The Graph Of Quadratic Function F Has Zeros Of -8 And 4 And A Maximum At (-2,18). What Is The Value Of

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Understanding the properties of quadratic functions is fundamental in algebra and calculus. When given specific points and features of a quadratic graph—such as zeros and a maximum point—it becomes possible to determine the equation of the parabola. In this article, we explore the process of finding the quadratic function $f(x)$ given its zeros at -8 and 4, and a maximum point at $(-2, 18)$. We will analyze the key concepts involved, develop the step-by-step solution, and interpret the significance of the resulting quadratic function.

Understanding the Given Data: Zeros and Maximum Point

What Are Zeros of a Quadratic Function?

- Zeros (or roots) of a quadratic function are the points where the graph intersects the x-axis.
- For the function $f(x)$, zeros at -8 and 4 imply:
- $f(-8) = 0$
- $f(4) = 0$

Maximum Point and Its Significance

- A maximum point on the parabola indicates the vertex of the quadratic when the parabola opens downward.
- The maximum point is given at $(-2, 18)$, meaning:
- The vertex of the parabola is at $(-2, 18)$.
- The parabola reaches its highest point at this coordinate.

Formulating the Quadratic Function

Standard and Factored Forms

- The quadratic function can be expressed in various forms:
- Standard form: $f(x) = ax^2 + bx + c$
- Factored form: $f(x) = a(x - r_1)(x - r_2)$, where r_1, r_2 are zeros.
- Vertex form: $f(x) = a(x - h)^2 + k$, where (h, k) is the vertex.

Choosing the Appropriate Form

- Since the zeros are known (-8 and 4), the factored form is particularly convenient:
$$
f(x) = a(x + 8)(x - 4)
$$
- The goal is to find the value of a using the vertex point.

Determining the Leading Coefficient a

Using the Vertex Point

- The vertex form relates the vertex (h, k) to the quadratic:
$$
f(x) = a(x - h)^2 + k
$$
- We know the vertex is at $(-2, 18)$:
$$
f(x) = a(x + 2)^2 + 18
$$
- To confirm the value of a, we can use the fact that the quadratic must pass through one of the zeros, specifically at $x = -8$ or $x = 4$.

Substituting a Zero to Find a

- Choose $x = -8$, where $f(-8) = 0$:
$$
0 = a(-8 + 2)^2 + 18
$$
$$
0 = a(-6)^2 + 18
$$
\[

$$
0 = 36a + 18
$$

- Solve for a:

$$
36a = -18
$$

$$
a = -\frac{18}{36} = -\frac{1}{2}
$$

- To verify, substitute $x = 4$:

$$
f(4) = -\frac{1}{2}(4 + 2)^2 + 18 = -\frac{1}{2}(6)^2 + 18 = -\frac{1}{2}(36) + 18 = -18 + 18 = 0
$$

- The calculation confirms the value of $a = -\frac{1}{2}$.

Final Equation of the Quadratic Function

Expressing in Vertex Form

- The quadratic function is:

$$
f(x) = -\frac{1}{2}(x + 2)^2 + 18
$$

- This form clearly shows the vertex at $(-2, 18)$ and the parabola opening downward (since $a < 0$).

Expressing in Standard and Factored Forms

- Standard form:
- Expand the vertex form:

$$
f(x) = -\frac{1}{2}(x^2 + 4x + 4) + 18
$$

$$
f(x) = -\frac{1}{2}x^2 - 2x - 2 + 18
$$

$$
f(x) = -\frac{1}{2}x^2 - 2x + 16
$$

- Factored form:
- Recall the zeros at -8 and 4:

$$
f(x) = -\frac{1}{2}(x + 8)(x - 4)
$$
- Expanding:
$$
f(x) = -\frac{1}{2}(x^2 + 4x - 8x - 32)
$$
$$
f(x) = -\frac{1}{2}(x^2 - 4x - 32)
$$
$$
f(x) = -\frac{1}{2}x^2 + 2x + 16
$$
- Both the standard and factored forms are consistent and describe the same parabola.

Interpreting the Graph and the Function

Shape of the Parabola

- Since $a = -\frac{1}{2}$, the parabola opens downward.
- The maximum point at $(-2, 18)$ is the vertex, confirming the parabola's highest point.

Significance of the Zero Points

- The zeros at -8 and 4 indicate the parabola intersects the x-axis at these points.
- The distance between zeros is $4 - (-8) = 12$.

Vertex and Symmetry

- The axis of symmetry passes through the vertex:
$$
x = -2
$$
- The zeros are symmetric with respect to this line:
$$
\text{Distance from } -2 \text{ to } -8 \text{ is } 6
$$
$$
\text{Distance from } -2 \text{ to } 4 \text{ is } 6
$$
- This symmetry is characteristic of quadratic graphs.

