Solve The Given System Of Equations Using Either Gaussian Or Gauss-Jordan Elimination. (If There Is No

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Solving systems of equations is a fundamental task in algebra, essential for various fields such as engineering, physics, computer science, and mathematics. When dealing with multiple equations involving multiple variables, methods like Gaussian elimination and Gauss-Jordan elimination provide systematic approaches to find solutions efficiently. This comprehensive guide will introduce you to both techniques, explain their differences, and demonstrate step-by-step procedures for solving systems of equations. Whether you're a student looking to understand the core concepts or a professional seeking a refresher, this article aims to clarify these methods with detailed explanations and illustrative examples.

Understanding Systems of Equations

Before diving into elimination methods, it's important to understand what a system of equations entails.

What Is a System of Equations?

A system of equations consists of two or more equations with the same set of variables. The goal is to find the values of these variables that satisfy all equations simultaneously.

For example, consider the system:

$$
\begin{cases}
2x + 3y = 8 \\
x - y = 1
\end{cases}
$$

Here, the variables are x and y. The solutions are the pairs (x, y) that make both equations true.

Types of Solutions

Depending on the system, solutions can be:

- **Unique solution:** Exactly one set of variable values satisfies all equations.
- **Infinite solutions:** The system is dependent, with infinitely many solutions.
- **No solution:** The equations are inconsistent and have no common solution.

Understanding the nature of the solutions helps in choosing the appropriate solving method.

Gaussian Elimination Method

Gaussian elimination is a systematic procedure that transforms the system's augmented matrix into an upper triangular form, facilitating straightforward back substitution to find solutions.

Step-by-Step Procedure

1. Write the augmented matrix of the system.
2. Use row operations (swap, multiply, add/subtract rows) to create zeros below the leading coefficient (pivot) in the first column.
3. Repeat for subsequent columns, reducing the matrix to upper triangular form.
4. Perform back substitution to solve for the variables.

Illustrative Example

Solve the system:

\[
\begin{cases}
x + 2y + z = 6 \\
2x + 3y + 3z = 14 \\
-x + y + 2z = 5
\end{cases}
\]

Step 1: Write the augmented matrix:

\[
\begin{bmatrix}
1 & 2 & 1 & | & 6 \\
2 & 3 & 3 & | & 14 \\
-1 & 1 & 2 & | & 5

\end{bmatrix}
\]

Step 2: Zero out below the first pivot (row 1, column 1):

- Row 2: \(R_2 - 2 R_1 \):

\[
(2 - 2 \times 1, 3 - 2 \times 2, 3 - 2 \times 1, 14 - 2 \times 6) = (0, -1, 1, 2)
\]

- Row 3: \(R_3 + R_1 \):

\[
(-1 + 1, 1 + 2, 2 + 1, 5 + 6) = (0, 3, 3, 11)
\]

Updated matrix:

\[
\begin{bmatrix}
1 & 2 & 1 & | & 6 \\
0 & -1 & 1 & | & 2 \\
0 & 3 & 3 & | & 11
\end{bmatrix}
\]

Step 3: Zero below the second pivot (row 2, column 2):

- Make pivot positive: multiply row 2 by -1:

\[
R_2: -1 \times R_2 \Rightarrow (0, 1, -1, -2)
\]

- Eliminate below: \(R_3 - 3 R_2 \):

\[
(0 - 0, 3 - 3 \times 1, 3 - 3 \times (-1), 11 - 3 \times (-2)) = (0, 0, 6, 17)
\]

Updated matrix:

\[
\begin{bmatrix}
1 & 2 & 1 & | & 6 \\
0 & 1 & -1 & | & -2 \\
0 & 0 & 6 & | & 17
\end{bmatrix}
\]

Step 4: Back substitution:

- From row 3: $6z = 17 \Rightarrow z = \frac{17}{6}$

- From row 2: $y - z = -2 \Rightarrow y = -2 + z = -2 + \frac{17}{6} = -2 + \frac{17}{6} = -\frac{12}{6} + \frac{17}{6} = \frac{5}{6}$

