The Equation Of A Circle Is X2+y212x+6y+20=0.What Is The Radius Of The Circle?Enter Your Answer In The

The Equation Of A Circle Is X2+y212x+6y+20=0.What Is The Radius Of The Circle?Enter Your Answer In The article aims to clarify how to find the radius of a circle given its general equation, specifically focusing on the equation $X^2 + y^2 + 12x + 6y + 20 = 0$. Understanding how to interpret and manipulate the equation of a circle is essential for students, teachers, and anyone interested in coordinate geometry. This comprehensive guide will walk you through the process step-by-step, highlighting key concepts and providing practical tips to solve similar problems efficiently.

Understanding the Equation of a Circle

The equation of a circle in the Cartesian coordinate plane is generally expressed as:

$$ (x - h)^2 + (y - k)^2 = r^2 $$

where:

- (h, k) is the center of the circle.
- r is the radius of the circle.

This is called the standard form of the circle's equation. When an equation is given in the expanded form, such as:

$$ x^2 + y^2 + Dx + Ey + F = 0 $$

it can be converted to standard form through a process called completing the square. Once in standard form, the radius can be directly obtained as:

$$ r = \sqrt{(h)^2 + (k)^2} $$

or simply the square root of the constant term on the right side of the equation.

Analyzing the Given Equation

The given equation is:

$$ x^2 + y^2 + 12x + 6y + 20 = 0 $$

Notice that this is in the general quadratic form:

$$ x^2 + y^2 + Dx + Ey + F = 0 $$

where:

- $D = 12$
- $E = 6$
- $F = 20$

Our goal is to convert this into the standard form to identify the center and the radius.

Step-by-Step Solution to Find the Radius

Step 1: Group x and y terms

Start by grouping the x and y terms:

$$ (x^2 + 12x) + (y^2 + 6y) = -20 $$

Step 2: Complete the square for x and y

Completing the square involves rewriting quadratic expressions in the form $(x + a)^2$.

For x:

- Take half of the coefficient of x (which is 12):

$$ \frac{12}{2} = 6 $$

- Square it:

$$ 6^2 = 36 $$

For y:

- Take half of the coefficient of y (which is 6):

$$ \frac{6}{2} = 3 $$

- Square it:

$$ 3^2 = 9 $$

Now, add these squares inside the respective groups, but to keep the equation balanced, we also subtract these same values.

$$ (x^2 + 12x + 36) + (y^2 + 6y + 9) = -20 + 36 + 9 $$

This simplifies to:

$$ (x + 6)^2 + (y + 3)^2 = 25 $$

Step 3: Write the equation in standard form

The equation becomes:

$$ (x + 6)^2 + (y + 3)^2 = 25 $$

This is now in the standard form:

$$ (x - h)^2 + (y - k)^2 = r^2 $$

where:

- $h = -6$
- $k = -3$
- $r^2 = 25$

Finding the Radius of the Circle

Given the standard form, the radius r is:

$$ r = \sqrt{25} = 5 $$

Therefore, the radius of the circle is 5.

Key Points to Remember

- Converting the general quadratic form of a circle to standard form involves completing the square.

- The coefficients of x and y are used to determine the terms needed to complete the square.
- The constant term on the right side of the standard form is the square of the radius.
- The center of the circle can be identified from the standard form as $(-h, -k)$.

Additional Tips for Solving Circle Equations

- Always check the signs before completing the square; remember, adding the square terms inside the equation requires adding the same to the other side.
- If the right side of the equation is not positive, the given equation may not represent a circle.
- Practice with various equations to become comfortable with completing the square and identifying the circle's properties.

Conclusion

Understanding how to find the radius of a circle from its algebraic equation is a fundamental skill in coordinate geometry. The key steps involve rewriting the equation in standard form through completing the square, then extracting the radius from the constant term. For the given equation:

$$ x^2 + y^2 + 12x + 6y + 20 = 0 $$

the process reveals that the circle's radius is 5 units. Mastery of these techniques allows for quick analysis of circles in various contexts, including geometry problems, real-world applications, and mathematical modeling.

