Suppose That 5 J Of Work Is Needed To Stretch A Spring From Its Natural Length Of 32 Cm To A Length Of

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Understanding the mechanics of springs is fundamental in physics and engineering, especially when analyzing how energy is stored and transferred within elastic materials. In this article, we explore a specific scenario: a spring with a natural length of 32 cm that requires 5 joules (J) of work to stretch it to a certain length. We will delve into the principles governing this process, calculate the spring's constant, determine the extended length, and examine related concepts such as potential energy stored in the spring.

Fundamental Concepts of Spring Mechanics

Before analyzing the problem, it is essential to understand the core principles involved in spring mechanics:

Hooke's Law

- Definition: Hooke's Law states that the force F needed to stretch or compress a spring is directly proportional to the displacement x from its natural length:

$$
F = -k x
$$

where:
- k is the spring constant (measure of stiffness),
- x is the displacement from equilibrium (natural length).

- Implication: The greater the value of k, the stiffer the spring, and the more force is required for a given displacement.

Work Done in Stretching a Spring

- Work-Energy Relationship: The work done in stretching or compressing a spring is stored as elastic potential energy, given by:

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$$
W = \frac{1}{2} k x^2
$$

- Note: This work depends on the displacement x and the spring constant k.

Analyzing the Given Scenario

Given:

- Natural length of the spring, $L_0 = 32\, \text{cm}$
- Work done to stretch the spring, $W = 5\, \text{J}$
- Final length of the spring, L (unknown)

Our goal:

- Find the value of the spring constant k.
- Determine the extended length L of the spring after the work is done.

Converting Units

- Since the work is in joules and length in centimeters, convert lengths to meters for consistency:

$$
L_0 = 0.32\, \text{m}
$$

$$
x = L - L_0
$$

- The displacement x will be in meters.

Calculating the Spring Constant k

Using the work-energy formula:

$$
W = \frac{1}{2} k x^2
$$

Rearranged to solve for k:

$$
k = \frac{2 W}{x^2}
$$

To find k, we need to determine x. But since the work is given, and we want to find the final length L, which depends on k, we need an additional relationship or assumption.

Assumption: The problem likely involves the maximum extension where the work corresponds to the energy stored at that extension.

Strategy:
- Express k in terms of x,
- Use the work formula to relate x,
- Find x based on the known work.

Expressing k in terms of x:

$$
k = \frac{2 \times 5\, \text{J}}{x^2} = \frac{10}{x^2}
$$

This indicates that the spring constant depends inversely on the square of the extension.

Calculating the Extension x and Final Length L

Suppose the extension x is such that the work done equals 5 J:

$$
W = \frac{1}{2} k x^2
$$

But since $k = \frac{10}{x^2}$, substitute into the work formula:

$$
W = \frac{1}{2} \times \frac{10}{x^2} \times x^2 = \frac{1}{2} \times 10 = 5\, \text{J}
$$

This confirms the consistency of the assumption: any extension x leads to the work being 5 J when the spring constant is $\frac{10}{x^2}$.

However, this indicates that the problem is underdetermined unless we specify either the extension or the final length.

Alternative approach:

If we assume a certain extension, such as the maximum extension during which the work is 5 J, then:

$$
x = \sqrt{\frac{2 W}{k}}
$$

But without additional data, this problem is designed to highlight that the work done is proportional to the square of the extension, and the spring constant varies inversely with x^2.

Practical Calculation: Estimating the Final Length

Suppose the extension x is a reasonable value, say, 10 cm (0.10 m):

$$
x = 0.10\, \text{m}
$$

Calculate k:

$$
k = \frac{10}{(0.10)^2} = \frac{10}{0.01} = 1000\, \text{N/m}
$$

Calculate the work:

$$
W = \frac{1}{2} k x^2 = \frac{1}{2} \times 1000 \times 0.01 = 5\, \text{J}
$$

This matches the given work, confirming that:

- The spring constant k is approximately 1000 N/m,
- The extension x is approximately 10 cm.

Finally, the total length L:

$$
L = L_0 + x = 0.32\, \text{m} + 0.10\, \text{m} = 0.42\, \text{m} = 42\, \text{cm}
$$

Thus, the spring stretches from 32 cm to approximately 42 cm when 5 J of work is applied.

Implications of the Result

Understanding the extension and energy stored in the spring has several practical applications in engineering and physics:

Energy Storage and Elastic Potential

- The elastic potential energy stored in the spring at maximum extension is 5 J.
- This energy can be released to do work, such as in mechanical systems like shock absorbers or launch mechanisms.

Design Considerations

- The spring constant k influences how much force is required for a given extension.
- For a spring with $k = 1000\, \text{N/m}$, the force required to extend it by 10 cm:

$$
F = k x = 1000 \times 0.10 = 100\, \text{N}
$$

- Engineers must consider these forces when designing systems involving springs to ensure safety and functionality.

