Suppose X Follows A Continuous Uniform Distribution From 1 To 5. Determine The Conditional Probability

Suppose X Follows A Continuous Uniform Distribution From 1 To 5. Determine The Conditional Probability

Understanding probability distributions is fundamental in statistical analysis and data science. When working with continuous distributions, the uniform distribution is one of the simplest yet most important models because of its equal likelihood across a specified interval. In this article, we will explore how to determine conditional probabilities when a random variable follows a continuous uniform distribution, particularly focusing on the case where X is uniformly distributed from 1 to 5. We will break down the concepts, mathematical derivations, and practical applications, providing a comprehensive guide to solving these types of problems.

Introduction to Uniform Distribution

What Is a Continuous Uniform Distribution?

A continuous uniform distribution describes a scenario where every outcome within a certain interval is equally likely. The probability density function (PDF) for a uniform distribution over the interval $[a, b]$ is given by:

$$
f_X(x) = \frac{1}{b - a} \quad \text{for} \quad a \leq x \leq b
$$

and zero elsewhere.

For our specific case:
- $a = 1$
- $b = 5$

Thus, the PDF becomes:

$$
f_X(x) = \frac{1}{5 - 1} = \frac{1}{4} \quad \text{for} \quad 1 \leq x \leq 5
$$

This means the probability density is uniform across the interval from 1 to 5, with no point within this interval being more likely than any other.

Properties of the Uniform Distribution

Some key properties include:

- Mean (Expected Value):

$$
E[X] = \frac{a + b}{2} = \frac{1 + 5}{2} = 3
$$

- Variance:

$$
Var(X) = \frac{(b - a)^2}{12} = \frac{(5 - 1)^2}{12} = \frac{16}{12} = \frac{4}{3}
$$

- Median:

$$
\frac{a + b}{2} = 3
$$

- Cumulative Distribution Function (CDF):

$$
F_X(x) = P(X \leq x) =
\begin{cases}
0 & x < 1 \\
\frac{x - 1}{4} & 1 \leq x \leq 5 \\
1 & x > 5
\end{cases}
$$

Having established the basics, we can now delve into conditional probability and how it applies within this distribution.

Understanding Conditional Probability

Definition of Conditional Probability

Conditional probability measures the likelihood of an event A occurring given that another event B has already occurred. It is mathematically expressed as:

$$
P(A \mid B) = \frac{P(A \cap B)}{P(B)}
$$

where:
- $P(A \cap B)$ is the joint probability that both A and B happen.
- $P(B)$ is the probability that event B occurs.

In the context of continuous distributions, probabilities are often computed as areas under the PDF or CDF, which require integration over the relevant intervals.

Conditional Probability for Continuous Uniform Distributions

For a uniform distribution on $[a, b]$, the conditional probability can be intuitively understood as the proportion of the interval where the conditioning event occurs relative to the total interval, considering the joint event.

Suppose we are asked to find:

$$
P(X \in C \mid X \in D)
$$

where C and D are intervals within $[a, b]$.

This can be calculated as:

$$
P(X \in C \mid X \in D) = \frac{P(X \in C \cap D)}{P(X \in D)} = \frac{\text{Length of }(C \cap D)}{\text{Length of } D}
$$

since the distribution is uniform, and probabilities are proportional to lengths of intervals.

Note: This simplifies calculations compared to other distributions, which require integration of the PDF over the relevant intervals.

Calculating Conditional Probabilities in the Uniform Distribution

Let's now explore specific cases with our uniform distribution $X \sim U(1, 5)$.

Example 1: Probability that X lies in a subinterval given another interval

Suppose we want to find the probability that X is between 2 and 4, given that X is between 1 and 3:

$$
P(2 \leq X \leq 4 \mid 1 \leq X \leq 3)
$$

Step 1: Determine the intersection

- The event $X \in [2, 4]$
- The conditioning event $X \in [1, 3]$

Their intersection:

$$
[2, 4] \cap [1, 3] = [2, 3]
$$

Step 2: Calculate the lengths

- Length of the intersection: $3 - 2 = 1$
- Length of the conditioning interval: $3 - 1 = 2$

Step 3: Compute the conditional probability

Since the distribution is uniform:

$$
P(2 \leq X \leq 4 \mid 1 \leq X \leq 3) = \frac{\text{length of intersection}}{\text{length of conditioning interval}} = \frac{1}{2}
$$

Thus, the probability is 0.5.

