Suppose That 3 J Of Work Is Needed To Stretch A Spring From Its Natural Length Of 24 Cm To A Length Of

Suppose That 3 J Of Work Is Needed To Stretch A Spring From Its Natural Length Of 24 Cm To A Length Of 28 cm. This scenario provides an excellent opportunity to explore the principles of work, elastic potential energy, and Hooke's Law in the context of spring mechanics. Understanding these concepts not only deepens your grasp of physics but also enhances your ability to solve real-world problems involving elastic materials and energy transfer.

Understanding the Fundamentals of Spring Mechanics

Hooke's Law and Spring Force

Hooke's Law states that the force needed to stretch or compress a spring is directly proportional to the displacement from its natural length. Mathematically, this is expressed as:

$$F = kx$$

where:

- F is the force applied to the spring,
- k is the spring constant (a measure of the spring's stiffness),
- x is the displacement from the natural length.

In this scenario:

- The natural length of the spring, $L_0 = 24 \text{ cm}$,
- The stretched length, $L = 28 \text{ cm}$,
- The displacement, $x = L - L_0 = 4 \text{ cm} = 0.04 \text{ m}$.

Calculating the Spring Constant (k)

Work Done in Stretching the Spring

The work W done to stretch a spring from its natural length to a certain extension is given by the elastic potential energy stored in the spring:

$$W = \frac{1}{2} k x^2$$

Given that:

- The work done, $W = 3 \text{ J}$,
- The displacement, $x = 0.04 \text{ m}$.

We can rearrange the formula to find k:

$$k = \frac{2W}{x^2}$$

Substituting the known values:

$$k = \frac{2 \times 3}{(0.04)^2} = \frac{6}{0.0016}$$

$$k = 3750 \text{ N/m}$$

This high value indicates a very stiff spring.

Determining the Elastic Potential Energy at Different Extensions

Elastic Potential Energy Formula

The elastic potential energy U stored in a stretched or compressed spring is:

$$U = \frac{1}{2} k x^2$$

Using the calculated spring constant, we can find the energy stored at any extension.

Energy at the Given Extension

At $x = 0.04 \text{ m}$:

$$U = \frac{1}{2} \times 3750 \times (0.04)^2$$

$$U = 0.5 \times 3750 \times 0.0016$$

$$U = 3 \text{ J}$$

which confirms our initial data.

Energy at a Different Extension

Suppose we want to find the energy stored when the spring is stretched to 30 cm:

- New length, $L = 30 \text{ cm}$,
- Displacement, $x = 30 - 24 = 6 \text{ cm} = 0.06 \text{ m}$.

Calculating:

$$U = \frac{1}{2} \times 3750 \times (0.06)^2$$

$$U = 0.5 \times 3750 \times 0.0036$$

$$U = 6.75 \text{ J}$$

This demonstrates that the elastic potential energy increases quadratically with extension.

Understanding the Work-Energy Relationship

The Work-Energy Theorem

The work done on a spring to stretch it is stored as elastic potential energy:

$$W = U$$

In our case, 3 J of work results in 3 J of stored energy, which aligns with the work-energy theorem.

Implications for Mechanical Systems

- The amount of work needed to stretch or compress a spring depends on the extension and the spring constant.
- The energy stored can be released to do work, such as moving objects or generating vibrations.

Practical Applications of Spring Mechanics

Designing Mechanical Devices

Understanding the relationship between work, energy, and spring constant aids in designing:

- Shock absorbers,
- Suspension systems,
- Mechanical clocks,
- Trampolines.

Energy Efficiency Considerations

Knowing how much energy a spring can store helps optimize energy transfer in devices, ensuring efficiency and safety.

Sample Problems and Solutions

Problem 1: Find the work required to stretch the spring from its natural length to 30 cm.

Solution:

- Displacement $x = 0.06 \text{ m}$,
- Using $k = 3750 \text{ N/m}$:

$$ W = \frac{1}{2} k x^2 = 0.5 \times 3750 \times 0.0036 = 6.75 \text{ J} $$

Answer: 6.75 Joules.

Problem 2: What is the maximum elastic potential energy stored when the spring is stretched to 40 cm?

Solution:

- Displacement $x = 40 - 24 = 16 \text{ cm} = 0.16 \text{ m}$,
- Calculate energy:

$$ U = \frac{1}{2} \times 3750 \times (0.16)^2 = 0.5 \times 3750 \times 0.0256 = 48 \text{ J} $$

Answer: 48 Joules.

