The Graph Of Y = X2 - 4x Is Shown On The Grid.By Drawing The Line Y = X -4,solve The Equations: Y = X-

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Introduction to the Graphs and Equations

Understanding the graphical representation of equations is fundamental in algebra and coordinate geometry. In this article, we examine the parabola represented by the equation $y = x^2 - 4x$ and analyze how it interacts with the straight line given by $y = x - 4$. By drawing the line on the same grid, we can visually determine the solutions to the system of equations, which correspond to the points where the line and the parabola intersect. These points are the solutions to the equations when set equal to each other.

Understanding the Parabola $y = x^2 - 4x$

Shape and Features of the Parabola

The equation $y = x^2 - 4x$ is a quadratic function, which graphs as a parabola opening upwards. To analyze its features:

- The quadratic is in standard form: $y = ax^2 + bx + c$, with $a = 1$, $b = -4$, and $c = 0$.
- The axis of symmetry is given by the formula $x = -\frac{b}{2a}$.

Calculating the vertex:

$$
x = -\frac{-4}{2 \times 1} = \frac{4}{2} = 2
$$

Substituting $x = 2$ into the equation to find the vertex's y-coordinate:

$$
y = (2)^2 - 4 \times 2 = 4 - 8 = -4
$$

\]

Thus, the vertex of the parabola is at point $(2, -4)$.

Plotting the Parabola

To sketch the parabola:

- Plot the vertex at $(2, -4)$.
- Find additional points by substituting various x-values:

x	$y = x^2 - 4x$	Point
0	0 - 0 = 0	(0, 0)
1	1 - 4 = -3	(1, -3)
3	9 - 12 = -3	(3, -3)
4	16 - 16 = 0	(4, 0)

Plotting these points shows the parabola's shape and helps visualize where it intersects with other lines.

Understanding the Line $y = x - 4$

Features of the Line

The line $y = x - 4$ is a straight line with:

- Slope $m = 1$ (since coefficient of x is 1).
- Y-intercept at $(0, -4)$.

This means the line crosses the y-axis at $(0, -4)$ and rises one unit vertically for each unit moved horizontally.

Plotting the Line

To sketch this line:

- Mark the point $(0, -4)$ on the grid.
- Use the slope to find another point, for example:

When $x = 2$:

$$
y = 2 - 4 = -2
$$

So, a second point is $(2, -2)$.

Plot these points and draw a straight line passing through them.

Finding the Intersection Points: Solving the System of Equations

Setting the Equations Equal

To find where the parabola and the line intersect, set:

$$
x^2 - 4x = x - 4
$$

This simplifies to:

$$
x^2 - 4x - x + 4 = 0
$$
$$
x^2 - 5x + 4 = 0
$$

This is a quadratic equation that can be solved using factorization, completing the square, or quadratic formula.

Solving the Quadratic Equation

Using the quadratic formula:

$$
x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}
$$

where $a = 1$, $b = -5$, $c = 4$:

$$
x = \frac{-(-5) \pm \sqrt{(-5)^2 - 4 \times 1 \times 4}}{2 \times 1} = \frac{5 \pm \sqrt{25 - 16}}{2}
$$

$$
x = \frac{5 \pm \sqrt{9}}{2} = \frac{5 \pm 3}{2}
$$

Thus,

- When $+3$:

$$
x = \frac{5 + 3}{2} = \frac{8}{2} = 4
$$

- When -3:

$$
x = \frac{5 - 3}{2} = \frac{2}{2} = 1
$$

The x-coordinates of the intersection points are $x = 1$ and $x = 4$.

Finding Corresponding y-Values

Substitute back into the line equation $y = x - 4$:

- For $x = 1$:

$$
y = 1 - 4 = -3
$$

- For $x = 4$:

$$
y = 4 - 4 = 0
$$

Intersection Points

The points where the parabola and the line intersect are:

$$
(1, -3) \quad \text{and} \quad (4, 0)
$$

Visual Representation and Interpretation

Graphical Summary

- The parabola opens upwards with vertex at $(2, -4)$.
- The line passes through $(0, -4)$ and $(2, -2)$.
- The intersection points $(1, -3)$ and $(4, 0)$ lie on both graphs.

Significance of the Intersection Points

- They represent solutions where both equations are satisfied simultaneously.
- The points can be viewed as solutions to the system of equations:

$$
\begin{cases}
y = x^2 - 4x \\
y = x - 4
\end{cases}
$$

- The solutions are important in various applications, including physics, engineering, and optimization problems, where understanding the points of intersection is crucial.

Additional Considerations and Applications

Algebraic vs. Graphical Solutions

While solving algebraically provides exact points of intersection, graphing offers a visual insight into the behavior of the functions. Combining both methods enhances understanding and accuracy.

