The Graph Of Y = Cos X For 0 < X < 360 Isshown Below.Calculate The Two Solutions To Cos X = 0.74

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Understanding the behavior of the cosine function is fundamental in trigonometry, especially when analyzing its graph within specific intervals. In this article, we will explore the graph of \(y = \cos x \) for \(0 < x < 360^\circ \), and focus on solving the equation \(\cos x = 0.74 \). Specifically, we will determine the two solutions within this interval where the cosine of \(x \) equals 0.74. This detailed analysis not only enhances comprehension of the cosine graph but also provides step-by-step guidance on solving such equations using inverse cosine functions and the properties of the cosine graph.

Understanding the Cosine Function and Its Graph

Basic Properties of the Cosine Function

The cosine function is a fundamental trigonometric function defined on the real numbers. Some key properties include:

- **Periodicity:** \(\cos x \) has a period of 360°, meaning it repeats every 360 degrees.
- **Range:** The output of \(\cos x \) ranges between -1 and 1, inclusive, i.e., \(-1 \leq \cos x \leq 1 \).
- **Symmetry:** \(\cos x \) is an even function, satisfying \(\cos(-x) = \cos x \).

Graph Characteristics for \(0^\circ < x < 360^\circ \)

The graph of \(y = \cos x \) over this interval can be summarized as:

- Starting at \(x = 0^\circ \), \(\cos 0^\circ = 1 \).
- Decreasing to \(\cos 180^\circ = -1 \), reaching its minimum at \(180^\circ \).
- Increasing back to \(\cos 360^\circ = 1 \), completing one full cycle.

This wave-like shape is symmetric about the vertical axis passing through the maximum at $x=0^\circ$ and $x=360^\circ$.

Solving the Equation $\cos x = 0.74$

Theoretical Background

Given the equation:
$$\cos x = 0.74$$
Our goal is to find all solutions x within $0^\circ < x < 360^\circ$.

Since $\cos x$ is positive and less than 1, the solutions will be located in the first and fourth quadrants, where cosine values are positive.

Step-by-Step Solution Process

1. **Calculate the inverse cosine of 0.74:**

   Use a calculator or computational tool to find:
   $$x_1 = \cos^{-1}(0.74)$$

   Make sure your calculator is in degree mode if working in degrees.

   For example:
   $$x_1 \approx \cos^{-1}(0.74) \approx 42.21^\circ$$

   This is the principal value, corresponding to the first quadrant solution.

2. **Determine the second solution:**

   Since cosine is positive in the first and fourth quadrants, the second solution in the interval $0^\circ < x < 360^\circ$ is:
   $$x_2 = 360^\circ - x_1 \approx 360^\circ - 42.21^\circ = 317.79^\circ$$

   These two values, x_1 and x_2, satisfy the original equation within the specified interval.

3. **Verify the solutions:**

   Check that both solutions are within the interval:
   $$0^\circ < 42.21^\circ < 360^\circ$$
   $$0^\circ < 317.79^\circ < 360^\circ$$

   Both are valid solutions.

Summary of the Solutions

The solutions to $\cos x = 0.74$ on $0^\circ < x < 360^\circ$ are approximately:

- $x \approx 42.21^\circ$
- $x \approx 317.79^\circ$

Visualizing the Solutions on the Graph

Graphical Representation of $y = \cos x$

To better understand the solutions, sketch or visualize the graph of $y = \cos x$ over the interval 0° to 360°. Key points include:

- Maximum at $x=0^\circ$ and $x=360^\circ$, $y=1$.
- Minimum at $x=180^\circ$, $y=-1$.
- Crossings at $y=0$ at $x=90^\circ$ and $x=270^\circ$.

Where the graph intersects the horizontal line $y=0.74$, these points correspond to the solutions $x \approx 42.21^\circ$ and 317.79°.

Using Graphs to Confirm Solutions

Plotting $y = \cos x$ and the line $y=0.74$ can visually confirm the approximate locations of the solutions. This visualization aids learners in understanding how the cosine function behaves and where solutions are situated in relation to key points.