- From row 1: $x + 2 y + z = 6$

$$
x = 6 - 2 y - z = 6 - 2 \times \frac{5}{6} - \frac{17}{6} = 6 - \frac{10}{6} - \frac{17}{6} = 6 - \frac{27}{6} = 6 - 4.5 = 1.5
$$

Final Solution:

$$
x = 1.5, \quad y = \frac{5}{6}, \quad z = \frac{17}{6}
$$

Gauss-Jordan Elimination Method

Gauss-Jordan elimination extends Gaussian elimination by transforming the matrix into reduced row echelon form (RREF), where the leading coefficients are 1 and all other entries in the pivot columns are zero. This method provides direct solutions without back substitution.

Step-by-Step Procedure

1. Form the augmented matrix of your system.
2. Use row operations to create leading 1s in each row (pivot positions).
3. Zero out all other entries in the pivot columns by row operations.
4. Continue until the matrix is in RREF, enabling the direct reading of solutions.

Applying Gauss-Jordan to the Example

Using the same system:

$$
\begin{cases}
x + 2y + z = 6 \\
2x + 3y + 3z = 14 \\
-x + y + 2z = 5
\end{cases}
$$

Step 1: Start with the augmented matrix (as before):

$$
\begin{bmatrix}
1 & 2 & 1 & | & 6 \\
2 & 3 & 3 & | & 14 \\
-1 & 1 & 2 & | & 5
\end{bmatrix}
$$

Step 2: Make zeros in column 1 below and above row 1:

- Row 2: $R_2 - 2 R_1$:

$$
(0, -1, 1, 2)
$$

- Row 3: $R_3 + R_1$:

$$
(0, 3, 3, 11)
$$

- Normalize row 2: multiply by -1:

$$
R_2: 0, 1, -1, -2
$$

- Zero out above row 2:

- Row 1: $R_1 - 2 R_2$:

$$
(1 - 2 \times 0, 2 - 2 \times 1, 1 - 2 \times (-1), 6 - 2 \times (-2)) = (1, 0, 3, 10)
$$

- Row 3: $R_3 - 3 R_2$:

$$
(0, 0, 6, 17)
$$

Updated matrix:

$$
\begin{bmatrix}
1 & 0 & 3 & | & 10 \\
0 & 1 & -1 & | & -2 \\
0 & 0 & 6 & | & 17
$$

\end{bmatrix}
\]

Step 3: Normalize row 3:

\[
R_3: \frac{1}{6} R_3 \Rightarrow (0, 0, 1, \frac{17}{6})
\]

Step 4: Zero out above row 3:

- Row 1: \(R_1 - 3 R_3 \):

\[
(1, 0, 0, 10 - 3

Frequently Asked Questions

What is the main goal of Gaussian and Gauss-Jordan elimination methods when solving systems of equations?

The main goal is to systematically reduce the system to a form where the solutions can be easily identified, typically to row-echelon form (Gaussian) or reduced row-echelon form (Gauss-Jordan), enabling straightforward back-substitution or direct reading of solutions.

How do you determine if a system of equations has no solution using Gaussian or Gauss-Jordan elimination?

You can identify inconsistency when a row reduces to a form where all variables have zero coefficients but the constant term is non-zero (e.g., 0 = c, where c ≠ 0), indicating the system has no solution.

What is the difference between Gaussian elimination and Gauss-Jordan elimination?

Gaussian elimination reduces the system to row-echelon form, making back-substitution possible, while Gauss-Jordan elimination further reduces the system to reduced row-echelon form, allowing solutions to be read directly without back-substitution.

Can Gaussian or Gauss-Jordan elimination be used for systems with infinite solutions?

Yes, these methods can reveal infinite solutions when the system reduces to a form with free variables, indicating the system is dependent and has infinitely many solutions.

What are common pitfalls to avoid when applying Gaussian or Gauss-Jordan elimination?

Common pitfalls include incorrect row operations, division by zero, and misinterpreting inconsistent or dependent systems, which can lead to incorrect solutions or missed solutions.

How do you handle a system with more variables than equations during elimination?

