Suppose That A Firms Production Function Is Q = 10L1/2K1/2. The Cost Of A Unit Of Labor Is $20 And The

Suppose That A Firms Production Function Is Q = 10L1/2K1/2. The Cost Of A Unit Of Labor Is $20 And The firm aims to optimize production costs while maintaining a desired output level. Understanding the relationship between inputs, costs, and output is essential for effective decision-making in microeconomics. In this article, we will explore the production function, derive the cost minimization problem, analyze the firm's input choices, and discuss the implications for business strategy.

Understanding the Production Function Q = $10L^{1/2}K^{1/2}$

What Does the Production Function Represent?

The production function Q = 10L1/2K1/2 describes the relationship between the inputs—labor (L) and capital (K)—and the resulting output (Q). It indicates that output increases with input quantities but at a diminishing rate, characteristic of the square root function.

Properties of the Production Function

- Cobb-Douglas Form: This function is a specific case of the Cobb-Douglas production function, which is widely used due to its simplicity and ability to model returns to scale.
- Returns to Scale: To assess whether the firm experiences increasing, decreasing, or constant returns to scale, analyze the sum of the exponents:

\[
1/2 + 1/2 = 1
\]

Since the sum equals 1, the production function exhibits constant returns to scale. Doubling inputs doubles output, providing predictability in scaling operations.

- Marginal Products:
- Marginal Product of Labor (MPL):

\[
MP_L = \frac{\partial Q}{\partial L} = 10 \times \frac{1}{2}L^{-1/2}K^{1/2} =

$$
5 \frac{K^{1/2}}{L^{1/2}}
$$

- Marginal Product of Capital (MPK):

$$
MP_K = \frac{\partial Q}{\partial K} = 10 \times \frac{1}{2}K^{-1/2}L^{1/2} = 5 \frac{L^{1/2}}{K^{1/2}}
$$

These marginal products show how additional units of labor or capital influence output, declining as input quantities increase.

Cost Structure and Optimization

Input Costs

Given the cost of labor per unit, the firm faces:

- Labor Cost (w): $20 per unit
- Capital Cost (r): (Assuming as given or to be determined; if not specified, we can analyze generally or assume a specific value)

The total cost (C) of inputs is:

$$
C = wL + rK
$$

The goal is to produce a given output level Q at the minimum possible cost.

Formulating the Cost Minimization Problem

To minimize costs for a given output Q, the firm solves:

$$
\min_{L,K} C = 20L + rK
$$

subject to:

$$
Q = 10L^{1/2}K^{1/2}
$$

This is a constrained optimization problem that can be approached using methods like Lagrangian multipliers.

Deriving the Cost-Minimizing Input Combination

Setting Up the Lagrangian

Define the Lagrangian:

\[
\mathcal{L} = 20L + rK + \lambda \left(Q - 10L^{1/2}K^{1/2} \right)
\]

where λ is the Lagrange multiplier.

First-Order Conditions

Differentiate with respect to L, K, and λ:

1. With respect to L:

\[
\frac{\partial \mathcal{L}}{\partial L} = 20 - \lambda \times 10 \times
\frac{1}{2} L^{-1/2} K^{1/2} = 0
\]
\[
20 = 5 \lambda \frac{K^{1/2}}{L^{1/2}}
\]

2. With respect to K:

\[
\frac{\partial \mathcal{L}}{\partial K} = r - \lambda \times 10 \times
\frac{1}{2} K^{-1/2} L^{1/2} = 0
\]
\[
r = 5 \lambda \frac{L^{1/2}}{K^{1/2}}
\]

3. With respect to λ:

\[
Q - 10 L^{1/2} K^{1/2} = 0
\]

Solving for Input Ratios

Dividing the first equation by the second:

\[

\frac{20}{r} = \frac{K^{1/2}/L^{1/2}}{L^{1/2}/K^{1/2}} =
\frac{K^{1/2}}{L^{1/2}} \times \frac{K^{1/2}}{L^{1/2}} = \frac{K}{L}
\]

Thus:

\[
\frac{K}{L} = \frac{20}{r}
\]
or equivalently:

\[
K = \frac{20}{r} L
\]

This ratio indicates the optimal proportion of capital to labor to minimize cost for producing output Q.

