

The Cross-section Of A Prism Is An Equilateral Triangle Of Side 6cm. The Length Of The Prism Is 20 Cm.

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Understanding the properties and characteristics of prisms is essential in geometry, especially when analyzing their cross-sections and dimensions. In this comprehensive guide, we will explore the details of a prism with a cross-section that is an equilateral triangle of side 6 cm and a length of 20 cm. We will delve into the geometric properties, volume, surface area, and practical applications of such a prism, providing clear explanations and calculations to enhance your understanding.

Introduction to Prisms and Cross-Sections

Before diving into the specifics of the prism in question, it is important to understand the basic concepts related to prisms and cross-sections.

What Is a Prism?

A prism is a three-dimensional geometric solid with two parallel, congruent bases connected by rectangular faces. The shape of the bases determines the type of prism:

- Triangular prism
- Rectangular prism
- Pentagonal prism
- Hexagonal prism, and so on.

Cross-Section of a Prism

A cross-section is the intersection of a solid with a plane. When a plane cuts through a prism parallel to its bases, the cross-section is identical to the shape of the bases. If the cutting plane is inclined or at an angle, the cross-section may be a different shape.

In our case, the cross-section of the prism is an equilateral triangle, which indicates the shape of the base.

Details of the Prism with an Equilateral Triangle Base

Given:

- Cross-section (base shape): Equilateral triangle
- Side length of the equilateral triangle: 6 cm

- Length (height) of the prism: 20 cm

This information defines a right prism with an equilateral triangular base.

Properties of the Equilateral Triangle Base

An equilateral triangle has several specific properties:

- All sides are equal in length.
- All internal angles are 60° .
- The altitude (height) can be calculated using the side length.

For a side length $(a = 6\text{ cm})$:

$$h_{\text{triangle}} = \frac{\sqrt{3}}{2} \times a = \frac{\sqrt{3}}{2} \times 6$$

Calculating:

$$h_{\text{triangle}} = 3\sqrt{3} \approx 3 \times 1.732 = 5.196\text{ cm}$$

- The area of the equilateral triangle:

$$A_{\text{triangle}} = \frac{\sqrt{3}}{4} \times a^2 = \frac{\sqrt{3}}{4} \times 36$$

Calculating:

$$A_{\text{triangle}} = 9\sqrt{3} \approx 9 \times 1.732 = 15.588\text{ cm}^2$$

Calculating the Volume of the Prism

The volume of a prism is given by the product of the area of its base and its length.

Formula for Volume:

$$V = A_{\text{base}} \times \text{length}$$

Given:

- $A_{\text{base}} = 15.588\text{ cm}^2$
- Length $(L = 20\text{ cm})$

Calculating:

$$V = 15.588 \times 20 = 311.76\text{ cm}^3$$

Therefore, the volume of the prism is approximately 311.76 cubic centimeters.

Calculating the Surface Area of the Prism

The surface area of a prism includes the areas of all its faces.

Components of Surface Area:

- Two triangular bases
- Three rectangular lateral faces

Area of the Triangular Bases:

Since there are two bases:

$$A_{\text{total, bases}} = 2 \times 15.588 = 31.176 \text{ cm}^2$$

Area of the Lateral Faces:

Each lateral face is a rectangle with:

- One side equal to the side of the triangle (6 cm)
- The other side equal to the length of the prism (20 cm)

Calculating the area of each rectangle:

$$A_{\text{rect}} = \text{side} \times \text{length} = 6 \times 20 = 120 \text{ cm}^2$$

Since there are three such faces:

$$A_{\text{lateral}} = 3 \times 120 = 360 \text{ cm}^2$$

Total Surface Area:

$$SA = A_{\text{bases}} + A_{\text{lateral}} = 31.176 + 360 = 391.176 \text{ cm}^2$$

Hence, the total surface area of the prism is approximately 391.18 square centimeters.

Visualizing and Sketching the Prism

Creating a visual representation is helpful to understand the structure:

- Draw the equilateral triangle base with sides of 6 cm.
- Extend lines perpendicular to the base for 20 cm.
- Connect corresponding vertices to form the prism.

This visualization aids in comprehending the three-dimensional structure and the relationships between its faces.

Practical Applications of Equilateral Triangular Prisms

Understanding such geometric shapes is useful in various fields:

- Architecture: designing truss structures and roof supports.
- Engineering: manufacturing of components with triangular cross-sections.
- Art and Design: creating aesthetic objects with precise geometric forms.

- Education: teaching geometry concepts and spatial reasoning.