Frequently Asked Questions (FAQs)

1. **What is the standard form of a circle's equation?**
   It is $(x - h)^2 + (y - k)^2 = r^2$, where (h, k) is the center and r is the radius.

2. **How do I complete the square for the x and y terms?**
   Take half of the coefficient of the linear term, square it, and add inside the parentheses, adjusting the other side of the equation accordingly.

3. **Can the equation have a negative right side?**

No, a circle's equation in standard form has a positive number on the right side, representing r^2.

4. **What if the equation is not in quadratic form?**
   You need to manipulate it into quadratic form and complete the square to find the circle's parameters.

In summary, converting the given equation to standard form reveals that the circle has a radius of 5 units. Understanding this process enhances your ability to analyze and interpret circle equations in algebra and coordinate geometry effectively.

Frequently Asked Questions

What is the standard form of the circle equation given as $x^2 + y^2 + 12x + 6y + 20 = 0$?

The standard form of the circle equation is $(x + 6)^2 + (y + 3)^2 = r^2$.

How do you find the radius of a circle from its equation in the form $x^2 + y^2 + Ax + By + C = 0$?

First, complete the square for x and y terms to rewrite the equation in standard form: $(x + h)^2 + (y + k)^2 = r^2$. The radius is then $\sqrt{(r^2)}$.

Given the circle equation $x^2 + y^2 + 12x + 6y + 20 = 0$, what is the value of the radius?

After completing the square, the radius is $\sqrt{(25)} = 5$.

What are the steps to determine the radius from the circle's general equation?

1. Group x and y terms. 2. Complete the square for both. 3. Rewrite in standard form to identify the radius as the square root of the constant term.

Can you show the detailed calculation to find the radius of the circle from the equation $x^2 + y^2 + 12x + 6y + 20 = 0$?

Yes. Complete the square:
For x: $x^2 + 12x = (x + 6)^2 - 36$

For y: $y^2 + 6y = (y + 3)^2 - 9$
Rewrite the equation:
$(x + 6)^2 - 36 + (y + 3)^2 - 9 + 20 = 0$
Simplify:
$(x + 6)^2 + (y + 3)^2 = 25$
Radius = $\sqrt{25} = 5$.

What is the importance of completing the square when finding the radius of a circle from its equation?

Completing the square transforms the general quadratic form into the standard form, allowing direct identification of the circle's center and radius.

If the circle's equation is $x^2 + y^2 + 14x + 8y + 36 = 0$, what is its radius?

Complete the square:
$(x + 7)^2 - 49 + (y + 4)^2 - 16 + 36 = 0$
Simplify:
$(x + 7)^2 + (y + 4)^2 = 29$
Radius = $\sqrt{29}$.

How would you enter the radius of the circle in a numerical form after calculation?

Calculate the square root of the constant term after completing the square; for example, if the constant is 25, enter 5.

What is the final answer for the radius of the circle given by the equation $x^2 + y^2 + 12x + 6y + 20 = 0$?

The radius of the circle is 5.

Additional Resources

Understanding the Equation of a Circle and Calculating Its Radius

When delving into the world of conic sections, the circle stands out as one of the most fundamental and symmetric shapes in geometry. Its properties are not only aesthetically pleasing but also mathematically significant, making it a core concept in algebra, geometry, and even advanced fields like calculus and physics. A common challenge faced by students and enthusiasts alike is deciphering the standard forms of circle equations and extracting key parameters such as the radius from a given algebraic expression.

This comprehensive review aims to elucidate how to analyze the provided equation:

$X^2 + y^2 + 12x + 6y + 20 = 0$

and determine the radius of the circle it represents. We will explore the theory behind circle equations, demonstrate the step-by-step process of completing the square, and interpret the resulting standard form to find the radius.

Fundamental Concepts of the Equation of a Circle

Standard Form of a Circle Equation

The most common and recognizable form of a circle's equation in the Cartesian plane is:

$(x - h)^2 + (y - k)^2 = r^2$

where:

- (h, k) are the coordinates of the circle's center.
- r is the radius of the circle.