Calculating the Optimal Input Quantities

Expressing K in Terms of L

From the input ratio:

\[
K = \frac{20}{r} L
\]

Substitute into the production function:

\[
Q = 10 L^{1/2} \left(\frac{20}{r} L \right)^{1/2}
\]

Simplify:

\[
Q = 10 L^{1/2} \times \left(\frac{20}{r} \right)^{1/2} L^{1/2} = 10 \left(
\frac{20}{r} \right)^{1/2} L
\]

Solve for L:

\[
L = \frac{Q}{10} \left(\frac{r}{20} \right)^{1/2}
\]

Similarly, K:

$$
K = \frac{20}{r} L = \frac{20}{r} \times \frac{Q}{10} \left(\frac{r}{20} \right)^{1/2}
$$

Simplify:

$$
K = 2 \times \frac{Q}{r} \times \left(\frac{r}{20} \right)^{1/2} = \frac{2Q}{r} \times \frac{\sqrt{r}}{\sqrt{20}} = \frac{2Q}{\sqrt{20} \times \sqrt{r}}
$$

Summary of Optimal Inputs:

$$
L^ = \frac{Q}{10} \left(\frac{r}{20} \right)^{1/2}
$$

$$
K^ = \frac{2Q}{\sqrt{20} \times \sqrt{r}}
$$

Note: These formulas depend on the capital cost r, which can be specified or estimated based on market conditions.

Calculating the Minimum Cost

Using the optimal input quantities, total cost becomes:

$$
C_{min} = 20 L^ + r K^
$$

Substitute the expressions:

$$
C_{min} = 20 \times \frac{Q}{10} \left(\frac{r}{20} \right)^{1/2} + r \times \frac{2Q}{\sqrt{20} \times \sqrt{r}}
$$

Simplify:

$$
C_{min} = 2Q \left(\frac{r}{20} \right)^{1/2} + 2Q \frac{\sqrt{r}}{\sqrt{20}}
$$

Note that:

$$
\left(\frac{r}{20} \right)^{1/2} = \frac{\sqrt{r}}{\sqrt{20}}
$$

Thus,

$$
C_{min} = 2Q \times \frac{\sqrt{r}}{\sqrt{20}} + 2Q \times \frac{\sqrt{r}}{\sqrt{20}} = 4Q \times \frac{\sqrt{r}}{\sqrt{20}}
$$

Or,

$$
C_{min} = 4Q \frac{\sqrt{r}}{\sqrt{20}}
$$

This formula provides the minimum cost for producing output Q given input prices.

Implications for Business Strategy

Cost Efficiency and Input Choices

By understanding the relationship between input costs and output, firms can adjust their input proportions to achieve cost efficiency. For example, if the cost of capital (r) increases, the optimal amount of capital K decreases,

Frequently Asked Questions

What is the production function given in the problem?

The production function is $Q = 10L^{1/2}K^{1/2}$.

How do you interpret the exponents in the production function $Q = 10L^{1/2}K^{1/2}$?

The exponents indicate that both labor (L) and capital (K) have a square root relationship with output Q, suggesting diminishing returns to each input individually.

What is the cost of one unit of labor in this scenario?

The cost of one unit of labor is $20.

How can the firm minimize costs given the production function and input prices?

The firm can minimize costs by choosing the optimal combination of labor and capital that produces a given level of output at the lowest possible cost, typically using cost minimization techniques such as setting the marginal rate of technical substitution equal to the input price ratio.

What is the marginal product of labor (MP_L) for this production function?

The marginal product of labor is $MP_L = 10 (1/2) L^{-1/2} K^{1/2} = 5 K^{1/2} / L^{1/2}$.