Such systems often have free variables, indicating infinite solutions. During elimination, identify these free variables, and express the solutions parametrically in terms of these variables.

What should you do if, during elimination, you encounter a zero pivot element?

You should swap rows to bring a non-zero element into the pivot position or check for special cases like dependent or inconsistent systems, to proceed with elimination or conclude the nature of the solutions.

Is it always necessary to use Gauss-Jordan elimination for solving systems, or are there situations where Gaussian elimination suffices?

Gaussian elimination suffices when you only need to find solutions via back-substitution, but Gauss-Jordan elimination is preferred for directly obtaining the solutions in reduced form, especially for small systems or when multiple solutions are involved.

How can you verify the correctness of the solutions obtained from elimination methods?

You can substitute the solutions back into the original equations to verify that all equations are satisfied, ensuring the correctness of the solutions.

Additional Resources

Gaussian Elimination and Gauss-Jordan Method: A Comprehensive Guide to Solving Systems of Linear Equations

When tackling systems of linear equations, mathematicians and engineers alike rely on systematic, reliable methods to find solutions efficiently and accurately. Among these techniques, Gaussian elimination and Gauss-Jordan elimination stand out as foundational algorithms that transform complex systems into simpler, solvable forms. Whether you're a student delving into linear algebra for the first time or a professional needing robust computational tools, understanding these methods is crucial. In this article, we explore these techniques in detail, examining their processes, advantages, limitations, and practical applications.

Understanding the Foundations: What Are Gaussian and Gauss-Jordan Elimination?

Before diving into the step-by-step procedures, it's essential to grasp what these methods aim to accomplish.

Gaussian Elimination is a method that reduces a system of linear equations to an upper triangular form, making it straightforward to perform back substitution to find solutions. It systematically eliminates variables from equations, starting from the top, resulting in a simpler equivalent system.

Gauss-Jordan Elimination extends Gaussian elimination further by reducing the matrix to reduced row-echelon form (RREF). This form simplifies the process of reading off solutions directly, often eliminating the need for back substitution.

Both methods operate on the augmented matrix of the system, which consolidates all coefficients and constants into a single matrix structure, enabling algorithmic manipulation.

Step-by-Step Process of Gaussian Elimination

2.1 Representing the System as a Matrix

Suppose you have a system of linear equations:

\[
\begin{cases}
a_{11}x_1 + a_{12}x_2 + \dots + a_{1n}x_n = b_1 \\
a_{21}x_1 + a_{22}x_2 + \dots + a_{2n}x_n = b_2 \\
\vdots \\
a_{m1}x_1 + a_{m2}x_2 + \dots + a_{mn}x_n = b_m
\end{cases}
\]

This can be represented as an augmented matrix:

\[
\left[
\begin{array}{cccc|c}

a_{11} & a_{12} & \dots & a_{1n} & b_1 \\
a_{21} & a_{22} & \dots & a_{2n} & b_2 \\
\vdots & \vdots & & \vdots & \vdots \\
a_{m1} & a_{m2} & \dots & a_{mn} & b_m
\end{array}
\right]
\]

2.2 Forward Elimination

The goal here is to create zeros below the leading coefficients (pivots) of each column.

- Pivot Selection: For each column, select a pivot element (preferably the largest in absolute value for numerical stability).
- Row Operations:
- Swap rows if necessary to bring the pivot into position.
- Normalize the pivot row so that the pivot element becomes 1.
- Use row operations to eliminate the variable from all rows below the pivot.

This process proceeds down the matrix, transforming it into an upper triangular matrix where all entries below the main diagonal are zeros.

2.3 Back Substitution

Once in upper triangular form, solutions are obtained starting from the last equation, substituting known values back into the previous equations to find all variables systematically.

Extending to Gauss-Jordan Elimination

While Gaussian elimination stops at upper triangular form, Gauss-Jordan elimination continues to reduce the matrix further into the reduced row-echelon form (RREF), characterized by:

- Leading 1s in each row (pivots).
- Zeros in all other entries in the pivot columns.

This process simplifies the solution process dramatically, often allowing direct reading of solutions.

