

Speed Of A Body Spinning About An Axis Increases From Rest To 100 Rpm In 5 Second If A Constant Torque

Speed Of A Body Spinning About An Axis Increases From Rest To 100 Rpm In 5 Second If A Constant Torque is a fascinating phenomenon that illustrates fundamental principles of rotational dynamics. Understanding how torque influences angular acceleration, the relationships between torque, moment of inertia, and angular velocity, and the practical implications of these concepts is essential for students, engineers, and enthusiasts alike. This article explores the physics behind this scenario, providing a comprehensive overview of the concepts, calculations, and real-world applications involved.

Fundamentals of Rotational Motion

Angular Displacement, Velocity, and Acceleration

In rotational motion, the key quantities include:

- Angular Displacement (θ): The angle through which a body rotates, measured in radians.
- Angular Velocity (ω): The rate of change of angular displacement, typically in radians per second (rad/s).
- Angular Acceleration (α): The rate of change of angular velocity, measured in radians per second squared (rad/s²).

These quantities are interconnected through the equations of rotational kinematics, analogous to linear motion.

Torque and Moment of Inertia

The rotational equivalent of force is torque (τ), which causes an angular acceleration in a body. The moment of inertia (I) measures an object's resistance to changes in its rotational motion, depending on the mass distribution relative to the axis of rotation.

The fundamental relationship connecting torque, moment of inertia, and angular acceleration is:

$$\tau = I \alpha$$

This equation indicates that for a given torque, the angular acceleration is inversely proportional to the body's moment of inertia.

Understanding the Scenario: From Rest to 100 RPM in 5 Seconds

Converting Units for Consistency

Before delving into calculations, it's essential to convert all quantities into SI units:

- Final Angular Velocity (ω_f): 100 revolutions per minute (rpm).

To convert rpm to rad/s:

$$\omega_f = 100 \text{ rpm} \times \frac{2\pi \text{ radians}}{1 \text{ revolution}} \times \frac{1 \text{ minute}}{60 \text{ seconds}} = \frac{100 \times 2\pi}{60} \approx 10.47 \text{ rad/s}$$

- Time (t): 5 seconds (given).

- Initial Angular Velocity (ω_i): 0 rad/s (since starting from rest).

Calculating Angular Acceleration

Using the kinematic equation:

$$\omega_f = \omega_i + \alpha t$$

Since ($\omega_i = 0$):

$$\alpha = \frac{\omega_f - \omega_i}{t} = \frac{10.47}{5} \approx 2.09 \text{ rad/s}^2$$

This constant angular acceleration indicates how rapidly the body gains rotational speed under a steady torque.

Determining the Torque Required

Role of Moment of Inertia

To find the torque, the moment of inertia (I) of the body must be known. The calculation varies depending on the geometry of the object:

- Solid Sphere: $I = \frac{2}{5} M R^2$
- Solid Cylinder: $I = \frac{1}{2} M R^2$
- Hollow Cylinder or Ring: $I = M R^2$

Where:

- M: Mass of the object
- R: Radius of the object

For this analysis, assume the object is a solid cylinder with known mass and radius.

Calculating Torque

Using the fundamental relation:

$$\tau = I \alpha$$

Suppose, for example, the body has:

- Mass (M): 10 kg
- Radius (R): 0.5 m

Then:

$$I = \frac{1}{2} \times 10 \times (0.5)^2 = 0.5 \times 10 \times 0.25 = 1.25 \text{ kg}\cdot\text{m}^2$$

And the torque:

$$\tau = 1.25 \times 2.09 \approx 2.61 \text{ N}\cdot\text{m}$$

Thus, a torque of approximately 2.61 N·m is required to spin the body from rest to 100 rpm in 5 seconds.

Implications and Practical Applications

Designing Rotational Systems

Understanding the relationship between torque, moment of inertia, and angular acceleration is critical in designing machinery such as turbines, flywheels, and rotors. Engineers can use these principles to select appropriate motors and torque requirements to achieve desired acceleration profiles.

Energy Considerations

The work done by the torque to accelerate the body is stored as rotational kinetic energy:

$$KE_{\text{rot}} = \frac{1}{2} I \omega_f^2$$

Using the earlier example:

$$KE_{\text{rot}} = 0.5 \times 1.25 \times (10.47)^2 \approx 68.6 \text{ Joules}$$

This energy must be supplied by the torque source over the 5 seconds, implying an average power:

$$P_{\text{avg}} = \frac{KE_{\text{rot}}}{t} \approx \frac{68.6}{5} \approx 13.72 \text{ Watts}$$

This calculation helps in selecting power sources for rotational systems.

Limitations and Real-World Factors

In practice, several factors influence the torque and acceleration:

- Friction and Damping: Resistance from bearings, air, or internal friction reduces acceleration.
- Variable Torque: Real motors may not provide perfectly constant torque.
- Structural Constraints: The body must withstand the stresses generated during acceleration.

Designers must account for these factors to ensure system reliability and safety.

Extensions and Advanced Concepts

Angular Momentum and Conservation

Angular momentum ($L = I \omega$) plays a crucial role in rotational dynamics. In closed systems with no external torque, angular momentum remains conserved.