This form makes it straightforward to identify the center and radius once the equation is in standard form.

General Form of a Circle Equation

The general quadratic form of a circle's equation is:

$x^2 + y^2 + Dx + Ey + F = 0$

where D, E, and F are constants.

To convert this into the standard form, we need to complete the square for both x and y terms. This process will reveal the center coordinates and the radius.

Analyzing the Given Equation

Let’s examine the specific equation provided:

$X^2 + y^2 + 12x + 6y + 20 = 0$

Note: The original problem mentions X2 + y212x + 6y + 20 = 0, but this appears to be a typographical error. Based on context and standard equations, it likely intends to be:

$x^2 + y^2 + 12x + 6y + 20 = 0$

Assuming this correction, we proceed with this form.

Step-by-Step Solution: Finding the Radius

Step 1: Rewrite the Equation

The equation is:

$x^2 + y^2 + 12x + 6y + 20 = 0$

Our goal is to rewrite it in the standard form:

$(x - h)^2 + (y - k)^2 = r^2$

To do this, we need to complete the square for the x and y terms.

Step 2: Group x and y Terms

Group the x and y terms together:

$(x^2 + 12x) + (y^2 + 6y) = -20$

Step 3: Complete the Square

For the x terms:

- Take half of the coefficient of x, which is 12/2 = 6.
- Square it: $6^2 = 36$.

For the y terms:

- Take half of the coefficient of y, which is 6/2 = 3.

- Square it: $3^2 = 9$.

Add these squares to both sides of the equation to maintain equality:

$(x^2 + 12x + 36) + (y^2 + 6y + 9) = -20 + 36 + 9$

Simplify the right side:

$-20 + 36 + 9 = 25$

Step 4: Write in Completed Square Form

Express the grouped terms as perfect squares:

$(x + 6)^2 + (y + 3)^2 = 25$

This is the standard form of the circle's equation.

Interpreting the Standard Form

From the standard form:

$(x + 6)^2 + (y + 3)^2 = 25$

we identify:

- Center: $(-6, -3)$, since $(x - h)^2$ and $(y - k)^2$ are in the form, and the signs are opposite to the signs inside the parentheses.
- Radius: $\sqrt{25} = 5$

Conclusion: What Is the Radius?

The radius of the circle is 5 units.

Additional Insights and Related Concepts

Implications of the Center and Radius

Knowing the center and radius allows for various geometric and algebraic applications:

- Drawing the circle accurately on a coordinate plane.
- Determining points lying on the circle.
- Calculating distances between the circle and other geometric objects.
- Finding the area (πr^2) and circumference ($2\pi r$) of the circle.

Common Mistakes to Avoid

- Omitting to complete the square properly can lead to incorrect radius calculations.
- Confusing the signs when rewriting the circle in standard form.
- Forgetting to take the square root of the right side to find the radius.

Extension: General Approach to Equations of Circles

For more complex equations, the steps involve:

1. Grouping x and y terms.
2. Completing the square for each variable.
3. Rearranging to get the standard form.
4. Identifying the center and radius from the standard form.

Summary

In summary, starting from the quadratic form of the circle's equation:

$x^2 + y^2 + 12x + 6y + 20 = 0$

we completed the square for both x and y to arrive at:

$(x + 6)^2 + (y + 3)^2 = 25$

which indicates:

- The center of the circle is at $(-6, -3)$.
- The radius is $\sqrt{25} = 5$.

This process exemplifies how algebraic manipulation transforms a general quadratic equation into a form that directly reveals the geometric properties of the circle.

Final Remarks

Understanding the equation of a circle and its parameters is a foundational skill in coordinate geometry. It bridges algebraic manipulation with geometric intuition, enabling students and professionals to analyze and interpret circles efficiently. The key steps involve recognizing the general quadratic form, completing the square, and interpreting the standard form to extract the circle's center and radius.

Whether you're solving for the radius, plotting the circle, or analyzing geometric relationships, mastery of these techniques is invaluable. Keep practicing with various equations, and over time, identifying the circle's key features will become second nature.

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