Conclusion: What Is The Value Of?

- The original question asks, “What is the value of”—presumably referring to the quadratic function at a specific point or a particular parameter.
- Based on the analysis, the quadratic function is:
$$
\boxed{f(x) = -\frac{1}{2}x^2 - 2x + 16}
$$
- If the question pertains to the value of $f(x)$ at a given point, such as $x = 0$:
$$
f(0) = -\frac{1}{2}(0)^2 - 2(0) + 16 = 16
$$
- Alternatively, if the question asks for the maximum value, it is:
$$
\boxed{\text{Maximum value } = 18}
$$
occurring at the vertex $(-2, 18)$.

Summary

- The quadratic function with zeros at -8 and 4, and a maximum at $(-2, 18)$, is:
$$
\boxed{f(x) = -\frac{1}{2}(x + 2)^2 + 18}
$$
- Its standard form is:
$$
f(x) = -\frac{1}{2}x^2 - 2x + 16
$$
- Its factored form is:
$$
f(x) = -\frac{1}{2}(x + 8)(x - 4)
$$
- The maximum value of the function is 18, achieved at $x = -2$.

This comprehensive approach demonstrates how to derive the quadratic function from key features of its graph, emphasizing the importance of understanding zeros, vertex, and the parabola's symmetry in algebraic analysis.

Frequently Asked Questions

What is the quadratic function F(x) with zeros at -8 and 4 and a maximum point at (-2, 18)?

The quadratic function is $F(x) = -3(x + 8)(x - 4)$.

How do you find the quadratic function when given zeros and a maximum point?

Start by writing the factored form using zeros, then determine the leading coefficient using the maximum point or other given data.

Given zeros at -8 and 4, and a maximum at (-2, 18), what is the vertex form of the quadratic function?

The vertex form is $F(x) = a(x + 2)^2 + 18$, with a determined by the known zeros and maximum point.

What is the value of the quadratic function F(x) at x = -2, given the maximum at (-2, 18)?

$F(-2) = 18$, since that is the maximum point provided.

How can we verify the quadratic function's coefficients with the given zeros and maximum point?

By substituting the zeros into the factored form and using the maximum point to solve for the leading coefficient, ensuring the function passes through all known points.

Additional Resources

The Graph Of Quadratic Function F Has Zeros Of -8 And 4 And A Maximum At (-2,18). What Is The Value Of

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Introduction

Quadratic functions are foundational elements in algebra and calculus, serving as the building blocks for understanding more complex functions. Their graphs are parabolas, which exhibit specific features such as zeros (roots), vertex points (maximum or minimum), and axes of symmetry. When analyzing a quadratic function based on given characteristics, such as zeros and vertex points, it becomes possible to reconstruct the function's equation and interpret its properties comprehensively.

This investigative article explores the problem: The graph of quadratic function F has zeros at -8 and 4, and a maximum at (-2, 18). What is the value of ...? The precise question appears to be incomplete, but given the context, it likely pertains to determining the quadratic function's explicit formula, its coefficients, or the value of the function at a specific point.

The analysis will delve into the fundamentals of quadratic functions, interpret the given zeros and maximum point, and methodically derive the function's equation through algebraic techniques. This process illuminates the interplay between graph features and algebraic expressions, providing a comprehensive understanding suitable for educational, research, and review contexts.

Understanding the Basic Properties of Quadratic Functions

The General Form

A quadratic function can generally be expressed as:

$$f(x) = ax^2 + bx + c$$

where:
- $a \neq 0$,
- b and c are real numbers.

The graph of $f(x)$ is a parabola opening upward if $a > 0$ and downward if $a < 0$.

Vertex and Axis of Symmetry

The vertex (h, k) of the parabola occurs at:

$$h = -\frac{b}{2a}$$

and

$$k = f(h)$$

The axis of symmetry is the vertical line $x = h$.

Interpreting the Given Data

The problem states:

- The zeros (roots) of the quadratic function F are at $x = -8$ and $x = 4$.
- The parabola has a maximum at $(-2, 18)$.

From these, we can infer:

- The zeros are symmetric with respect to the parabola's axis of symmetry.
- The maximum point indicates the vertex of the parabola, which is the highest point on the

graph (since the parabola opens downward).

Deducing the Quadratic Equation

Step 1: Forming the Factored Form

Given the zeros at $x = -8$ and $x = 4$:

$$f(x) = k(x + 8)(x - 4)$$

where k is a constant scalar factor to be determined.

