The Equation $2x^2 + X - 1 = 0$ Has Two Solutions. Find An Equation Of The Form $Ax^2 + Bx + C = 0$, Which

The Equation $2x^2 + X - 1 = 0$ Has Two Solutions. Find An Equation Of The Form $Ax^2 + Bx + C = 0$, Which

Understanding quadratics and their solutions is fundamental in algebra. The given quadratic equation, $2x^2 + X - 1 = 0$, presents an interesting problem: it is known to have two solutions, and the task is to find an equation of the standard quadratic form, $Ax^2 + Bx + C = 0$, that satisfies certain conditions derived from or related to the original equation. This article delves into the principles behind quadratic equations, explores the conditions for having two solutions, and guides you through constructing such an equation systematically.

Analyzing the Given Equation and Its Solutions

Understanding the Provided Equation

The provided quadratic equation appears to be written as $2x^2 + X - 1 = 0$. It's important to clarify that the notation 'X' likely refers to the variable x, assuming a typographical inconsistency. So, the equation probably is:

- $2x^2 + x - 1 = 0$

This standard form allows us to analyze the properties of the solutions.

Number of Solutions and Discriminant

For any quadratic equation $Ax^2 + Bx + C = 0$, the number of solutions depends on the discriminant, D, given by:

$$D = B^2 - 4AC$$

- If $D > 0$, the equation has two distinct real solutions.
- If $D = 0$, the solutions are real and equal (a repeated root).
- If $D < 0$, the solutions are complex conjugates.

Given that the original equation has two solutions, it implies:

$$D = B^2 - 4AC > 0$$

For the specific equation $2x^2 + x - 1 = 0$, the discriminant is:

$$D = (1)^2 - 4 \times 2 \times (-1) = 1 - (-8) = 1 + 8 = 9$$

Since $9 > 0$, the equation indeed has two distinct real solutions.

Constructing a Related Quadratic Equation with Specific Properties

The core of the problem is to find an equation of the form $Ax^2 + Bx + C = 0$, which shares some property or satisfies a particular condition derived from or related to the original equation that has two solutions.

Possible Conditions for the New Equation

While the problem statement is partial, typical conditions for such problems include:

- The new equation has the same solutions as the original.
- The new equation has solutions related to the original solutions (e.g., reciprocals, negatives).
- The new equation has solutions that satisfy certain sum or product conditions.

In this context, let's explore these possibilities and how to construct such an equation.

Approach 1: Same Solutions as the Original Equation

Using the Roots of the Original Equation

Suppose the original quadratic has roots α and β. We know:

$$\alpha + \beta = -\frac{B}{A}$$
$$\alpha \beta = \frac{C}{A}$$

For the original equation $2x^2 + x - 1=0$:

- $A = 2$
- $B = 1$
- $C = -1$

Calculating sum and product:

$$ \alpha + \beta = -\frac{1}{2} $$
$$ \alpha \beta = -\frac{1}{2} $$

Any quadratic with the same roots will be of the form:

$$ x^2 - (\alpha + \beta) x + \alpha \beta = 0 $$

or scaled accordingly.

Constructing an Equation with the Same Roots

Multiplying through by an arbitrary non-zero constant K, the quadratic becomes:

$$ Kx^2 - K (\alpha + \beta) x + K \alpha \beta = 0 $$

or explicitly:

$$ Kx^2 + \frac{K}{2} x - \frac{K}{2} = 0 $$

Choosing $K = 2$ for simplicity:

$$ 2x^2 + x - 1 = 0 $$

which is identical to the original. Alternatively, choosing other values of K gives similar equations, scaled versions of the original.

Conclusion: Any quadratic with roots α and β can be written as:

$$ x^2 - (\alpha + \beta) x + \alpha \beta = 0 $$

which, in this case, is:

$$ x^2 + \frac{1}{2} x - \frac{1}{2} = 0 $$

Multiplying by 2, we get:

$$ 2x^2 + x - 1 = 0 $$

which is our original equation.

Approach 2: Equation with Roots Related to the Original Roots

Suppose you want a quadratic equation whose solutions are related to the original roots \(

α and β. Common relations include:

- The roots are reciprocals: $\frac{1}{\alpha}$ and $\frac{1}{\beta}$
- The roots are negatives: $-\alpha$, $-\beta$
- The roots are shifted by a constant

Let's analyze the reciprocal roots case.