Real-World Applications

- Physics: Determining points where two motion paths intersect.
- Economics: Finding equilibrium points where supply and demand curves meet.

- Engineering: Analyzing systems where different forces or signals intersect.

Practice Problems

To deepen understanding, consider solving similar systems:

- $y = x^2 + 2x$ and $y = -x + 3$.
- $y = 2x^2 - 5x + 1$ and $y = x + 4$.

Conclusion

The process of graphing quadratic and linear equations and analyzing their points of intersection is a fundamental skill in algebra. By understanding the features of the parabola $y = x^2 - 4x$ and the line $y = x - 4$, and then solving the system algebraically, we find that the solutions are the points $(1, -3)$ and $(4, 0)$. These points represent where the line and parabola intersect on the grid, illustrating the powerful connection between algebraic solutions and their graphical representations. Mastery of these concepts provides a solid foundation for tackling more complex problems in mathematics and its many applications.

Frequently Asked Questions

How do you find the points of intersection between the parabola $y = x^2 - 4x$ and the line $y = x - 4$?

To find the intersection points, set $x^2 - 4x$ equal to $x - 4$ and solve for x: $x^2 - 4x = x - 4$. Simplify to $x^2 - 5x + 4 = 0$, then factor or use the quadratic formula to find the x-values of the intersection points.

What are the steps to solve the equations $y = x^2 - 4x$ and $y = x - 4$ simultaneously?

Set the two equations equal: $x^2 - 4x = x - 4$. Rearrange to form a quadratic equation: $x^2 - 5x + 4 = 0$. Solve for x using factoring or quadratic formula, then substitute back to find y-values.

What is the significance of the line $y = x - 4$ in relation to the parabola $y = x^2 - 4x$?

The line $y = x - 4$ acts as a secant line intersecting the parabola at points where the equations satisfy each other, helping to find the intersection points which are solutions to both equations.

How many solutions are there to the system of equations $y = x^2 - 4x$ and $y = x - 4$?

Depending on the discriminant of the quadratic equation formed, there can be 0, 1, or 2 solutions. Typically, if the line intersects the parabola at two points, there are two solutions; at one point, one solution; and if they do not intersect, zero solutions.

Can the graph of $y = x - 4$ be used to determine the nature of the solutions visually?

Yes, by plotting the line $y = x - 4$ and the parabola $y = x^2 - 4x$ on the same grid, the points where they intersect indicate the solutions to the system visually.

What is the role of substitution in solving the system of equations involving the parabola and the line?

Substitution involves replacing y in the parabola's equation with the expression from the line $y = x - 4$, leading to a quadratic equation in x. Solving this quadratic yields the x-coordinates of the intersection points, which can then be used to find the corresponding y-values.

Additional Resources

Comprehensive Analysis of the Graph of $Y = X^2 - 4X$ and Solving Related Equations

Understanding the graphical representation of quadratic and linear equations is fundamental in algebra and analytic geometry. In this detailed review, we explore the graph of the quadratic function $Y = X^2 - 4X$, analyze its properties, and examine how the line $Y = X - 4$ interacts with it. Additionally, we will demonstrate the process of solving the equations where these graphs intersect, including solving $Y = X - 4$ and the quadratic simultaneously.

Understanding the Quadratic Function: $Y = X^2 - 4X$

1. Basic Form and Characteristics

The given quadratic function, $Y = X^2 - 4X$, is expressed in standard quadratic form:

$$ Y = aX^2 + bX + c $$

Where:

- $a = 1$
- $b = -4$
- $c = 0$

This quadratic has a parabola opening upwards because $a > 0$.

2. Vertex of the Parabola

The vertex of a parabola in quadratic form can be found using:

$$ X_{vertex} = -\frac{b}{2a} $$

Calculating:

$$ X_{vertex} = -\frac{-4}{2 \times 1} = \frac{4}{2} = 2 $$

To find the corresponding Y-coordinate:

$$ Y_{vertex} = (2)^2 - 4 \times 2 = 4 - 8 = -4 $$

Therefore, the vertex is at (2, -4).

This point represents the minimum of the parabola, providing valuable insight into the shape of the graph.

3. Axis of Symmetry

The axis of symmetry passes through the vertex:

$$ X = 2 $$

This vertical line divides the parabola into two mirror images.

4. Intercepts

- Y-intercept: Set $X=0$:

$$ Y = 0^2 - 4 \times 0 = 0 $$

Y-intercept at (0, 0).

- X-intercepts: Solve $Y=0$:

$$ 0 = X^2 - 4X $$
$$ X(X - 4) = 0 $$
$$ \Rightarrow X=0 \quad \text{or} \quad X=4 $$

X-intercepts at (0, 0) and (4, 0).