Applications of Finding Solutions to $\cos x = 0.74$

Practical Contexts

Understanding how to solve for x in $\cos x = 0.74$ has several practical applications, including:

- Analyzing wave motion and oscillations in physics.
- Designing signals and waveforms in electrical engineering.
- Modeling periodic phenomena in environmental science or economics.
- Solving problems involving angles of elevation/depression in navigation and surveying.

Further Mathematical Exploration

Once familiar with solving $\cos x = c$ for various values of c, students can extend their skills to:

- Solving similar equations involving sine or tangent functions.
- Understanding the general solutions of trigonometric equations over different intervals.
- Applying identities and transformations to solve more complex trigonometric problems.

Conclusion

The process of solving $\cos x = 0.74$ within $0^\circ < x < 360^\circ$ highlights important aspects of trigonometry, including understanding the cosine graph, the use of inverse functions, and recognizing the symmetry of the cosine function. The approximate solutions—$x \approx 42.21^\circ$ and $x \approx 317.79^\circ$—are derived through inverse cosine calculations and understanding the symmetry of the cosine wave. Visualizing these solutions on the graph further solidifies comprehension, illustrating how the cosine function intersects specific horizontal lines within its period. Mastery of such problems is essential for students and professionals dealing with periodic functions and their applications across various scientific and engineering fields.

For further practice, students are encouraged to explore solving similar equations with different constants and intervals, as well as applying these concepts to real-world problems involving periodic phenomena.

Frequently Asked Questions

What is the given equation involving cosine in the interval 0° < X < 360°?

The equation is cos X = 0.74 within the interval 0° < X < 360°.

How do we find the solutions to cos X = 0.74 in the given interval?

We find the solutions by taking the inverse cosine (arccos) of 0.74 and then determining the two angles within 0° to 360°, using the symmetry of the cosine function.

What is the value of X when cos X = 0.74 using a calculator?

X ≈ arccos(0.74) ≈ 42.1°.

What is the second solution for X in the interval 0° < X < 360°?

The second solution is X ≈ 360° - 42.1° ≈ 317.9°.

Why are there two solutions for cos X = 0.74 within 0° to 360°?

Because cosine is positive in the first and fourth quadrants, resulting in two angles where cos X equals 0.74.

How can these solutions be verified graphically?

By plotting the graph of y = cos X and drawing a horizontal line at y = 0.74, then locating the points of intersection within the interval 0° to 360°.

Additional Resources

The Graph Of Y = Cos X For 0 < X < 360 Isshown Below. Calculate The Two Solutions To Cos X = 0.74

Understanding the behavior of trigonometric functions is fundamental in mathematics, engineering, physics, and numerous applied sciences. Among these, the cosine function stands out due to its periodic nature, symmetry, and wide range of applications. When analyzing the graph of $y = \cos x$ over the interval $0^\circ < x < 360^\circ$, one often encounters problems that involve finding specific solutions where the cosine value equals a given number—in this case, 0.74. This article delves into how to determine the two solutions to the equation $\cos x = 0.74$ within the specified interval, exploring the underlying principles, methods, and practical implications.

The Cosine Function: An Overview

Before diving into the specific problem, it's essential to understand the fundamental properties of the cosine function:

- Periodicity: The cosine function repeats every 360°, or 2π radians. This means that $\cos(x + 360^\circ) = \cos x$.

- Range: The cosine of any real number always lies between -1 and 1; thus, $-1 \leq \cos x \leq 1$.

- Symmetry: The cosine graph is symmetric about the y-axis, making it an even function, i.e., $\cos(-x) = \cos x$.

- Key Points: At specific angles, the cosine values are well-known:
- $\cos 0^\circ = 1$
- $\cos 90^\circ = 0$
- $\cos 180^\circ = -1$
- $\cos 270^\circ = 0$
- $\cos 360^\circ = 1$

Understanding these properties allows us to interpret the graph effectively and pinpoint solutions to equations involving cosine.

Visualizing the Graph of $y = \cos x$ from 0° to 360°

The graph of $y = \cos x$ over the interval $0^\circ < x < 360^\circ$ resembles a smooth, wave-like curve oscillating between 1 and -1. Key features include:

- Starting at $(0^\circ, 1)$.
- Descending toward 0 at 90°.
- Reaching -1 at 180°.
- Increasing back to 0 at 270°.
- Returning to 1 at 360°.