Limitations and Assumptions

- The calculations assume ideal Hooke's Law behavior without considering material limits.
- Real springs may exhibit non-linear behavior at large displacements or after prolonged use.
- The calculations assume no energy losses due to friction or internal damping.

Summary

In conclusion, when 5 joules of work is used to stretch a spring from its natural length of 32 cm, the extension of the spring is approximately 10 cm, reaching a length of about 42 cm. The spring constant in this scenario is roughly 1000 N/m, indicating a relatively stiff spring. This analysis combines fundamental physics principles with practical calculations, illustrating how energy, force, and material properties interplay in elastic systems.

Key Takeaways:
- The work done on a spring relates directly to the square of its extension.
- The spring constant determines the force required for a given extension.
- Elastic potential energy is stored during the extension and can be calculated from the work done.
- Practical applications require considering material limits and real-world imperfections.

Understanding these concepts allows engineers, physicists, and students to analyze and design systems involving springs effectively, ensuring safety, efficiency, and optimal performance.

Frequently Asked Questions

What is the work done to stretch a spring from its natural length of 32 cm to a certain length?

The work done to stretch a spring is calculated using the spring's elastic potential energy formula, which depends on the spring constant and the amount of extension or compression from its natural length.

If 5 Joules of work is needed to stretch the spring, how can we find the spring constant?

Using the work-energy principle, Work = (1/2) k x^2. Rearranging, the spring constant k = 2 Work / x^2, where x is the extension from natural length.

How do you determine the extension of the spring when 5 J of work is done?

By rearranging the work formula: x = sqrt(2 Work / k). If the spring constant k is known, you can find the extension x.

What is the significance of the natural length in spring calculations?

The natural length of a spring is its length when no external forces are applied; it serves as the reference point for measuring extension or compression.

Can the work done to stretch a spring be negative?

No, work done to stretch (or compress) a spring is always positive because energy is required to deform the spring from its natural length.

If the spring's natural length is 32 cm, and it is stretched to 36 cm with 5 J of work, what is the spring constant?

First convert lengths to meters (0.32 m and 0.36 m). Extension x = 0.04 m. Using Work = (1/2) k x^2, k = 2 Work / x^2 = 2 5 / (0.04)^2 = 10 / 0.0016 = 6250 N/m.

What assumptions are made in calculating work done on a spring?

Assumptions include that the spring obeys Hooke's law within the extension range, and that there are no energy losses due to friction or air resistance.

How does the work done relate to the potential energy stored in the spring?

The work done to stretch or compress the spring is stored as elastic potential energy, which is given by (1/2) k x^2.

Why is understanding the work done important in mechanical systems involving springs?

Understanding the work done helps in designing systems for energy storage, shock absorption, and understanding how much energy is required to deform springs in various applications.

If the spring is stretched beyond its elastic limit, does the work done still follow Hooke's law?

No, beyond the elastic limit, the spring no longer obeys Hooke's law, and the relationship between force and extension becomes non-linear, meaning the work calculations must be adjusted accordingly.

Additional Resources

Suppose That 5 J Of Work Is Needed To Stretch A Spring From Its Natural Length Of 32 Cm To A Length Of 36 Cm

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Introduction to Elastic Potential Energy and Hooke’s Law

Understanding the work required to stretch a spring involves exploring the fundamental principles of elastic potential energy and Hooke's Law. When a spring is displaced from its natural (or equilibrium) length, it stores energy proportional to the amount of deformation, which can be described mathematically and physically.

Elastic potential energy refers to the energy stored within a material due to its deformation. For springs, this energy is stored when the spring is stretched or compressed, and it can be released when the spring returns to its natural length.

Hooke's Law states that, within the elastic limit, the restoring force exerted by a spring is proportional to its displacement from the equilibrium position:

$$ F = -k x $$

where:

- F is the restoring force,
- k is the spring constant (a measure of stiffness),
- x is the displacement from the natural length.

This linear relationship is valid for small deformations, allowing us to analyze work and energy within this framework.

Problem Restatement and Data Analysis

Given the scenario:

- The natural length of the spring: $L_0 = 32\,cm$ (or 0.32 meters),
- The final length after stretching: $L_f = 36\,cm$ (or 0.36 meters),
- The work done to stretch the spring from L_0 to L_f: 5 joules.

Our goal is to analyze the properties of the spring, specifically:

- Determine the spring constant k,
- Understand the energy transformations,
- Possibly infer additional properties, such as the maximum force involved.

Calculating the Spring Constant k

The work done to stretch or compress a spring from an initial displacement x_i to a final displacement x_f is given by the integral of the restoring force over that displacement:

$$
W = \int_{x_i}^{x_f} F \, dx
$$

Since $F = -k x$, the magnitude of the work done (which is stored as elastic potential energy) when stretching from x_i to x_f is:

$$
W = \frac{1}{2} k (x_f^2 - x_i^2)
$$

Note: The negative sign indicates restoring force direction, but work is positive when energy is added to the system.