Graphical Representation of Y = X^2 - 4X

Plotting key points:

- Vertex: (2, -4)
- Intercepts: (0, 0) and (4, 0)
- Additional points (for accuracy), e.g., at X = 1:

$$ Y = 1 - 4 = -3 \Rightarrow (1, -3) $$

At X = 3:

$$ Y = 9 - 12 = -3 \Rightarrow (3, -3) $$

Plotting these points and drawing a smooth parabola gives a complete picture of the quadratic graph.

Analyzing the Line: Y = X - 4

1. Basic Features

The line Y = X - 4 is a linear function with:

- Slope: 1 (a 45-degree inclination)
- Y-intercept: at (0, -4)

2. Graphical Characteristics

- It is a straight line crossing the Y-axis at (0, -4).
- For X = 1:

$$Y = 1 - 4 = -3$$

- For $X = 4$:

$$Y = 4 - 4 = 0$$

Plotting these points helps visualize the line.

Finding the Intersection Points: Solving $Y = X^2 - 4X$ and $Y = X - 4$

The primary goal is to find the points where the parabola and the line intersect—that is, solutions to the system:

$$
\begin{cases}
Y = X^2 - 4X \\
Y = X - 4
\end{cases}
$$

Since both expressions are equal to Y, set them equal to each other:

$$X^2 - 4X = X - 4$$

1. Formulating the Equation

Bring all terms to one side to form a quadratic:

$$X^2 - 4X - X + 4 = 0$$
$$X^2 - 5X + 4 = 0$$

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2. Solving the Quadratic Equation

Use the quadratic formula:

$$X = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Where:

- $a = 1$
- $b = -5$
- $c = 4$

Calculations:

$$ X = \frac{-(-5) \pm \sqrt{(-5)^2 - 4 \times 1 \times 4}}{2 \times 1} $$
$$ X = \frac{5 \pm \sqrt{25 - 16}}{2} $$
$$ X = \frac{5 \pm \sqrt{9}}{2} $$
$$ X = \frac{5 \pm 3}{2} $$

Thus, two solutions:

- $X = \frac{5 + 3}{2} = \frac{8}{2} = 4$
- $X = \frac{5 - 3}{2} = \frac{2}{2} = 1$

3. Corresponding Y-values

Plug back into $Y = X - 4$:

- For $X=4$:

$$ Y = 4 - 4 = 0 $$

- For $X=1$:

$$ Y = 1 - 4 = -3 $$

Intersection points:

- $\boxed{(4, 0)}$
- $\boxed{(1, -3)}$

These are the precise points where the parabola and line cross each other.

Interpreting the Results and Graphical Significance

1. Visual Confirmation

Plotting the points:

- Line: passes through (0, -4), (1, -3), (4, 0)
- Parabola: passes through (0, 0), (1, -3), (2, -4), (3, -3), (4, 0)

The intersection points (1, -3) and (4, 0) confirm the solutions.

2. Geometric Meaning

- The solutions correspond to the points where the quadratic curve and the straight line meet.
- The parabola is symmetric about the line $X=2$, and the intersection points lie symmetrically with respect to this axis.

3. Algebraic vs. Graphical Solutions

- Algebraic approach provides exact solutions (X=1, 4).
- Graphical interpretation helps visualize the intersection and approximate points if needed.

Additional Insights and Applications

1. Role of Discriminant in Quadratic Solutions

The discriminant ($\Delta = b^2 - 4ac$):

$$
\Delta = 25 - 16 = 9
$$

Since $\Delta > 0$, the quadratic has two distinct real roots, meaning the line intersects the parabola at two points.

2. Use of Graphs in Real-World Contexts

- Quadratic functions model projectile motion, profit maximization, and physical phenomena.
- Linear functions often represent constant rates or relationships.
- Intersection points indicate equilibrium, optimality, or critical points in models.

3. Extending the Analysis

- Varying the line's slope or intercept alters intersection points.
- For example, changing $Y = X - 4$ to $Y = mX + c$ allows exploration of different scenarios.
- Analyzing the discriminant of the combined quadratic equations helps determine the nature and number of solutions.

Summary and Final Remarks

This comprehensive review underscores the importance of understanding the interplay between quadratic and linear functions. The key takeaways include:

- The graph of $Y = X^2 - 4X$ is a parabola opening upward with vertex at (2, -4) and intercepts at (0, 0) and (4, 0).
- The line $Y = X - 4$ intersects this parabola at two points: (1, -3) and (4, 0).
- Solving the system involves equating the two expressions for Y, leading to a quadratic in X, which can be solved via the quadratic formula.
- The solutions are confirmed both algebraically and graphically, demonstrating the harmony between algebraic methods and geometric visualization.

In educational and practical contexts, mastering these techniques enhances problem-solving skills, deepens comprehension of conic sections,

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