Summary and Key Takeaways

- The work required to stretch a spring relates directly to the elastic potential energy stored.
- The spring constant k can be determined from the work done and the extension.
- Elastic potential energy increases quadratically with extension, emphasizing the importance of spring stiffness in design.
- Practical applications range from vehicle suspension to mechanical toys, highlighting the relevance of these principles.

Final Remarks

Understanding the relationship between work, elastic potential energy, and spring properties is vital for engineers, physicists, and anyone involved in designing systems that utilize elastic components. The scenario of stretching a spring from 24 cm to 28 cm with a known work input allows us to derive the spring constant and predict energy storage at various extensions, enabling precise control and optimization of mechanical systems.

Keywords: spring constant, elastic potential energy, work done, Hooke's Law, mechanical energy, spring mechanics, energy storage, physics problem-solving, elastic energy calculations, spring design

Frequently Asked Questions

How do you calculate the work done to stretch a spring from its natural length to a certain length?

The work done is calculated using the spring's elastic potential energy formula: Work = $(1/2)\ k\ (\Delta x)^2$, where k is the spring constant and Δx is the change in length from the natural length.

If 3 Joules of work is required to stretch a spring from 24 cm to a certain length, how can we find the spring constant?

First, determine the extension Δx in meters, then use the work formula: Work = $(1/2)\ k\ \Delta x^2$. Rearranged, $k = (2\ \text{Work}) / \Delta x^2$. Plug in the values to find k.

What is the significance of the work value 3 Joules in understanding the spring's properties?

The 3 Joules indicates the energy needed to stretch the spring by a certain amount, which helps determine the spring constant and understand the spring's stiffness.

How does the initial length of the spring affect the work needed to stretch it?

The initial (natural) length serves as the baseline. The work required depends on the extension from this natural length, so knowing the change in length is essential for calculating work, regardless of the initial length.

Can the work done to stretch a spring be negative? Why or why not?

No, the work done to stretch a spring is always positive because energy is added to the system to increase its length. Negative work would imply energy is being removed or the spring is doing work on the environment.

If the spring is stretched beyond the initial problem, how does the work needed relate to the extension?

The work required increases quadratically with extension, following the formula: Work = (1/2) k Δx^2. Larger extensions require disproportionately more work due to the quadratic relationship.

Additional Resources

Suppose That 3 J Of Work Is Needed To Stretch A Spring From Its Natural Length Of 24 Cm To A Length Of

Imagine a scenario where a simple mechanical task involves the application of energy to deform a spring. Specifically, consider a spring whose natural (unstretched) length measures 24 centimeters, and we want to understand the amount of work required to stretch it to a certain length. In this case, it's given that the work needed to stretch the spring from its natural length to some extended length amounts to 3 joules. Such a problem might seem straightforward at first glance, but it opens the door to exploring fundamental concepts in physics and engineering—namely, the relationship between work, elastic potential energy, and the properties of springs.

In this article, we will delve deep into the principles underlying this problem, explain how to determine the extended length of the spring, and explore the physical significance of the work-energy relationship. We will also introduce the mathematical tools used to analyze spring behavior, such as Hooke's law, and illustrate the real-world applications of such calculations.

Understanding the Basics: Work, Energy, and Springs

The Concept of Work in Mechanical Systems

In physics, work is defined as the process of energy transfer when a force is applied to move an object over a distance. Mathematically, it is expressed as:

$$ W = \int \vec{F} \cdot d\vec{s} $$

where $\vec{F}$ is the force vector, and $d\vec{s}$ is the differential element of displacement.

In the context of stretching a spring, work is done by an external agent to overcome the elastic restoring force of the spring. This work results in the elastic potential energy stored within the spring once deformed.

Key point: For a linear elastic spring, the work done to stretch or compress it is stored as potential energy, which can be recovered when the spring returns to its natural length.

Hooke's Law and Spring Force

The behavior of most ideal springs is described by Hooke's Law, which states:

$$ F = -k x $$

where:

- F is the restoring force exerted by the spring,
- k is the spring constant (a measure of stiffness),
- x is the displacement from the natural length (positive when stretched, negative when compressed).

The negative sign indicates that the force exerted by the spring acts in the opposite direction of displacement.

Implication: The greater the spring constant k, the more force is needed to stretch or compress the spring by a certain amount.

Calculating the Work Done to Stretch the Spring

Work-Energy Relationship for Springs

The work done on a spring in stretching it from an initial length L_0 to a final length L can be expressed as the elastic potential energy stored in the spring:

$$ W = \frac{1}{2} k x^2 $$

where:

- $x = L - L_0$ is the extension from the natural length.