How do you calculate the total cost function for this firm?

The total cost function is $C = wL + rK$, where w is the wage rate ($20), and r is the cost of capital (not specified in the question). To fully determine it, the cost of capital must be known.

If the firm wants to produce Q units of output, how would you determine the optimal input combination?

Set up the cost minimization problem with the production function and input prices, then solve for L and K that satisfy both the production requirement and cost conditions, often using Lagrangian methods.

What is the concept of isoquants in this context?

Isoquants are curves representing different combinations of L and K that produce the same level of output Q; for this production function, they can be derived by solving $Q = 10L^{1/2}K^{1/2}$ for K in terms of L.

How does the symmetry in the production function influence the firm's input choices?

Since the production function is symmetric in L and K, the firm may choose input combinations where L and K are equal or proportional when input prices are equal, simplifying the cost minimization process.

Additional Resources

Suppose That A Firm’s Production Function Is Q = 10L^{1/2}K^{1/2}. The Cost Of A Unit Of Labor Is $20 And The

Understanding a firm's production function is fundamental to analyzing its cost structure, output levels, and overall efficiency. In this article, we delve into the specifics of the given production function $Q = 10L^{1/2}K^{1/2}$, explore its implications for cost minimization, and assess how input prices influence optimal input choices. This detailed review aims to clarify the operational and economic features of this production function, highlighting its advantages and limitations for managerial decision-making.

Overview of the Production Function $Q = 10L^{1/2}K^{1/2}$

Functional Form and Characteristics

The production function $Q = 10L^{1/2}K^{1/2}$ is a Cobb-Douglas type, characterized by its multiplicative structure and constant elasticity of substitution. Its key features include:

- Symmetry between inputs: Both labor (L) and capital (K) appear symmetrically, with equal exponents of 1/2.
- Returns to scale: Since the sum of exponents is 1 (0.5 + 0.5), the function exhibits constant returns to scale. Doubling both inputs doubles output.
- Diminishing marginal returns: Each input individually exhibits diminishing marginal returns, as indicated by the square root form.
- Ease of elasticity calculation: The exponents imply an elasticity of substitution of 1, meaning inputs can be substituted at a constant rate without affecting output.

This functional form is widely used because it captures the intuitive economic notion that increasing inputs yields less additional output proportionally, and it provides analytical simplicity.

Implications for Production and Efficiency

- Input substitutability: The unit elasticity of substitution allows for flexible input combinations to achieve desired output levels.
- Scale analysis: The constant returns to scale suggest that scaling inputs

proportionally leads to a proportional increase in output, which is favorable for firms aiming to expand production efficiently.
- Marginal products: Both marginal products decline as input levels increase, consistent with the law of diminishing returns.

Cost Analysis and Input Pricing

Given Input Prices

The problem states that the cost of a unit of labor is $20. To fully analyze costs, we need the price of capital (K). Assuming the price of capital per unit is r, the total cost C for inputs L and K is:

$$
C = 20L + rK
$$

The goal is to minimize this cost for a given level of output Q.

Cost Minimization Problem

To find the optimal input combination, the firm solves:

$$
\text{Minimize } C = 20L + rK
$$
$$
\text{subject to } Q = 10L^{1/2}K^{1/2}
$$

Using the method of Lagrange multipliers, set up the Lagrangian:

$$
\mathcal{L} = 20L + rK + \lambda(Q - 10L^{1/2}K^{1/2})
$$

Differentiating with respect to L, K, and λ:

$$
\frac{\partial \mathcal{L}}{\partial L} = 20 - \lambda \times 10 \times \frac{1}{2} L^{-1/2} K^{1/2} = 0
$$
\[

\frac{\partial \mathcal{L}}{\partial K} = r - \lambda \times 10 \times
\frac{1}{2} K^{-1/2} L^{1/2} = 0
\]
\[
\frac{\partial \mathcal{L}}{\partial \lambda} = Q - 10 L^{1/2} K^{1/2} = 0
\]