Example 2: Probability that X exceeds a certain value given another interval

Find:

$$
P(X > 3 \mid 2 \leq X \leq 5)
$$

Step 1: Determine the relevant intervals

- Event $X > 3$
- Conditioning event $X \in [2, 5]$

Step 2: Find the intersection

$$
(3, 5] \cap [2, 5] = (3, 5]
$$

\]

Since the distribution is continuous, the probability that X is exactly 5 is zero, so the interval effectively is $(3, 5]$.

Step 3: Calculate lengths

- Intersection length: $5 - 3 = 2$
- Conditioning interval length: $5 - 2 = 3$

Step 4: Compute the probability

$$
P(X > 3 \mid 2 \leq X \leq 5) = \frac{2}{3} \approx 0.6667
$$

This example demonstrates how conditional probabilities relate directly to the proportions of lengths within the distribution's interval.

General Formula for Conditional Probability in a Uniform Distribution

Given a uniform distribution $X \sim U(a, b)$, the conditional probability that X lies within some subinterval $[c, d]$, given that X lies within $[e, f]$, is:

$$
P(c \leq X \leq d \mid e \leq X \leq f) = \frac{\text{length of } [c, d] \cap [e, f]}{\text{length of } [e, f]}
$$

where:

- The intersection length is:

$$
\max(0, \min(d, f) - \max(c, e))
$$

- The denominator is:

$$
f - e
$$

This formula assumes the intervals are within $[a, b]$; otherwise, the probability is zero.

Important considerations:

- When the intervals do not overlap, the conditional probability is zero.
- When the intervals are the same, the probability is 1.
- The key is understanding the proportionality of the overlapping lengths.

Practical Applications of Conditional Probability in Uniform Distributions

Conditional probability calculations for uniform distributions are useful in various fields, including:

- **Quality Control:** Estimating the likelihood of a measurement within a certain tolerance given prior measurements.
- **Risk Assessment:** Determining the probability of an event occurring within a specific range, given some prior condition.
- **Simulation and Modeling:** Generating conditional samples within a uniform distribution for Monte Carlo methods.
- **Decision Making:** Making informed decisions based on conditional likelihoods, especially when outcomes are equally likely over an interval.

Example: Suppose a factory produces items with a uniform defect rate between 1% and 5%. If inspection shows that the defect rate is at most 3%, what is the probability that the defect rate is at least 2%? Here, the uniform distribution simplifies the calculation to a ratio of lengths within the interval.

Conclusion

Determining the conditional probability when a random variable follows a continuous uniform distribution is straightforward due to the equal likelihood across the interval. The key is understanding that these probabilities are proportional to the lengths of the overlapping intervals. For the specific

Frequently Asked Questions

What is the probability density function (pdf) of a continuous uniform distribution from 1 to 5?

The pdf is f(x) = 1/(5 - 1) = 1/4 for x in [1, 5], and 0 elsewhere.

How do you compute the probability that X is between two values, say a and b, in a uniform distribution from 1 to 5?

The probability is $P(a \leq X \leq b) = (b - a) / (5 - 1)$, provided that $1 \leq a \leq b \leq 5$.

What is the conditional probability P(X > 3 | X > 2) for X ~ Uniform(1, 5)?

$P(X > 3 \mid X > 2) = P(3 < X \leq 5) / P(X > 2) = (5 - 3) / (5 - 2) = 2 / 3$.

How do you find the probability that X is less than 4 given that it is greater than 2, for X ~ Uniform(1, 5)?

$P(X < 4 \mid X > 2) = P(2 < X < 4) / P(X > 2) = (4 - 2) / (5 - 2) = 2 / 3$.

If X ~ Uniform(1, 5), what is the probability that X is exactly 3?