Rotational Kinetic Energy and Power

Understanding how kinetic energy relates to angular velocity helps optimize systems for energy efficiency. Power calculations are vital for determining the capacity of motors and energy requirements.

Transient and Steady-State Behavior

While this scenario deals with a steady torque leading to uniform angular acceleration, real-world systems often involve transient effects, variable torques, and complex load conditions.

Conclusion

The increase of a body's rotational speed from rest to 100 rpm in 5 seconds under a constant torque exemplifies core principles of rotational dynamics. By analyzing the relationships between torque, moment of inertia, and angular acceleration, engineers can design efficient rotational systems tailored to specific performance criteria. Accurate calculations and an understanding of underlying physics are essential for optimizing machinery, ensuring safety, and improving energy efficiency in various industrial and technological applications.

Summary of Key Points:

- Conversion of units is essential for accurate calculations.
- The angular acceleration is directly proportional to the applied torque and inversely proportional to the body's moment of inertia.
- Knowledge of geometry and mass distribution is necessary to compute the moment of inertia.
- Practical applications span from designing engines and turbines to understanding energy storage mechanisms.
- Real-world systems require consideration of additional factors like friction, variable torque, and structural integrity.

By mastering these concepts, practitioners can effectively control and optimize rotational motion, leading to innovations and improvements across multiple fields.

Frequently Asked Questions

How does the speed of a spinning body change when a constant torque is applied from rest?

The speed increases uniformly over time, starting from rest and reaching a maximum value determined by the torque and moment of inertia, in this case reaching 100 RPM in 5 seconds.

What is the significance of constant torque in the acceleration of a spinning body?

Constant torque results in a uniform angular acceleration, meaning the body's angular velocity increases at a steady rate over time.

How can we determine the angular acceleration of the body in

this scenario?

Angular acceleration can be calculated by dividing the change in angular velocity by the time taken; here, from 0 to 100 RPM in 5 seconds.

What is the relation between torque, moment of inertia, and angular acceleration?

Torque (τ) equals the moment of inertia (I) multiplied by angular acceleration (α), expressed as $\tau = I \times \alpha$.

How do we convert the final speed from RPM to radians per second?

Multiply the RPM value by $2\pi/60$. For 100 RPM, the final angular velocity is $(100 \times 2\pi)/60 \approx 10.47$ rad/sec.

If the torque is constant and the angular acceleration is known, how long would it take to reach a higher speed, say 200 RPM?

Assuming constant angular acceleration, time to reach 200 RPM would be double the time to reach 100 RPM (i.e., 10 seconds), since the angular velocity increases linearly with time.

Additional Resources

Speed Of A Body Spinning About An Axis Increases From Rest To 100 Rpm In 5 Seconds If A Constant Torque

Understanding the dynamics of rotational motion is fundamental in physics and engineering, especially when analyzing how objects respond to applied torques. The scenario where the speed of a body spinning about an axis increases from rest to 100 rpm in 5 seconds under a constant torque offers a fascinating insight into rotational kinematics and dynamics. This situation encapsulates core principles such as angular acceleration, moment of inertia, torque, and angular velocity, providing a comprehensive case study for students and professionals alike.

Introduction to Rotational Dynamics and Key Concepts

Rotational motion involves objects spinning around an axis, with their behavior governed by analogous principles to linear motion but expressed in angular terms. When a torque is applied to a body, it causes angular acceleration, which in turn changes the angular velocity over time.

Key Concepts:

- Angular velocity (ω): Rate at which a body spins, typically measured in radians per second (rad/s) or revolutions per minute (rpm).
- Angular acceleration (α): The rate of change of angular velocity, measured in rad/s^2 .
- Moment of inertia (I): The rotational equivalent of mass, representing an object's resistance to changes in its rotational motion.
- Torque (τ): The rotational equivalent of force, causing angular acceleration.

Understanding the Scenario: From Rest to 100 rpm in 5 Seconds

The core problem involves a body initially at rest ($\omega_0 = 0$) being subjected to a constant torque that accelerates it to an angular velocity corresponding to 100 rpm within 5 seconds. This scenario is common in mechanical systems such as motors, flywheels, and rotating machinery.

Why is this important?

Analyzing such a problem helps engineers design systems with predictable acceleration profiles, ensuring components can withstand stresses and operate efficiently.

Converting Units and Initial Parameters

Before delving into calculations, it's essential to convert the given values into SI units:

- Final angular velocity (ω_f):

$$\left(100 \, \text{rpm} = \frac{100}{60} \, \text{rps} \approx 1.6667 \, \text{rps} \right)$$

Since 1 revolution = 2π radians,

$$\left(\omega_f = 1.6667 \, \text{rps} \times 2\pi \, \text{rad/rev} \approx 10.472 \, \text{rad/sec} \right)$$

- Time (t): 5 seconds

- Initial angular velocity (ω_0): 0 rad/sec (from rest)

Calculating Angular Acceleration

Given the initial and final angular velocities over a known time, the angular acceleration (α) can be determined:

$$\alpha = \frac{\omega_f - \omega_0}{t} = \frac{10.472 \, \text{rad/sec} - 0}{5 \, \text{sec}} = 2.0944 \, \text{rad/sec}^2$$

rad/sec^2
\\

This constant angular acceleration indicates how quickly the body spins up to the desired speed under the applied torque.

