The Graph Of Linear Function F Passes Through The Point (1, -8) And Has A Slope Of 2. What Is The Zero

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Understanding the properties of linear functions is fundamental in algebra and coordinate geometry. When analyzing a linear function, key features such as the slope, y-intercept, and zeros (roots) provide valuable insights into the graph's behavior. In this article, we explore how to determine the zero of a linear function given specific information about its point and slope. We will walk through the problem step-by-step, providing clear explanations and useful tips for solving similar problems.

What Is a Linear Function?

Before delving into the specifics of the problem, let's review what a linear function is.

Definition of a Linear Function

A linear function is a function that can be expressed in the form:

$$f(x) = mx + b$$

where:
- m is the slope of the line,
- b is the y-intercept, the point where the line crosses the y-axis.

The graph of a linear function is always a straight line.

Key Components of a Linear Function

- Slope (m): Indicates the steepness of the line. A positive slope means the line rises from left to right, while a negative slope means it falls.
- Y-intercept (b): The point where the line crosses the y-axis.
- Zero (Root): The value of x where the function equals zero, i.e., where the line crosses the x-axis.

Given Data and Problem Statement

Let's restate the problem in our own words:

> The linear function F passes through the point (1, -8) and has a slope of 2. What is the zero of this function?

The question asks us to find the value of x where $F(x) = 0$.

Step 1: Write the Equation of the Linear Function

Given the slope $m = 2$ and a point $(x_1, y_1) = (1, -8)$, we can find the equation of the line using the point-slope form.

Point-Slope Form of a Line

The point-slope form is:

$$ y - y_1 = m (x - x_1) $$

Substituting the known values:

$$ y - (-8) = 2 (x - 1) $$

which simplifies to:

$$ y + 8 = 2 (x - 1) $$

Convert to Slope-Intercept Form

Distribute the slope:

$$ y + 8 = 2x - 2 $$

Subtract 8 from both sides:

$$ y = 2x - 2 - 8 $$

$$ y = 2x - 10 $$

Thus, the equation of the linear function $F(x)$ is:

$$ F(x) = 2x - 10 $$

Step 2: Find the Zero of the Function

The zero of the function, also called the root, is the x-value where $F(x) = 0$:

$$ 0 = 2x - 10 $$

Now, solve for x:

$$ 2x = 10 $$

$$ x = \frac{10}{2} $$

$$ x = 5 $$

Therefore, the zero of the function $F(x)$ is at $x = 5$.

Summary of the Solution

To determine the zero of a linear function when given a point and slope:

1. Write the equation using the point-slope form.
2. Convert the equation to slope-intercept form.
3. Set $F(x) = 0$ and solve for x.

In our example, this process revealed the zero at $x = 5$.

Additional Tips for Solving Similar Problems

- Always verify the point lies on the derived line by substituting x and y.
- Remember that the zero of the function is the x-intercept of the graph.
- If the problem provides a different point and slope, follow the same steps, substituting the given values.
- For functions in the form $y = mx + b$, the zero is at $x = -b/m$.

Using the Formula for Zero in Slope-Intercept Form

Since $F(x) = mx + b$, the zero occurs where:

$$ 0 = mx + b $$

$$ x = -\frac{b}{m} $$

In our case, $b = -10$ and $m = 2$, so:

$$ x = -\frac{-10}{2} = \frac{10}{2} = 5 $$

which confirms our previous calculation.

Real-World Applications of Finding the Zero

Understanding the zero of a linear function has practical applications:

- Business and Economics: Finding the break-even point where revenue equals cost.
- Physics: Determining when a moving object crosses a reference point.
- Engineering: Calculating the point where a system's response reaches zero.

Conclusion

Determining the zero of a linear function is straightforward once you have the slope and a point on the line. By converting the information into the slope-intercept form and solving for x when $F(x) = 0$, you can efficiently find the root of the function. In our example, with a slope of 2 passing through (1, -8), the zero occurs at $x = 5$.

Remember, mastering these fundamental concepts not only helps with solving algebra problems but also builds a strong foundation for understanding more complex mathematical and real-world scenarios involving linear relationships.

Frequently Asked Questions

What is the equation of the linear function F that passes through (1, -8) with a slope of 2?

The equation is y = 2x - 10.

How do you find the zero of a linear function with a given slope and point?

Set y to 0 in the equation and solve for x to find the zero.

What is the zero of the linear function passing through (1, -8) with slope 2?

The zero is at x = 5.

Can you explain how to determine the x-intercept of the line?

Yes, by substituting y = 0 into the equation and solving for x, we find the x-intercept or zero.

Why is finding the zero of a linear function important?

Because it tells us where the graph crosses the x-axis, which is useful in many applications like break-even analysis.

What is the general method to find the zero of any linear function y = mx + b?

Set y to 0 and solve for x: 0 = mx + b, then x = -b/m.

Using the point (1, -8) and slope 2, what is the specific x-intercept?

The x-intercept is at x = 5.

Is the zero the same as the y-intercept for this linear function?

No, the zero is where the line crosses the x-axis, while the y-intercept is where it crosses the y-axis.

Additional Resources

The Graph Of Linear Function F Passes Through The Point (1, -8) And Has A Slope Of 2. What Is The Zero?