Examples of Practical Uses:

1. Structural beams with triangular cross-sections for strength and stability.
2. Optical devices utilizing prism shapes for light refraction.
3. Packaging and storage solutions requiring specific geometric configurations.

Summary of Key Calculations and Results

Property	Calculation	Result
Altitude of base triangle	$\left(\frac{\sqrt{3}}{4} \times 6^2\right)$	5.196 cm
Area of base triangle	$\left(\frac{\sqrt{3}}{4} \times 6^2\right)$	15.588 cm ²
Volume of prism	(15.588×20)	311.76 cm ³
Area of both bases	(2×15.588)	31.176 cm ²
Area of lateral faces	(3×120)	360 cm ²
Total surface area	$31.176 + 360$	391.176 cm ²

These calculations provide a comprehensive understanding of the dimensions and properties of the prism with an equilateral triangle base of side 6 cm and length 20 cm.

Conclusion

The prism with an equilateral triangle base of side 6 cm and length 20 cm is a classic example of a right prism with a regular polygonal base. Its geometric properties, such as volume and surface area, can be precisely calculated using fundamental formulas. Recognizing these properties aids in various practical and educational applications, from engineering designs to mathematical teaching. Whether you are a student, educator, or professional, understanding the structure and calculations related to such prisms enhances your grasp of three-dimensional geometry and spatial reasoning.

If you wish to explore further, consider variations such as changing the base shape, the side lengths, or the length of the prism to see how these factors influence the overall properties.

Frequently Asked Questions

What is the area of the cross-section of the prism?

The cross-section is an equilateral triangle with side length 6 cm. The area is $(\sqrt{3}/4) \times \text{side}^2 = (\sqrt{3}/4) \times 36 = 9\sqrt{3} \text{ cm}^2$.

What is the volume of the prism?

The volume is the area of the cross-section multiplied by the length of the prism: $9\sqrt{3} \text{ cm}^2 \times 20 \text{ cm} = 180\sqrt{3} \text{ cm}^3$.

How do you calculate the surface area of this prism?

The surface area includes the lateral surface area and the area of the two triangular bases. Lateral area = perimeter of base $\times$ length = $(3 \times 6) \times 20 = 360 \text{ cm}^2$. Total surface area = $2 \times$ area of triangle + lateral area = $2 \times 9\sqrt{3} + 360 \text{ cm}^2$.

What is the perimeter of the cross-sectional triangle?

Since the cross-section is an equilateral triangle with side 6 cm, its perimeter is $3 \times 6 = 18 \text{ cm}$.

If the prism is cut perpendicular to its length, what shape will the cross-section be?

The cross-section will remain an equilateral triangle of side 6 cm, as the prism has a uniform cross-sectional shape along its length.

What is the height of the equilateral triangle cross-section?

The height h of an equilateral triangle with side s is $(\sqrt{3}/2) \times s = (\sqrt{3}/2) \times 6 = 3\sqrt{3} \text{ cm}$.

Additional Resources

The Cross-section Of A Prism Is An Equilateral Triangle Of Side 6cm. The Length Of The Prism Is 20 Cm.

Understanding the geometric properties and volume calculations of three-dimensional objects forms a fundamental part of mathematics and engineering disciplines. Among these objects, prisms hold a special place due to their practical applications in construction, manufacturing, and design. This article explores a specific type of prism characterized by an equilateral triangular cross-section with sides measuring 6 centimeters and a length of 20 centimeters. We will delve into its geometric features, surface area, volume, and real-world applications, providing a comprehensive yet accessible overview for students, educators, and professionals alike.

Defining the Prism: Basic Geometric Framework

What Is a Prism?

A prism is a three-dimensional geometric solid that features two congruent, parallel bases connected by rectangular faces. The shape of the bases determines the classification of the prism: rectangular, triangular, pentagonal, etc. The sides connecting corresponding edges of the bases are called lateral faces.

In this context, the prism under discussion has an equilateral triangular base, making it a triangular prism.

Characteristics of the Equilateral Triangle Cross-section

An equilateral triangle is a triangle with all three sides equal in length. Its properties include:

- Equal sides and angles, with each interior angle measuring 60° .
- Symmetry about its height, median, and angle bisectors, which coincide.

Given the side length of 6 cm, the equilateral triangle's properties will influence the prism's surface area and volume calculations.

Dimensions of the Prism

- Base: Equilateral triangle with side length = 6 cm.
- Length (Height of the Prism): 20 cm.

These dimensions define the shape entirely and allow for precise mathematical computations.