Equation with Roots as Reciprocals

The roots $\frac{1}{\alpha}$ and $\frac{1}{\beta}$ have sum and product:

$$S' = \frac{1}{\alpha} + \frac{1}{\beta} = \frac{\alpha + \beta}{\alpha \beta}$$
$$P' = \frac{1}{\alpha \beta}$$

From earlier, $\alpha + \beta = -\frac{1}{2}$, and $\alpha \beta = -\frac{1}{2}$. Therefore:

$$S' = \frac{-\frac{1}{2}}{-\frac{1}{2}} = 1$$
$$P' = \frac{1}{-\frac{1}{2}} = -2$$

The quadratic with roots $\frac{1}{\alpha}$ and $\frac{1}{\beta}$:

$$x^2 - S' x + P' = 0$$
$$x^2 - 1 x - 2 = 0$$
or simply:

$$x^2 - x - 2 = 0$$

This quadratic has solutions that are reciprocals of the original roots.

Approach 3: Constructing an Equation with Prescribed Roots

Suppose the problem asks for any quadratic $Ax^2 + Bx + C = 0$ with two solutions that satisfy certain criteria, such as:

- The sum of solutions is S.
- The product of solutions is P.

Given the original solutions, the sum and product are known, so any quadratic with those roots or related roots can be constructed.

Examples of Such Equations

- Using the original roots:

$$ x^2 + \frac{1}{2} x - \frac{1}{2} = 0 $$

- Using reciprocals:

$$ x^2 - x - 2 = 0 $$

- Using shifted roots:

Suppose the roots are shifted by a constant k, then the roots become $\alpha + k$ and $\beta + k$. The sum and product change accordingly, leading to a new quadratic.

Summary and Final Remarks

In this exploration, we've examined how the original quadratic equation $2x^2 + x - 1=0$, which has two real solutions, can be used to construct related quadratic equations with specific properties.

Key points include:

- The discriminant determines the number and nature of solutions.
- Roots can be expressed explicitly via the quadratic formula:

$$ x = \frac{-B \pm \sqrt{D}}{2A} $$

- Equations with the same roots can be constructed by manipulating the sum and product of roots.
- Roots related through inversion or shifting lead to new quadratics with predictable solutions.
- The flexibility in choosing the coefficient A, B, and C allows for a variety of equations sharing certain solution properties.

Practical steps for constructing such equations:

1. Determine the roots of the original equation using the quadratic formula.
2. Calculate sum and product of the roots.
3. Use Vieta's formulas to construct new quadratics with desired roots or properties.
4. Adjust coefficients to meet specific criteria or to generate scaled versions.

Final note: The process of constructing quadratic equations with particular solution characteristics is a fundamental skill in algebra, useful in various applications such as solving word problems, designing functions with specific behaviors, and understanding the relationships between roots and coefficients.

In summary, starting from the original quadratic $2x^2 + x - 1=0$, which has two solutions, we

Frequently Asked Questions

Given the quadratic equation $2x^2 + x - 1 = 0$ has two solutions, how can we find a similar equation with different coefficients but also two solutions?

You can choose different values for A, B, and C such that the discriminant $D = B^2 - 4AC$ is positive. For example, setting A=3, B=4, C=1 gives $3x^2 + 4x + 1 = 0$ with two solutions because its discriminant is $16 - 12 = 4 > 0$.

What condition must an quadratic equation satisfy to have two real solutions?

The discriminant, $D = B^2 - 4AC$, must be greater than zero ($D > 0$).

If $2x^2 + x - 1 = 0$ has two solutions, can we find an equation of the form $Ax^2 + Bx + C = 0$ with different coefficients that also has two solutions?

Yes. For instance, $5x^2 + 3x + 2 = 0$ has a discriminant of $3^2 - 452 = 9 - 40 = -31$, which is negative; so it doesn't have two real solutions. To ensure two solutions, choose coefficients such that the discriminant is positive. For example, $4x^2 + 3x + 1 = 0$ has $9 - 16 = -7$; still negative. Instead, $4x^2 + 5x + 1 = 0$ has $25 - 16 = 9 > 0$, so it has two solutions.

How do you determine the coefficients A, B, and C for a quadratic equation to ensure it has two solutions?

Calculate the discriminant $D = B^2 - 4AC$. To have two solutions, set A, B, and C so that $D > 0$. For example, choosing A=1, B=4, C=1 yields $D=16-4=12 > 0$, so the quadratic has two solutions.

Can you give an example of a quadratic equation of the form $Ax^2 + Bx + C=0$ that has two solutions, different from $2x^2 + x - 1=0$?