Within this interval, the graph crosses the horizontal line $y = 0.74$ twice: once while descending from 1 to -1 and once while ascending back from -1 to 1. Our goal is to find the precise x-values where $\cos x = 0.74$.

Mathematical Approach to Find the Solutions

Given the equation:

$$
\cos x = 0.74
$$

we aim to determine the two solutions within $0^\circ < x < 360^\circ$. To do this, we employ the inverse cosine function, often denoted as $\cos^{-1}$ or arccos, which yields the principal value of x in the range $0^\circ \leq x \leq 180^\circ$.

Step 1: Calculate the Principal Value

Using a scientific calculator or computer software:

$$
x_1 = \cos^{-1}(0.74)
$$

Calculating this:

$$
x_1 \approx \boxed{42.07^\circ}
$$

This is the angle in the first half of the interval, where the cosine is positive and equals 0.74.

Step 2: Find the Second Solution

Since the cosine function is symmetric about the y-axis, the second solution in the interval $0^\circ < x < 360^\circ$ corresponds to the angle:

$$
x_2 = 360^\circ - x_1
$$

Calculating:

$$
x_2 \approx 360^\circ - 42.07^\circ = \boxed{317.93^\circ}
$$

These two solutions, $x \approx 42.07^\circ$ and $x \approx 317.93^\circ$, are the points where the graph of $y = \cos x$ intersects the line $y = 0.74$.

Confirming the Solutions and Their Significance

To verify the accuracy of these solutions, one can substitute them back into the original equation:

- For $x \approx 42.07^\circ$:

$$
\cos 42.07^\circ \approx 0.74
$$

- For $x \approx 317.93^\circ$:

$$
\cos 317.93^\circ \approx 0.74
$$

Both are consistent with the initial problem statement.

Understanding these solutions has practical importance. For example:

- Engineering: When designing oscillatory systems, knowing the points where a wave reaches a specific amplitude can be crucial.
- Physics: In wave mechanics, the phase points corresponding to particular values influence interference patterns.
- Navigation and Signal Processing: Precise angle calculations are necessary for accurate positioning and data analysis.

Broader Context: Using Inverse Cosine and Its Limitations

While the inverse cosine function is a powerful tool, it's essential to recognize its domain and range:

- Domain of $\cos^{-1}$: $-1 \leq y \leq 1$
- Range of $\cos^{-1}$: $0^\circ \leq x \leq 180^\circ$

This means that for any value y within $[-1, 1]$, the principal value $\theta = \cos^{-1}(y)$ will be between 0° and 180°. For solutions outside this range, the periodicity and symmetry of the cosine function are used to find all possible solutions within the specified interval.

In our case, since 0.74 is within the range, the principal value suffices, and the second solution is obtained via the symmetry property.

Practical Applications and Further Exploration

Determining solutions to cosine equations isn't just an academic exercise; it has real-world implications across multiple fields:

- Signal Analysis: Finding points where a signal reaches a certain amplitude.
- Mechanical Vibrations: Calculating phase angles at which systems reach specific states.
- Astronomy: Determining angles of celestial bodies relative to observers.

Moreover, this method extends beyond degrees to radians, where similar calculations are performed using π-based units, requiring adjustments in the calculator or software settings.

Summary and Final Remarks

In conclusion, solving $\cos x = 0.74$ within $0^\circ < x < 360^\circ$ involves:

- Recognizing the properties of the cosine function.
- Using the inverse cosine to find the principal solution $x_1 \approx 42.07^\circ$.
- Applying symmetry to find the second solution $x_2 \approx 317.93^\circ$.
- Verifying the solutions by substitution.

These solutions exemplify the elegance and utility of trigonometry in analyzing periodic functions and solving real-world problems involving angles and amplitudes. Mastery of these techniques enhances our ability to interpret wave phenomena, design oscillatory systems, and navigate the complex relationships between angles and their corresponding ratios.

By understanding the foundational principles and applying systematic methods, students and professionals alike can confidently tackle similar problems, making trigonometry an indispensable tool in the mathematical toolkit.

End of Article

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