Determining the Moment of Inertia and Torque

The crux of the problem involves understanding how torque relates to the body's rotational inertia and angular acceleration.

Fundamental relation:

$$\tau = I \alpha$$

Where:

- τ = torque (Nm)
- I = moment of inertia ($\text{kg}\cdot\text{m}^2$)
- α = angular acceleration (rad/sec^2)

Without specific details about the body's shape, mass, or distribution, we typically analyze the relation between torque, inertia, and acceleration, rather than exact numerical values.

Assuming the moment of inertia I is known, the torque required is:

$$\tau = I \times 2.0944 \text{ rad/sec}^2$$

Hence:

- The larger the moment of inertia, the greater the torque needed to achieve the same angular acceleration.
- Conversely, for a given torque, a larger inertia results in a slower speed increase.

Features and Implications of a Constant Torque

Applying constant torque simplifies analysis but also reflects real-world scenarios such as electric motors operating at steady torque levels.

Features:

- Produces a uniform angular acceleration.
- The angular velocity increases linearly with time:

$$\omega(t) = \omega_0 + \alpha t$$

- The kinetic energy of the rotating body increases quadratically with angular velocity:

$$KE = \frac{1}{2} I \omega^2$$

- The power delivered varies with time:

$$P(t) = \tau \times \omega(t)$$

Implications:

- The torque must be sufficiently large to overcome initial inertia and any resistive forces (e.g., friction).
- Maintaining a constant torque in practical systems often requires precise control mechanisms.

Analyzing Energy and Power

As the body accelerates, its rotational kinetic energy increases. The change in kinetic energy from rest to the final speed is:

$$\Delta KE = \frac{1}{2} I \omega_f^2$$

Since torque is constant, the work done by the torque over time (which equals energy transferred) is:

$$W = \tau \times \theta$$

Where θ is the angular displacement during acceleration:

$$\theta = \frac{1}{2} \alpha t^2 = \frac{1}{2} \times 2.0944 \times 25 = 26.18 \text{ rad}$$

The work done (and thus energy supplied):

$$W = \tau \times 26.18$$

Matching this with the kinetic energy:

$$\frac{1}{2} I \omega_f^2 = \tau \times \theta$$

Rearranged:

$$\tau = \frac{\frac{1}{2} I \omega_f^2}{\theta}$$

This relation helps determine the torque needed for a given inertia to reach the target speed in the specified time.

Practical Applications and Engineering Considerations

Understanding how a body accelerates under constant torque is vital in designing various mechanical and electrical systems.

Applications:

- Electric motors: ensuring they can reach desired speeds within specific timeframes.
- Flywheels: energy storage devices that spin up under controlled torque.
- Robotics: controlling joint accelerations to prevent mechanical stress.
- Automotive engineering: accelerating wheels or drive shafts during startup.

Pros and Cons of Applying Constant Torque:

Pros:

- Simplifies control algorithms for motor operation.
- Easier to predict system behavior.
- Facilitates steady acceleration profiles.

Cons:

- Real-world systems often experience varying torques.
- Friction, air resistance, and other resistive forces can alter acceleration.
- Sudden changes in torque can cause mechanical stress.

Advanced Topics and Considerations

For more in-depth analysis, several factors come into play:

- Friction and Damping: Real systems experience resistive forces that require additional torque to overcome, affecting acceleration time.
- Variable Torque Applications: Many systems do not operate under constant torque but follow specific torque profiles for efficiency.
- Non-uniform Inertia: Bodies with complex mass distributions require integral calculus to determine their moment of inertia.
- Safety and Mechanical Limits: Rapid acceleration can induce stresses; design must account for material strength and fatigue.

Conclusion

The scenario where speed of a body spinning about an axis increases from rest to 100 rpm in 5 seconds under a constant torque exemplifies fundamental principles of rotational dynamics. By understanding the relationships between torque, moment of inertia, angular acceleration, and energy, engineers can design systems that achieve desired acceleration profiles safely and efficiently. While the analysis assumes ideal conditions, incorporating real-world factors like friction and variable loads is essential for practical applications. This case underscores the importance of precise calculations and control mechanisms in the realm of rotational motion, influencing fields from mechanical engineering to robotics and beyond.

Summary of Key Points:

- The body accelerates uniformly due to constant torque, reaching 100 rpm in 5 seconds.
- Conversion of units is critical for accurate calculations.
- The angular acceleration is directly proportional to the applied torque and inversely proportional to the moment of inertia.
- Energy considerations reveal how kinetic energy increases with the square of angular velocity.
- Practical systems must account for resistive forces and safety constraints.

This comprehensive analysis not only enhances understanding of rotational dynamics but also provides a foundation for designing systems with predictable and controlled accelerations.

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