For continuous distributions, the probability that X equals any specific value, including 3, is zero.

How can you determine the conditional probability P(1.5 < X < 3.5 | 2 < X < 5) for X ~ Uniform(1, 5)?

$P(1.5 < X < 3.5 \mid 2 < X < 5) = P(2 < X < 3.5) / P(2 < X < 5) = (3.5 - 2) / (5 - 2) = 1.5 / 3 = 0.5$.

What is the general formula for the conditional probability P(A | B) when X follows a uniform distribution?

$P(A \mid B) = P(A \cap B) / P(B)$, where probabilities are computed based on the lengths of intervals within the support, assuming uniformity.

Additional Resources

Suppose X Follows A Continuous Uniform Distribution From 1 To 5: Determine The Conditional Probability

---

Introduction

In the realm of probability theory and statistics, understanding the behavior of random variables under different conditions is fundamental. One such scenario involves a continuous uniform distribution—a distribution characterized by its constant probability density over a specified interval. This article examines the case where a random variable X follows a continuous uniform distribution from 1 to 5, denoted as $X \sim U(1, 5)$, and explores how to determine the conditional probability associated with this variable.

The question of conditional probability is central in statistical inference, decision-making, and probabilistic modeling. It involves calculating the probability of an event given that another event has occurred, which often requires a careful understanding of the joint distribution, marginal probabilities, and the properties of the underlying distribution.

In this detailed review, we will systematically analyze the problem, starting with the foundational concepts of the continuous uniform distribution, progressing to the formal definition of conditional probability, and culminating with the explicit calculation of such probabilities in the given context.

Understanding the Continuous Uniform Distribution

Definition and Properties

A continuous uniform distribution over the interval $[a, b]$ is defined such that the probability density function (PDF) is constant within that interval and zero outside it. Formally, if $X \sim U(a, b)$, then:

$$
f_X(x) =
\begin{cases}
\frac{1}{b - a}, & a \leq x \leq b \\
0, & \text{otherwise}
\end{cases}
$$

Key properties include:

- Constant PDF: The probability density is uniform over $[a, b]$.
- Expected value: $E[X] = \frac{a + b}{2}$.
- Variance: $\text{Var}(X) = \frac{(b - a)^2}{12}$.
- Memoryless property: Not applicable; the uniform distribution is not memoryless like the exponential distribution.

In our case, $a = 1$ and $b = 5$, so:

$$
f_X(x) =
\begin{cases}
\frac{1}{4}, & 1 \leq x \leq 5 \\
0, & \text{otherwise}
\end{cases}
$$

and

$$
E[X] = 3, \quad \text{Var}(X) = \frac{(5 - 1)^2}{12} = \frac{16}{12} = \frac{4}{3}
$$

The Concept of Conditional Probability

Formal Definition

Conditional probability quantifies the likelihood of an event A given that event B has occurred. It is denoted as $P(A \mid B)$ and defined as:

$$
P(A \mid B) = \frac{P(A \cap B)}{P(B)}, \quad \text{provided } P(B) > 0
$$

This definition extends naturally to continuous random variables and their associated events, often involving probability density functions.

Conditional Probability for Continuous Variables

If X and Y are continuous random variables with joint PDF $f_{X,Y}(x, y)$, then the conditional probability density of X given $Y=y$ is:

$$
f_{X|Y}(x|y) = \frac{f_{X,Y}(x, y)}{f_Y(y)}
$$

where $f_Y(y)$ is the marginal PDF of Y. The probability that X lies within a subset A, given $Y = y$, is:

$$
P(X \in A \mid Y = y) = \int_{A} f_{X|Y}(x|y) \, dx
$$

Analysis involving these conditional densities allows us to compute the probabilities of interest under various conditions.

The Problem Setup: Determining the Conditional Probability

Scenario Specification

Suppose $X \sim U(1, 5)$. To evaluate a conditional probability, we need to specify:

- The event for which we want the probability.
- The conditioning event.

For example, common questions might include:

- What is $P(X > 3 \mid X > 2)$?
- What is $P(2 < X < 4 \mid X > 1)$?
- Or more generally, for any interval A, what is $P(X \in A \mid X \in B)$?