Yes. For example, $3x^2 - 2x + 1=0$ has a discriminant of $(-2)^2 - 431 = 4 - 12 = -8$, which is negative. To have two solutions, choose coefficients like $3x^2 + 4x + 1=0$, which has discriminant $16 - 12=4>0$.

If the original equation has two solutions, does modifying the coefficients always result in an equation with two solutions?

Not necessarily. Modifying the coefficients can lead to a discriminant that is zero or negative, which means the equation has one or no real solutions. To ensure two solutions, the coefficients must be chosen such that the discriminant remains positive.

What is the relationship between the coefficients A, B, C and the number of solutions of the quadratic equation?

The key factor is the discriminant D = B^2 - 4AC. If D > 0, the quadratic has two real solutions; if D = 0, one real solution; and if D < 0, no real solutions.

How can we write a general quadratic equation of the form Ax^2 + Bx + C=0 that guarantees two solutions?

Choose any A, B, and C such that the discriminant D = B^2 - 4AC > 0. For example, A=1, B=5, C=1 gives 25 - 4=21 > 0, ensuring two solutions.

Is it possible to find an equation similar to 2x^2 + x - 1=0 that has only one solution?

Yes. When the discriminant D = 0, the quadratic has exactly one real solution (a repeated root). For example, 2x^2 + 2x - 1=0 has discriminant 4 - 42-1=4+8=12 > 0, so two solutions. To get one solution, set coefficients so D=0, for example, 2x^2 + x - 0.5=0 has discriminant 1 - 42(-0.5)=1+4=5>0, so two solutions. For a single solution, try 2x^2 + x + 0.25=0, where discriminant is 1 - 420.25=1-2= -1<0, no real solutions. So, to get one real solution, coefficients must satisfy B^2=4AC.

Additional Resources

The Equation 2x^2 + X - 1 = 0 Has Two Solutions. Find An Equation Of The Form Ax^2 + Bx + C = 0, Which is a thought-provoking problem that delves into the fundamental principles of quadratic equations. This statement invites learners and mathematicians alike to explore the nature of quadratic solutions, understand the conditions for two real roots, and ultimately craft a new quadratic equation with specific characteristics. Such exercises are not only academic but also enhance problem-solving skills, deepen algebraic understanding, and prepare students for more complex mathematical applications. In this comprehensive review, we will analyze the core concepts behind quadratic equations, interpret the original problem, and guide you through the process of constructing a new quadratic equation of the form Ax^2 + Bx + C = 0 that satisfies particular solution criteria.

Understanding Quadratic Equations and Their Solutions

What Is a Quadratic Equation?

A quadratic equation is a second-degree polynomial equation of the form:

$$ Ax^2 + Bx + C = 0 $$

where A, B, and C are constants, with $A \neq 0$. The solutions to this equation, known as roots or zeros, are the values of x that satisfy the equation.

Methods of Solving Quadratic Equations

Quadratic equations can be solved through various methods, each with its advantages:

- Factoring: Useful when the quadratic factors neatly into binomials.
- Completing the Square: Converts the quadratic into a perfect square trinomial.
- Quadratic Formula: A universal method applicable to all quadratics:

$$
x = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A}
$$

The discriminant, $D = B^2 - 4AC$, determines the nature of the roots:
- If $D > 0$, there are two distinct real solutions.
- If $D = 0$, there is exactly one real solution (a repeated root).
- If $D < 0$, solutions are complex conjugates.

Analyzing the Original Equation: 2x^2 + X - 1 = 0

Clarifying the Equation

The problem statement appears to have a typographical inconsistency with the variable notation: "2x^2 + X - 1 = 0". Typically, in algebra, standard practice is to use a consistent variable notation. It's reasonable to interpret "X" as a variable x, making the equation:

$$ 2x^2 + x - 1 = 0 $$

Alternatively, if "X" is a coefficient, the problem would require clarification. Given the context, the most logical assumption is that the original quadratic is:

$$ 2x^2 + x - 1 = 0 $$

This is a standard quadratic with coefficients $A=2$, $B=1$, and $C=-1$.

Solution and Discriminant of the Original Equation

Calculating the discriminant:

$$
D = B^2 - 4AC = (1)^2 - 4 \times 2 \times (-1) = 1 + 8 = 9
$$

Since $D = 9 > 0$, the quadratic has two distinct real solutions:

$$
x = \frac{-1 \pm \sqrt{9}}{2 \times 2} = \frac{-1 \pm 3}{4}
$$

This yields solutions:

$$
x = \frac{-1 + 3}{4} = \frac{2}{4} = \frac{1}{2}
$$

and

$$
x = \frac{-1 - 3}{4} = \frac{-4}{4} = -1
$$

Thus, the original quadratic equation has two solutions, $x = \frac{1}{2}$ and $x = -1$.

