

Suppose We Are Making A 95% Confidence Interval For The Population Mean From A Sample Of Size 15. What

Suppose We Are Making A 95% Confidence Interval For The Population Mean From A Sample Of Size 15. What does this mean, and how do we go about calculating and interpreting such an interval? Confidence intervals are fundamental tools in statistics that allow us to estimate a population parameter—like the mean—based on a sample. When working with small sample sizes, such as 15 observations, specific considerations come into play, especially regarding the distribution used and the accuracy of the estimate. This article explores the process of constructing a 95% confidence interval for a population mean from a small sample size, focusing on key concepts, calculation steps, and interpretation.

Understanding Confidence Intervals and Their Importance

What Is a Confidence Interval?

A confidence interval (CI) provides a range of values within which we expect the true population parameter (like the mean) to lie with a certain level of confidence. For a 95% confidence interval, we are saying that if we were to take many samples and construct a CI from each, approximately 95% of those intervals would contain the true population mean.

Why Use Confidence Intervals?

- Estimate Precision: They give a measure of the estimate's precision.
- Incorporate Uncertainty: They account for variability in the sample data.
- Guide Decision-Making: They help infer the population parameter for research, policy, and business decisions.

Key Concepts When Working With Small Samples

Sample Size and Its Impact

A sample size of 15 is considered small in statistical terms. Small samples tend to produce wider confidence intervals, reflecting greater uncertainty about the population mean.

Distribution Assumptions

- When the sample size is small (<30), and the population standard deviation is unknown, the t-distribution is used instead of the normal distribution.
- The t-distribution accounts for additional variability due to small sample sizes.

Sample Standard Deviation as an Estimate

Since the population standard deviation is often unknown, we estimate it using the sample standard deviation (s).

Steps to Construct a 95% Confidence Interval for the Population Mean (Sample Size = 15)

1. Collect and Summarize Data

- Obtain a random sample of size $n = 15$.
- Calculate the sample mean ($\bar{x}$).
- Calculate the sample standard deviation (s).

2. Determine the Appropriate Distribution

- Since $n < 30$ and σ (population standard deviation) is unknown, use the t-distribution with $n - 1 = 14$ degrees of freedom.

3. Find the Critical t-Value

- For a 95% confidence level, identify the t-value corresponding to 14 degrees of freedom.
- This value can be found using statistical tables or software.
- Typically, $t_{\{0.975,14\}} \approx 2.145$.

4. Calculate the Margin of Error (ME)

The margin of error quantifies the maximum expected difference between the sample mean and the true population mean.

$$\text{ME} = t_{\{\alpha/2, \text{df}\}} \times \frac{s}{\sqrt{n}}$$

Where:

- $t_{\{\alpha/2, \text{df}\}}$ = critical t-value
- s = sample standard deviation
- n = sample size

5. Construct the Confidence Interval

- The confidence interval is:

$$\left[\bar{x} - \text{ME}, \bar{x} + \text{ME} \right]$$

- This interval estimates the range within which the true population mean is likely to fall with 95% confidence.

Practical Example of Constructing the Confidence Interval

Suppose we collected a sample with the following data:

- Sample size $(n = 15)$
- Sample mean $(\bar{x} = 50)$
- Sample standard deviation $(s = 8)$

Step-by-step calculation:

1. Find the critical t-value:

- For 14 degrees of freedom and 95% confidence, $(t_{0.975,14} \approx 2.145)$.

2. Calculate standard error:

$$SE = \frac{s}{\sqrt{n}} = \frac{8}{\sqrt{15}} \approx \frac{8}{3.873} \approx 2.065$$

3. Calculate margin of error:

$$ME = 2.145 \times 2.065 \approx 4.429$$

4. Construct the interval:

$$(50 - 4.429, 50 + 4.429) = (45.571, 54.429)$$

Interpretation:

We are 95% confident that the true population mean lies between approximately 45.57 and 54.43.

Interpreting the Confidence Interval

What Does This Interval Tell Us?

- The calculated interval provides an estimated range for the population mean based on the sample.
- It reflects the uncertainty inherent in sampling, especially with small sample sizes.

Limitations and Considerations

- The accuracy depends on the sample being representative.
- The interval is only valid if the data are approximately normally distributed, especially critical for small samples.
- Outliers can significantly influence the sample mean and standard deviation, thus affecting the interval.