This formula assumes an ideal spring with no energy loss due to friction or other dissipative forces.

Note: Since the work done is stored as elastic potential energy, the work required to stretch the spring from its natural length to a certain extension is exactly equal to the increase in elastic potential energy.

Given Data and Objective

The problem states:

- The natural length of the spring: $L_0 = 24\, \text{cm}$
- Work required to stretch from natural length: $W = 3\, \text{J}$
- The goal: Find the extended length L.

From these, we need to determine:

1. The extension $x = L - L_0$,
2. The spring constant k.

Step-by-Step Solution

Step 1: Express the Known Quantities

- Convert natural length to meters for SI consistency:

$$ L_0 = 24\, \text{cm} = 0.24\, \text{m} $$

- The work done:

$$ W = 3\, \text{J} $$

- The extension:

$$ x = L - L_0 $$

Step 2: Use the Work-Energy Formula

Since the work done to stretch the spring is stored as elastic potential energy:

$$ W = \frac{1}{2} k x^2 $$

Rearranged to solve for k:

$$ k = \frac{2W}{x^2} $$

But both k and x are unknown at this point. To proceed, recognize that the work done depends on both k and x, and it's typical to express one in terms of the other.

Step 3: Express k in Terms of x and Known Quantities

Rearranged:

$$ k = \frac{2 \times 3}{x^2} = \frac{6}{x^2} $$

But we still have two unknowns (k and x). To find x, consider the physical constraints.

Alternatively, since the problem only states the work and the natural length, and asks for the extended length, we need an additional assumption or insight.

Assumption: The problem expects us to find the extension x such that the work required is 3 J, assuming the spring behaves ideally.

Step 4: Recognize the Relationship Between k, x, and Work

The key insight is that the amount of work to stretch the spring from its natural length to extension x is:

$$ W = \frac{1}{2} k x^2 $$

And the spring constant k is related to the physical properties of the spring, which are not directly given.

Therefore, to proceed, the problem might assume that the spring's stiffness k can be independently determined or that the question aims to find the extension x based solely on the work done.

Determining the Extended Length: Additional Assumptions and Calculation

Since the problem does not specify the final length or the spring constant directly, but gives the work done, we can interpret the problem as asking: What is the extension x corresponding to a work of 3 J?

Method:

1. Assume the initial length $L_0 = 0.24\, \text{m}$.
2. Use the work formula to relate x and the work:

$$ 3 = \frac{1}{2} k x^2 $$

3. Recognize that without k, we cannot directly find x. But if we consider typical spring constants or interpret the problem as seeking the extension corresponding to 3 J of work, then the relationship can be rearranged as:

$$ x = \sqrt{\frac{2W}{k}} $$

Potential Approach: Expressing x in terms of k

Suppose that the spring has a certain known stiffness k. For illustrative purposes, assume k is a typical value, or more precisely, the problem expects the calculation of x in terms of k.

Alternatively, if the problem intends for us to find the final length L, then:

$$L = L_0 + x$$

Given the lack of explicit k, the most straightforward way forward is to present the relationship:

$$x = \sqrt{\frac{2 \times 3}{k}}$$

and

$$L = 0.24 + x$$

Physical Significance and Practical Implications

The Elastic Potential Energy and Work Done

The calculation highlights how the energy stored in a spring is directly proportional to the square of the extension. This quadratic relationship underscores a fundamental principle: doubling the extension quadruples the stored energy, making the spring's behavior highly sensitive to displacement.

In practical applications, engineers must carefully choose spring constants to balance the required energy absorption with the physical limits of materials. For example, in automotive suspensions or machinery, springs are designed to handle specific energy levels without permanent deformation.

Safety and Material Limits

Understanding the work-energy relationship is crucial for safety. Excessive stretching can lead to material fatigue or failure. The work calculation informs engineers about the maximum safe extension for a spring based on its material properties and intended use.

Real-World Applications of Spring Work Calculations

- Design of Mechanical Systems: Precise control of spring extensions ensures reliable operation in devices like watches, automotive suspensions, or industrial machinery.
- Energy Storage Devices: Springs are used in energy harvesting and shock absorption systems, where knowing the work capacity and elastic limits is vital.
- Educational Demonstrations: Experiments involving measuring the work done to stretch springs help students visualize fundamental physics principles.

Summary and Final Thoughts

The problem statement—"Suppose that 3 J of work is needed to stretch a spring from its natural length of 24 cm to a certain length"—serves

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