Understanding the fundamental aspects of linear functions is essential in both academic and real-world contexts. Whether you're analyzing profit trends, predicting future values, or simply honing your algebra skills, grasping how to interpret the graph of a linear function can be incredibly valuable. In this article, we explore a specific linear function, given its passing point and slope, and determine its zero—also known as the root or x-intercept. This process involves analyzing the basic properties of linear equations, deriving their explicit formulas, and applying algebraic methods to find the point where the function crosses the x-axis.

The Fundamentals of Linear Functions

Before diving into the specifics of our function, it's crucial to understand what a linear function is and how it behaves.

What Is a Linear Function?

A linear function is a mathematical expression that models a straight line when graphed on a coordinate plane. Its general form is:

$$ y = mx + b $$

where:
- m is the slope of the line, indicating its steepness and direction (positive slope means increasing, negative means decreasing).
- b is the y-intercept, which is the point where the line crosses the y-axis.

Linear functions are characterized by their constant rate of change; for every unit increase in x, the change in y remains the same.

Key Features of Linear Graphs

- Slope (m): Determines the angle and direction of the line.
- Y-intercept (b): The point where the line crosses the y-axis.
- X-intercept or Zero: The point where the graph crosses the x-axis; this is where $y=0$.

Understanding these components helps in constructing the equation of the line when certain points or slopes are known.

Given Data: Point and Slope

In our case, the linear function F passes through the point $(1, -8)$ with a slope of 2.

- Point: $(x_1, y_1) = (1, -8)$
- Slope: $m = 2$

Using this information, the goal is to find the function's equation, then determine its zero.

Deriving the Equation of the Linear Function

The process involves applying the point-slope form of a line equation, which is especially useful when a point and slope are known.

The Point-Slope Form

$$
y - y_1 = m(x - x_1)
$$

Plugging in the known values:

$$
y - (-8) = 2(x - 1)
$$

Simplify:

$$
y + 8 = 2x - 2
$$

Subtract 8 from both sides:

$$
y = 2x - 2 - 8
$$

$$
y = 2x - 10
$$

Thus, the explicit equation of the linear function F is:

$$
F(x) = 2x - 10
$$

This formula fully describes the line's behavior, allowing us to analyze its properties further.

Understanding the Zero of the Function

The zero of a linear function, also called the root or x-intercept, is the value of x when $y = 0$. In other words, it's the point at which the line crosses the x-axis.

Why Is Finding the Zero Important?

- It indicates the input value that results in an output of zero.

- It helps in solving equations where the output is zero.
- It provides insights into the behavior of the function.
- It's essential in graphing, as it marks the point where the line intersects the x-axis.

Calculating the Zero of $F(x) = 2x - 10$

To find the zero, set $F(x)$ equal to zero:

$$
0 = 2x - 10
$$

Solve for x:

$$
2x = 10
$$

$$
x = \frac{10}{2} = 5
$$

Hence, the zero of the function is at $x=5$.

The point corresponding to this zero is $(5, 0)$, which can be verified by substituting back into the function:

$$
F(5) = 2(5) - 10 = 10 - 10 = 0
$$

This confirms that the line crosses the x-axis at $x=5$.

Visualizing the Function and Its Zero

Graphing the line provides an intuitive understanding of the function's behavior.

Graphing Steps:

1. Plot the known point: $(1, -8)$
2. Plot the zero point: $(5, 0)$
3. Determine additional points: For example, choose $x=0$:

$$
F(0) = 2(0) - 10 = -10

\]

So, $(0, -10)$ is another point on the line.

4. Draw the line: Connect these points with a straight line, noting that the slope of 2 indicates an upward incline.

This visual confirms the line passing through $(1, -8)$ and crossing the x-axis at $(5, 0)$.

Practical Applications and Significance

Understanding how to find the zero of a linear function has numerous applications:

- Business: Determining break-even points where profit (or loss) becomes zero.
- Physics: Calculating moments when a moving object reaches a specific position.
- Economics: Identifying equilibrium points where supply equals demand.
- Engineering: Analyzing system responses at critical points.

Moreover, this skill forms the foundation for more advanced topics in calculus, statistics, and data analysis.

Summary and Key Takeaways

- The given linear function passes through $(1, -8)$ with a slope of 2.
- Using the point-slope form, the function's explicit form is $F(x) = 2x - 10$.
- The zero of the function is found by setting $F(x) = 0$, resulting in $x=5$.
- The x-intercept point is $(5, 0)$.
- Visualizing the graph confirms the line's behavior and its intersection with the x-axis.

Final Thoughts

Mastering the process of deriving a linear function from known points and slopes, and then finding its zero, equips learners with essential algebraic tools. This particular problem exemplifies fundamental concepts that recur across numerous disciplines and real-world scenarios. Whether analyzing data trends or solving equations, understanding the properties of linear functions is a critical foundational skill in mathematics and beyond.

By systematically applying algebraic methods and visual reasoning, students and professionals alike can interpret and manipulate linear models with confidence, enabling more effective problem-solving and decision-making.

In conclusion, the zero of the linear function passing through \((1, -8) \) with a slope of 2 is at \(x=5 \), marking the point where the graph crosses the x-axis. This straightforward yet powerful calculation underscores the importance of understanding linear equations and their graphical representations in a variety of contexts.

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