Step 2: Finding the Axis of Symmetry

The zeros are at -8 and 4. The axis of symmetry lies midway between these zeros:

$$x_{axis} = \frac{-8 + 4}{2} = \frac{-4}{2} = -2$$

This matches the x-coordinate of the maximum point, indicating the parabola's vertex is at $x = -2$. This confirms the parabola opens downward (since the maximum occurs at the vertex).

Step 3: Using the Vertex to Find k

The vertex point is given as $(-2, 18)$. Plugging $x = -2$ into the factored form:

$$f(-2) = k(-2 + 8)(-2 - 4)$$

Calculate:

$$(-2 + 8) = 6$$
$$(-2 - 4) = -6$$

So,

$$18 = f(-2) = k \times 6 \times (-6) = k \times (-36)$$

Solve for k:

$$k = \frac{18}{-36} = -\frac{1}{2}$$

Final Equation of the Quadratic Function

Based on the above, the quadratic function is:

$$f(x) = -\frac{1}{2}(x + 8)(x - 4)$$

Expanding:

$$
\begin{aligned}
f(x) &= -\frac{1}{2} [x^2 - 4x + 8x - 32] \\
&= -\frac{1}{2} [x^2 + 4x - 32] \\
&= -\frac{1}{2}x^2 - 2x + 16
\end{aligned}
$$

Therefore, the explicit form of the quadratic function is:

$$ \boxed{f(x) = -\frac{1}{2}x^2 - 2x + 16} $$

Verifying the Derived Function

To ensure correctness, verify the key features:

- Zeros:

Set $f(x) = 0$:

$$
-\frac{1}{2}x^2 - 2x + 16 = 0
$$

Multiply through by -2:

$$
x^2 + 4x - 32 = 0
$$

Solve via quadratic formula:

$$
x = \frac{-4 \pm \sqrt{(4)^2 - 4 \times 1 \times (-32)}}{2 \times 1} = \frac{-4 \pm \sqrt{16 + 128}}{2} = \frac{-4 \pm \sqrt{144}}{2}
$$

$$
x = \frac{-4 \pm 12}{2}
$$

Solutions:

- $x = \frac{-4 + 12}{2} = \frac{8}{2} = 4$
- $x = \frac{-4 - 12}{2} = \frac{-16}{2} = -8$

Matches the given zeros.

- Vertex:

The vertex's x-coordinate:

$$ h = -\frac{b}{2a} $$

In standard form, $a = -\frac{1}{2}$, $b = -2$:

$$
h = -\frac{-2}{2 \times -\frac{1}{2}} = \frac{2}{-1} = -2
$$

Substitute $x = -2$ into $f(x)$:

$$
f(-2) = -\frac{1}{2} \times (-2)^2 - 2 \times (-2) + 16 = -\frac{1}{2} \times 4 + 4 + 16 = -2 + 4 + 16 = 18
$$

Matches the maximum point, confirming the correctness of the function.

Additional Insights and Context

Nature of the Parabola

- Since the leading coefficient $a = -\frac{1}{2}$ is negative, the parabola opens downward, consistent with the maximum at $(-2, 18)$.
- The zeros are at -8 and 4, symmetric about the vertex at $x = -2$.

Significance of the Vertex

The vertex point indicates the maximum height of the parabola and provides critical insight into the function's behavior. Its coordinates at $(-2, 18)$ reveal the peak of the parabola, which often corresponds to optimal solutions in applied contexts such as physics and economics.

Potential Applications and Broader Implications

Understanding the precise form of quadratic functions based on graphical features is essential in various disciplines:

- Physics: For projectile motion modeling, where maximum height and zeros correspond to launch and impact points.
- Economics: To model profit functions, where the maximum profit point is analogous to the vertex.
- Engineering: In structural analysis, where parabolic curves model beams and arches.

The methodology demonstrated—deriving the quadratic function from key features—serves as a valuable tool in data analysis, optimization, and problem-solving scenarios.

Conclusion

By carefully analyzing the zeros at \(-8 \) and \(4 \), and the maximum at \((-2, 18) \), we've reconstructed the quadratic function:

\[
\boxed{
f(x) = -\frac{1}{2}x^2 - 2x + 16
}
\]

This function encapsulates the parabola's shape, symmetry, and maximum point, exemplifying how geometric features translate into algebraic expressions. The process underscores the importance of understanding the interplay between graph features and algebraic forms, a fundamental skill in mathematics education, research, and applied sciences.

Final Note

While the original question appears incomplete—"What is the value of..."—the most logical interpretation pertains to the explicit formula of the quadratic function or its value at specific points. This analysis provides a comprehensive answer, demonstrating the depth of reasoning and algebraic manipulation involved in such problems.

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