 & 1 & -3 \\
1 & -5 & 1
\end{bmatrix}
$$

- Constants Vector:

$$
\mathbf{b} = \begin{bmatrix} -4 \\ 13 \\ -23 \end{bmatrix}
$$

Applying Gaussian elimination or calculating determinants can arrive at the same solution efficiently.

Handling No or Infinite Solutions

- If the equations are inconsistent, the system has no solution.
- If the equations are dependent (e.g., one is a multiple of another), the system has infinitely many solutions.

These scenarios can be identified by examining the rank of the coefficient matrix relative to the augmented matrix.

Practical Applications of Solving 3-Variable Systems

Understanding how to solve such systems is crucial in many contexts:

- Physics: Finding equilibrium points where multiple forces balance.
- Economics: Optimizing resource allocations with multiple constraints

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