Additional Tips for Accurate Confidence Interval Estimation

- **Ensure Random Sampling:** To make valid inferences, your sample should be randomly selected from the population.
- **Check for Normality:** Use plots or tests to verify the data are approximately normally distributed, especially critical for small samples.
- **Use Correct Distribution:** Always use the t-distribution for small samples when the population standard deviation is unknown.
- **Report the Confidence Level:** Clearly specify the confidence level (e.g., 95%) when presenting the interval.

Conclusion

Constructing a 95% confidence interval for the population mean from a small sample size like 15 involves understanding the underlying assumptions, correctly selecting the t-distribution, and carefully performing calculations. This process provides a statistically sound estimate of the population mean while acknowledging the inherent uncertainty of sampling. By following the outlined steps and considerations, researchers and analysts can make informed inferences and communicate their findings with confidence. Remember, the key is not just in calculating the interval but in interpreting what it means within the context of your data and research question.

Frequently Asked Questions

What does a 95% confidence interval for the population mean represent?

It indicates that we are 95% confident that the true population mean lies within the calculated interval based on our sample data.

How is the confidence level (95%) related to the confidence interval?

The confidence level determines the proportion of such intervals that would contain the true population mean if the sampling were repeated multiple times.

What is the significance of the sample size being 15 in constructing the confidence interval?

A sample size of 15 is relatively small, which may lead to wider confidence intervals due to increased variability and the need to use the t-distribution instead of the normal distribution.

Which distribution should be used when constructing a 95% confidence interval with a sample size of 15?

The t-distribution should be used because the sample size is small ($n < 30$) and the population standard deviation is typically unknown.

How do you calculate the margin of error for this confidence interval?

The margin of error is calculated as the t-critical value multiplied by the standard error of the sample mean.

What is the formula for the standard error in this context?

Standard error = sample standard deviation divided by the square root of the sample size ($SE = s/\sqrt{n}$).

If the sample mean is 50 with a sample standard deviation of 10, what would be the approximate margin of error for the 95% confidence interval with $n=15$?

First, find the t-critical value for 14 degrees of freedom at 95% confidence (~ 2.145). Then, margin of error = $2.145 (10/\sqrt{15}) \approx 2.145 \cdot 2.58 \approx 5.54$.

What are the potential limitations of constructing a

confidence interval with a small sample size like 15?

Small sample sizes can lead to less reliable estimates, wider confidence intervals, and increased impact of outliers or variability in the data.

How can the accuracy of the confidence interval be improved when working with small samples?

Increasing the sample size, ensuring data quality, and verifying the data's normality can improve the accuracy and reliability of the confidence interval.

Why is it important to check the data distribution before constructing a confidence interval for the mean?

Because the validity of the t-distribution-based confidence interval relies on the data being approximately normally distributed, especially with small samples.

Additional Resources

Suppose We Are Making A 95% Confidence Interval For The Population Mean From A Sample Of Size 15. What Does This Mean For Researchers, Statisticians, and Data Enthusiasts?

In the realm of statistical analysis, drawing meaningful conclusions about a population based on sample data is both an art and a science. When researchers collect a small sample, such as 15 observations, they often aim to estimate the true average (mean) of the entire population with a certain level of confidence. A common target is a 95% confidence interval, which provides a range believed to contain the population mean with high probability. But what exactly does constructing this interval entail? How is it calculated? What assumptions underpin it? And what are the practical implications of working with a small sample size? This article delves into these questions, unpacking the statistical concepts and procedures involved in creating a 95% confidence interval for a population mean from a sample of 15 observations.

Understanding Confidence Intervals: The Basics

What Is a Confidence Interval?

A confidence interval (CI) is a range of values, derived from sample data, that is believed to contain the true population parameter — in this case, the population mean — with a specified level of confidence. The '95%' in a 95% confidence interval indicates that if we were to repeat the sampling process numerous times, approximately 95% of those intervals would contain the true population mean.

Key features of confidence intervals include:

- Range of plausible values: Instead of providing just a point estimate (like the sample mean), the interval offers a range that reflects the uncertainty inherent in sampling.
- Confidence level: The probability (e.g., 95%) reflects the method's long-term success rate, not the probability that the parameter lies within any particular interval computed from a single sample.

Why Use Confidence Intervals?

Confidence intervals serve several purposes:

- Quantify uncertainty: They communicate the precision of an estimate.
- Support decision-making: They provide a basis for comparing groups or assessing the effect of variables.
- Inform further research: Wide intervals suggest more data are needed for precise estimates; narrow intervals indicate more certainty.