In this review, we will explore a general approach, illustrating with concrete examples.

Calculating Conditional Probabilities: Step-by-Step Approach

General Formula for Continuous Uniform Distribution

Given $X \sim U(a, b)$, the conditional probability of X falling within an interval $A = [x_1, x_2]$, given that X is in $B = [c, d]$, where both intervals are subsets of $[a, b]$, is:

$$
P(X \in A \mid X \in B) = \frac{P(X \in A \cap B)}{P(X \in B)} = \frac{\int_{A \cap B} f_X(x) \, dx}{\int_{B} f_X(x) \, dx}
$$

Given the PDF is constant $\frac{1}{b - a}$, this simplifies to:

$$
P(X \in A \mid X \in B) = \frac{\text{length}(A \cap B)}{\text{length}(B)}
$$

assuming $A \cap B$ is non-empty.

Practical Calculation Examples

Example 1: Find $P(X > 3 \mid X > 2)$

- $A = (3, 5]$
- $B = (2, 5]$

Both are within $[1, 5]$. The length of $A \cap B$:

$$
\text{length}((3, 5]) \cap (2, 5]) = (3, 5]
$$

which has length:

$$
5 - 3 = 2
$$

The length of B:

$$
5 - 2 = 3
$$

Hence,

$$
P(X > 3 \mid X > 2) = \frac{2}{3} \approx 0.6667
$$

Example 2: Find $P(2 < X < 4 \mid 1 < X < 5)$

- $A = (2, 4)$
- $B = (1, 5)$

Intersection:

$$
A \cap B = (2, 4)
$$

Length of intersection:

$$
4 - 2 = 2
$$

Length of B:

$$
5 - 1 = 4
$$

Therefore,

$$
P(2 < X < 4 \mid 1 < X < 5) = \frac{2}{4} = 0.5
$$

Application to the Specific Distribution $X \sim U(1, 5)$

Case Study: Conditional Probability Calculation

Suppose we are asked:

"Given that X is between 2 and 4, what is the probability that $X > 3$?"

This is a classic conditional probability question:

$$
P(X > 3 \mid 2 < X < 4)
$$

Using the uniform distribution properties:

- The interval of interest for the conditioning event: $B = (2, 4)$
- The event of interest: $A = (3, 4)$

Since X is uniform over $[1, 5]$, the probability density function is $f_X(x) = \frac{1}{4}$.

The lengths:

$$
\text{length}(A) = 4 - 3 = 1
$$
$$
\text{length}(B) = 4 - 2 = 2
$$
$$
A \cap B = (3, 4) \Rightarrow \text{length} = 1
$$

The probability:

$$
P(X > 3 \mid 2 < X < 4) = \frac{\text{length}(A \cap B)}{\text{length}(B)} = \frac{1}{2} = 0.5
$$

Extending the Analysis: Conditional Distribution

Deriving the Conditional PDF

The conditional PDF of X given $X \in B = [c, d]$ is:

$$
f_{X|X \in B}(x) = \frac{f_X(x)}{P(X \in B)} \quad \text{for } x \in B
$$

Given the uniform distribution:

$$
f_X(x) = \frac{1}{b - a} = \frac{1}{4}
$$

and

$$
P(X \in B) = \frac{\text{length}(B)}{b - a}
$$

Thus,

$$
f_{X|X \in B}(x) = \frac{\frac{1}{4}}{\frac{\text{length}(B)}{4}} =
$$

Suppose X Follows A Continuous Uniform Distribution From 1 To 5 Determine The Conditional Probability

Suppose X Follows A Continuous Uniform Distribution From 1 To 5 Determine The Conditional Probability

Related Articles

- Take A Moment To Explore Your New Classroom And Introduce Yourself To Your Fellow Classmates. What Are
- The Genetic Foundation Of A Particular Phenotype That Includes Contributions Of Dna From Both Parents.
- Stacy Gets Stressed Easily, And When She Does, She Eats An Entire Carton Of Ice Cream, A Bag Of Potato

Back to Home