Constructing a New Quadratic Equation with Specific Solution Features

General Approach

Given the solutions of the original quadratic, the task is to find an equation of the form:

$$ A x^2 + B x + C = 0 $$

which has particular solution properties, such as:

- Having the same solutions as the original ($x = \frac{1}{2}$ and $x = -1$)

- Having solutions that are related or transformed versions
- Ensuring the quadratic has two solutions (real and distinct)

The key is understanding how solutions relate to the coefficients via Vieta's formulas:

$$
x_1 + x_2 = -\frac{B}{A}
$$
$$
x_1 x_2 = \frac{C}{A}
$$

Using these relations, we can reverse engineer the desired quadratic.

Constructing an Equation with the Same Solutions

Suppose the goal is to find an equation that shares the same roots, $x = \frac{1}{2}$ and $x = -1$. The quadratic with these roots, by Vieta's formula, is:

$$
(x - \frac{1}{2})(x + 1) = 0
$$

Expanding:

$$
x^2 + x - \frac{1}{2}x - \frac{1}{2} = 0
$$

Simplify:

$$
x^2 + \left(1 - \frac{1}{2}\right) x - \frac{1}{2} = 0
$$
$$
x^2 + \frac{1}{2} x - \frac{1}{2} = 0
$$

Multiplying through by 2 to clear fractions:

$$
2x^2 + x - 1 = 0
$$

Interestingly, this is the original quadratic! Therefore, the original quadratic already has roots $x = \frac{1}{2}$ and $x = -1$.

Features of this quadratic:
- Coefficients: $A=2$, $B=1$, $C=-1$
- Solutions: as calculated, $x = \frac{1}{2}$ and $x = -1$

- Discriminant: 9, confirming two solutions

Developing Variations of the Quadratic Equation

Creating an Equation with Different Coefficients but Same Solutions

Suppose we want to generate an equation of the form $A x^2 + B x + C = 0$ that has the same solutions but different coefficients. Since roots are determined by the ratio of coefficients, scale the original quadratic by any non-zero constant k:

$$
k(2x^2 + x - 1) = 0
$$

which simplifies to:

$$
(2k) x^2 + k x - k = 0
$$

This quadratic shares the same roots because scaling does not affect roots, only the leading coefficient and other coefficients proportionally.

For example, choosing $k=3$:

$$
6x^2 + 3x - 3 = 0
$$

This quadratic has solutions $x = \frac{1}{2}$ and $x = -1$, just scaled.

Features:
- The solutions remain unchanged.
- The coefficients are scaled by k.
- The quadratic's shape (parabola) is scaled vertically.

Constructing a Quadratic with Different Roots

Suppose the goal is to create a quadratic with two solutions but different from the original roots, for example, roots at $x=1$ and $x=2$.

Using Vieta's:

\[

$$
x_1 + x_2 = 1 + 2 = 3
$$

$$
x_1 x_2 = 1 \times 2 = 2
$$

The quadratic with these roots is:

$$
(x - 1)(x - 2) = 0
$$

Expanding:

$$
x^2 - 3x + 2 = 0
$$

This quadratic has two solutions, both real and distinct, and differs from the original. It demonstrates the flexibility of constructing quadratics with specified roots.

Features, Pros, and Cons of Different Quadratic Constructions

Features:

- Scaling: Multiplied quadratic equations preserve roots but change coefficients proportionally.
- Root manipulation: Selecting roots intentionally allows for custom quadratic equations.
- Discriminant control: Adjusting coefficients can influence whether roots are real or complex.

Pros:

- Flexibility in designing quadratic equations with desired roots.
- Clear relationship between roots and coefficients via Vieta’s formulas.
- Ability to generate infinitely many equations with specified solution sets.

Cons:

- Changing coefficients without understanding roots may lead to incorrect assumptions about solution nature.
- Not all quadratic equations with arbitrary coefficients have real solutions.
- Scaling can sometimes produce coefficients difficult to interpret in applied contexts.

Conclusion: Mastering the Creation and Analysis of Quadratic Equations

The original problem, involving the quadratic $2x^2 + x - 1 = 0$, provides a rich foundation for understanding quadratic solutions and their properties.

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