The Scenario: Sample Size of 15

Implications of Small Sample Size

A sample size of 15 is considered small in statistical terms. This influences the methodology used and the confidence we can place in the resulting interval. With small samples:

- The estimate of the population mean can be more variable.
- The sampling distribution of the mean is less likely to be perfectly normal, especially if the underlying data are not normally distributed.
- The accuracy of the confidence interval depends heavily on the assumption of data distribution and variance estimation.

Assumptions Necessary for Valid Confidence Intervals

Constructing a valid confidence interval typically relies on certain assumptions:

- Independence: The observations are independent of each other.
- Normality: The data are approximately normally distributed, especially important when $n < 30$.
- Known or estimated variance: The population variance is unknown and must be estimated from the sample.

When these assumptions hold, the confidence interval can be accurately calculated. If not, the interval may be misleading, and alternative methods or transformations may be necessary.

Calculating the 95% Confidence Interval for the Mean

Step 1: Gather Sample Data

Suppose the sample of 15 observations yields:

- Sample mean ($\bar{x}$)
- Sample standard deviation (s)

For illustration, assume:

- $\bar{x} = 50$
- $s = 10$

Step 2: Determine the Appropriate Distribution

Because the sample size is small, and the population variance is unknown, the Student's t-distribution is used instead of the normal distribution.

The t-distribution accounts for additional uncertainty in estimating the population standard deviation from a small sample.

Step 3: Find the t-Value

The critical t-value depends on:

- The desired confidence level (95%)
- The degrees of freedom ($df = n - 1 = 14$)

Using a t-distribution table or calculator, the two-tailed critical value for 95% confidence and 14 degrees of freedom is approximately 2.145.

Step 4: Compute the Margin of Error (ME)

The margin of error quantifies the maximum expected difference between the sample mean and the true population mean:

$$ME = t_{\alpha/2, df} \times \frac{s}{\sqrt{n}}$$

Plugging in the numbers:

$$ME = 2.145 \times \frac{10}{\sqrt{15}} \approx 2.145 \times 2.582 \approx 5.535$$

Step 5: Construct the Confidence Interval

The interval is:

$$[50 - 5.535, 50 + 5.535]$$

$\bar{x} \pm \text{ME}$

$\downarrow$

$\downarrow$

50 ± 5.535

$\downarrow$

which yields:

- Lower bound: $(50 - 5.535 = 44.465)$
- Upper bound: $(50 + 5.535 = 55.535)$

Therefore, the 95% confidence interval is approximately (44.47, 55.53).

Interpreting the Confidence Interval

What Does This Interval Tell Us?

Based on the sample, we are 95% confident that the true population mean lies between approximately 44.47 and 55.53. This does not mean there's a 95% probability that the population mean is within this specific interval; rather, it reflects the reliability of the estimation method across many samples.

Limitations and Considerations

- Sample Representativeness: The interval is only as good as the sample. Biases or non-representativeness can skew results.
- Normality Assumption: If the data are heavily skewed or contain outliers, the t-interval may be inaccurate.
- Small Sample Size: While manageable with the t-distribution, small samples inherently carry more uncertainty.

Practical Implications and Best Practices

Ensuring Accurate Confidence Intervals

To improve the reliability of confidence intervals when working with small samples:

- Conduct normality assessments (e.g., Q-Q plots, Shapiro-Wilk test).
- Use data transformations if necessary to approximate normality.
- Consider non-parametric methods like the bootstrap if assumptions are violated.
- Increase sample size where feasible to reduce uncertainty.

Communicating Results Effectively

When reporting confidence intervals:

- Clearly specify the confidence level (e.g., 95%).
- Describe the assumptions and limitations.
- Highlight the degree of uncertainty, especially with small samples.

Real-World Applications

Confidence intervals are used across various fields:

- Medical research estimating average treatment effects.
- Quality control in manufacturing.
- Social sciences assessing average behaviors or attitudes.
- Environmental studies measuring pollutant levels.

In all cases, understanding the nuances of small-sample confidence intervals ensures that interpretations are accurate and decisions are well-informed.

Conclusion: The Power and Caution of Small-Sample Confidence Intervals

Constructing a 95% confidence interval for the population mean from a sample of size 15 involves careful statistical reasoning, primarily relying on the t-distribution due to the small sample size. While this method provides valuable insights into the possible range of the true mean, it also underscores the importance of assumptions, sample quality, and interpretation. Researchers and analysts must recognize that small samples can lead to wider intervals and higher uncertainty, emphasizing the need for robust data collection and cautious inference. Ultimately, confidence intervals serve as vital tools in translating sample data into meaningful, probabilistic statements about the larger population — provided their limitations are acknowledged and appropriately addressed.

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