From the first two derivatives, equate the expressions to eliminate \(
\lambda \):

\[
\frac{20}{\frac{10}{2} L^{-1/2} K^{1/2}} = \frac{r}{\frac{10}{2} K^{-1/2}
L^{1/2}}
\]

Simplify:

\[
\frac{20}{5 L^{-1/2} K^{1/2}} = \frac{r}{5 K^{-1/2} L^{1/2}}
\]

\[
\frac{20}{5} \times L^{1/2} K^{-1/2} = r \times L^{-1/2} K^{1/2}
\]

\[
4 L^{1/2} K^{-1/2} = r L^{-1/2} K^{1/2}
\]

Dividing both sides by \(L^{-1/2} K^{-1/2} \):

\[
4 \times \frac{L^{1/2} K^{-1/2}}{L^{-1/2} K^{-1/2}} = r \times \frac{L^{-1/2}
K^{1/2}}{L^{-1/2} K^{-1/2}}
\]

\[
4 \times L^{1/2 + 1/2} K^{-1/2 + 1/2} = r \times L^{-1/2 + 1/2} K^{1/2 + 1/2}
\]

Simplify exponents:

\[
4 \times L^{1} K^{0} = r \times L^{0} K^{1}
\]

\[
4L = rK
\]

So, the optimal input ratio is:

$$
K = \frac{4L}{r}
$$

This relationship indicates that the optimal capital input depends linearly on labor input and the capital price.

Deriving Cost Functions

Using the production function constraint:

$$
Q = 10L^{1/2}K^{1/2}
$$

Substitute K:

$$
Q = 10L^{1/2} \left(\frac{4L}{r}\right)^{1/2} = 10L^{1/2} \times \left(\frac{4L}{r}\right)^{1/2}
$$

$$
Q = 10L^{1/2} \times \left(\frac{4}{r}\right)^{1/2} L^{1/2} = 10 \times \left(\frac{4}{r}\right)^{1/2} \times L^{1/2 + 1/2} = 10 \times \left(\frac{4}{r}\right)^{1/2} \times L
$$

Solve for L:

$$
L = \frac{Q}{10} \times \left(\frac{r}{4}\right)^{1/2}
$$

Similarly, K:

$$
K = \frac{4L}{r} = \frac{4}{r} \times \frac{Q}{10} \times \left(\frac{r}{4}\right)^{1/2}
$$

Simplify:

$$
K = \frac{4Q}{10 r} \times \left(\frac{r}{4}\right)^{1/2} = \frac{2Q}{5 r} \times \left(\frac{r}{4}\right)^{1/2}
$$

Total cost:

$$
C = 20L + rK
$$

Substitute:

$$
C = 20 \times \frac{Q}{10} \times \left(\frac{r}{4}\right)^{1/2} + r \times \frac{2Q}{5 r} \times \left(\frac{r}{4}\right)^{1/2}
$$

Simplify:

$$
C = 2Q \times \left(\frac{r}{4}\right)^{1/2} + \frac{2Q}{5} \times \left(\frac{r}{4}\right)^{1/2}
$$

$$
C = \left(2Q + \frac{2Q}{5}\right) \times \left(\frac{r}{4}\right)^{1/2} = \frac{12Q}{5} \times \left(\frac{r}{4}\right)^{1/2}
$$

This expression shows the minimum cost for producing output Q given input prices.

Features and Analysis of the Production Function

Advantages

- Mathematical simplicity: The symmetry and exponents make analytical solutions straightforward.
- Flexibility in input substitution: The constant elasticity allows for predictable substitution between labor and capital.
- Constant returns to scale: Facilitates scalability and planning for expansion.
- Diminishing marginal returns: Reflects real-world production constraints.

Limitations and Challenges

- Assumption of perfect substitutability: The elasticity of substitution

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being 1 may not hold in all real-world scenarios.
- Simpl
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