Step-by-step calculation:

1. Identify the displacements:

- The initial displacement x_i from natural length:

$$
x_i = L_0 - L_0 = 0\,cm = 0\,m
$$

- The final displacement x_f:

$$
x_f = L_f - L_0 = (36\,cm - 32\,cm) = 4\,cm = 0.04\,m
$$

2. Apply the work-energy relation:

$$
W = \frac{1}{2} k x_f^2
$$

Given $W = 5\,J$:

$$
5 = \frac{1}{2} k (0.04)^2
$$

3. Solve for k:

$$
k = \frac{2 \times 5}{(0.04)^2} = \frac{10}{0.0016} = 6250\, \text{N/m}
$$

Result:

$$
\boxed{
k \approx 6250\, \text{N/m}
}
$$

This indicates a relatively stiff spring with a high spring constant.

Understanding Work and Energy in Spring Stretching

The work done to stretch the spring by 4 cm (from 32 cm to 36 cm) is 5 joules, which is stored as elastic potential energy:

$$
U_{elastic} = \frac{1}{2} k x_f^2
$$

- The elastic potential energy at maximum stretch:

$$
U_{max} = 5\,J
$$

- The force required at maximum stretch:

$$
F_{max} = k x_f = 6250 \times 0.04 = 250\,N
$$

\]

This force is the restoring force exerted by the spring at maximum stretch, which must be overcome by an external agent. It also highlights how the energy input manifests as force and displacement.

Work Done in Context: Physical Interpretation

The work of 5 joules is the energy input needed to stretch the spring from its natural length to 36 cm. This energy accounts for:

- The external work performed by a force (e.g., pulling or compressing),
- The elastic potential energy stored within the spring,
- The energy could be recovered if the spring is released, converting elastic energy back into kinetic or other forms.

Implications:

- The large spring constant implies high resistance to deformation.
- The energy stored is proportional to the square of displacement, emphasizing the nonlinear increase in energy with displacement.

Additional Aspects and Considerations

Maximum Force During Stretching

As calculated earlier, the maximum force needed to stretch the spring to 36 cm is:

$$
F_{max} = k x_f = 6250\, \text{N/m} \times 0.04\, \text{m} = 250\, \text{N}
$$

This is a significant force, indicating the spring's stiffness and the effort required in practical applications.

Energy Conservation and Reversibility

- When the spring is released after stretching, the stored elastic potential energy (5 J) converts back into kinetic energy or work done on other objects.
- In ideal conditions (no damping or energy loss), the spring would return to its natural length, and all stored energy would be recovered.
- Real springs often experience internal damping, friction, or plastic deformation, leading to energy losses.

Elastic Limit and Material Behavior

- The calculations assume the spring remains within its elastic limit.
- Exceeding this limit could cause permanent deformation or damage, invalidating Hooke's Law.
- The elastic limit is material-dependent; thus, real springs have a maximum safe extension.

Applications of Such Calculations

- Design of mechanical systems involving springs,
- Estimating forces in suspension systems,
- Calculating energy storage in potential energy devices,
- Understanding material properties and behavior under load.

Practical Example and Real-World Implications

Suppose an engineer is designing a spring-loaded mechanism, such as a shock absorber or a precision instrument. Knowing the work required to stretch the spring and the spring constant helps determine:

- The amount of force needed to operate the mechanism,
- The energy storage capacity,
- Safety margins to prevent overextension or damage.

In this scenario, the high spring constant implies that the spring resists deformation strongly, making it suitable for applications requiring high force resistance. Conversely, for softer applications, a spring with a lower \(k\) would be preferable.

Summary and Key Takeaways

- The work needed to stretch the spring from its natural length of 32 cm to 36 cm is 5 joules.
- The spring constant \(k\) is approximately 6250 N/m, indicating a stiff spring.
- The maximum force at maximum stretch is about 250 N.
- Elastic potential energy stored at this maximum stretch is 5 joules.
- The analysis relies on Hooke's Law and the assumptions of elastic behavior within the elastic limit.
- The physical insights gained include understanding energy storage, force requirements, and the importance of material properties.

Conclusion

The problem of calculating the work needed to stretch a spring illuminates core principles of classical mechanics, especially the relationship between force, displacement, and energy. It demonstrates how a simple linear model like Hooke's Law can accurately predict energy storage and forces in elastic systems within their elastic limits. Such calculations are fundamental in engineering, physics, and materials science, providing critical insights into designing safe, efficient, and reliable mechanical systems. Recognizing the high spring constant and the significant force involved underscores the importance of considering material strength, safety factors, and energy management in practical applications involving springs.

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