Geometric Properties of the Triangular Cross-section

Calculating the Area of the Equilateral Triangle Base

The area of an equilateral triangle with side length (a) is given by:

$$A_{\text{base}} = \frac{\sqrt{3}}{4} a^2$$

Plugging in $(a = 6\text{ cm})$:

$$A_{\text{base}} = \frac{\sqrt{3}}{4} \times 6^2 = \frac{\sqrt{3}}{4} \times 36 = 9\sqrt{3}\text{ cm}^2$$

Numerically,

$$A_{\text{base}} \approx 9 \times 1.732 = 15.588 \text{ cm}^2$$

This value represents the area of each triangular cross-section at the base of the prism.

Perimeter of the Cross-section

Since the cross-section is an equilateral triangle with side 6 cm:

$$P_{\text{triangle}} = 3 \times 6 = 18 \text{ cm}$$

This perimeter is useful when calculating lateral surface areas.

Surface Area of the Prism

The total surface area of a prism combines the areas of its two bases and its lateral faces.

Components of Surface Area

- Base Areas: Since the prism has two congruent bases,

$$A_{\text{total, bases}} = 2 \times 15.588 \approx 31.176 \text{ cm}^2$$

- Lateral Surface Area: Consists of three rectangles, each corresponding to a side of the triangle extended along the length of the prism.

Each rectangle has dimensions:

- Width: equal to the side of the triangle (6 cm).
- Length: equal to the prism's length (20 cm).

The lateral surface area (LSA):

$$LSA = P_{\text{triangle}} \times \text{length} = 18 \text{ cm} \times 20 \text{ cm} = 360 \text{ cm}^2$$

Total Surface Area (SA):

$$SA = 2A_{\text{base}} + LSA = 31.176 + 360 = 391.176 \text{ cm}^2$$

This surface area calculation is crucial for applications involving material estimation, such as covering or painting the prism.

Volume of the Prism

The volume of a prism is determined by the area of its base multiplied by its length:

$$V = A_{\text{base}} \times \text{length}$$

Using the previous calculation:

$$V = 15.588 \text{ cm}^2 \times 20 \text{ cm} = 311.76 \text{ cm}^3$$

This volume indicates the capacity of the prism, relevant in contexts like storage, manufacturing, or fluid containment.

Deeper Mathematical Insights and Variations

Surface Area and Volume in Context

These calculations serve as fundamental benchmarks in engineering design. For example:

- Construction: Knowing the surface area helps estimate the amount of material needed to manufacture the prism.
- Storage: Volume calculations inform capacity planning.
- Manufacturing Tolerances: Precise dimensions impact the fit and function of components.

Extensions and Variations

While this article addresses a prism with a fixed equilateral triangular cross-section, variations can include:

- Changing the side length of the triangle to see effects on volume and surface area.
- Varying the length to observe linear relationships.
- Considering non-equilateral triangular bases, which involve more complex calculations.

These variations demonstrate the versatility of geometric principles across different real-world applications.

Applications of an Equilateral Triangular Prism

Prisms with equilateral triangular bases are more than theoretical constructs—they feature prominently in various fields:

- Architecture: Used in roof designs and decorative elements due to their strength and aesthetic appeal.
- Manufacturing: Components like beams, supports, or containers often utilize triangular cross-sections for stability.
- Optics and Science: Certain prism shapes are used to manipulate light paths, making understanding their geometry essential.
- Education: Demonstrating properties of three-dimensional shapes and fostering spatial reasoning.

Practical Considerations and Real-World Construction

When translating these calculations to real-world scenarios, factors such as material thickness, manufacturing tolerances, and environmental conditions come into play. For instance:

- Material Selection: Depending on the application—metal, plastic, or wood—the fabrication process and precise measurements may vary.
- Structural Integrity: The equilateral triangle provides inherent stability, making such prisms suitable for load-bearing applications.
- Cost Estimation: Accurate surface area and volume calculations help determine material costs and manufacturing time.

Conclusion: The Significance of Geometric Precision

Understanding the properties of a prism with an equilateral triangle cross-section of side 6 cm and length 20 cm offers valuable insights into the intersection of geometry and practical application. From calculating surface areas for material estimation to understanding volume for capacity planning, these fundamental principles underpin many engineering and design projects. Mastery of such calculations not only enhances mathematical literacy but also equips professionals with the tools necessary for innovation and efficiency in diverse industries.

Whether in the classroom, the workshop, or the laboratory, grasping the detailed properties of such geometric structures fosters a deeper appreciation of the elegance and utility of mathematics in shaping our physical world.

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