2.1 Additional Operations

- For each pivot, normalize the row so that the pivot becomes 1.
- Use row operations to eliminate all other entries in the pivot column, both above and below the pivot.

2.2 Result Interpretation

In RREF, the matrix directly indicates the solutions:

- Each row corresponds to an equation with a single variable (if the system has a unique solution).
- Free variables are evident if the system has infinitely many solutions.
- Inconsistent systems are identified if a row reduces to [0 0 ... 0 | c] with c ≠ 0, indicating no solution.

Practical Example: Solving a System Using Gaussian Elimination

Let's consider a concrete example to illustrate the process.

System:

\[
\begin{cases}
2x + y - z = 8 \\
-3x - y + 2z = -11 \\
-2x + y + 2z = -3
\end{cases}
\]

Step 1: Write the augmented matrix

\[
\left[
\begin{array}{ccc|c}
2 & 1 & -1 & 8 \\
-3 & -1 & 2 & -11 \\
-2 & 1 & 2 & -3
\end{array}
\right]
\]

Step 2: Forward elimination

- Make the first pivot (row 1, column 1) equal to 1: divide row 1 by 2.

\[
R_1 \rightarrow \frac{1}{2} R_1
\]

\[
\left[
\begin{array}{ccc|c}

1 & 0.5 & -0.5 & 4 \\
-3 & -1 & 2 & -11 \\
-2 & 1 & 2 & -3
\end{array}
\right]
\]

- Eliminate below pivot:

\[
R_2 \rightarrow R_2 + 3 \times R_1
\]
\[
R_3 \rightarrow R_3 + 2 \times R_1
\]

Calculations:

- R2: \(-3 + 3 \times 1 = 0\), \(-1 + 3 \times 0.5 = 0.5\), \(2 + 3 \times (-0.5) = 0.5\), \(-11 + 3 \times 4 = 1\)

- R3: \(-2 + 2 \times 1 = 0\), \(1 + 2 \times 0.5 = 2\), \(2 + 2 \times (-0.5) = 1\), \(-3 + 2 \times 4 = 5\)

Updated matrix:

\[
\left[
\begin{array}{ccc|c}
1 & 0.5 & -0.5 & 4 \\
0 & 0.5 & 0.5 & 1 \\
0 & 2 & 1 & 5
\end{array}
\right]
\]

- Make the second pivot (row 2, column 2) equal to 1:

\[
R_2 \rightarrow 2 R_2
\]

\[
\left[
\begin{array}{ccc|c}
1 & 0.5 & -0.5 & 4 \\
0 & 1 & 1 & 2 \\
0 & 2 & 1 & 5
\end{array}
\right]
\]

- Use row 2 to eliminate entries below:

$$
R_3 \rightarrow R_3 - 2 \times R_2
$$

Calculations:

$$
0, 2 - 2 \times 1 = 0, 1 - 2 \times 1 = -1,\ 5 - 2 \times 2 = 1
$$

Updated matrix:

$$
\left[
\begin{array}{ccc|c}
1 & 0.5 & -0.5 & 4 \\
0 & 1 & 1 & 2 \\
0 & 0 & -1 & 1
\end{array}
\right]
$$

- Make the third pivot (row 3, column 3) equal to 1:

$$
R_3 \rightarrow - R_3
$$

$$
\left[
\begin{array}{ccc|c}
1 & 0.5 & -0.5 & 4 \\
0 & 1 & 1 & 2 \\
0 & 0 & 1 & -1
\end{array}
\right]
$$

Step 3: Back substitution

- From row 3: $z = -1$
- From row 2: $y + z = 2 \Rightarrow y - 1 = 2 \Rightarrow y = 3$
- From row 1: $x + 0.5 y - 0.5 z = 4 \Rightarrow x + 0.5 \times 3 - 0.5 \times (-1) = 4$

Calculate:

$$
x + 1.5 + 0.5 = 4 \Rightarrow x + 2 = 4 \Rightarrow x = 2
$$

Solution:

\[
\boxed{
x = 2,\quad y = 3,\quad z = -1
}

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