The Height Of A Door Is 1.8 Feet Longer Than It's With Its Front Area Is 1496.9 Ft. Find The Width And

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In the realm of geometry and practical measurements, solving real-world problems often involves understanding the relationships between different dimensions of objects. One such intriguing problem involves determining the width and height of a door based on given measurements and relationships. In this article, we will explore how to approach this problem systematically, analyze the given data, and derive the unknown dimensions step by step. Whether you're a student preparing for exams or someone interested in applying mathematical reasoning to everyday objects, this comprehensive guide will walk you through the process.

Understanding the Problem Statement

Before delving into calculations, it is essential to interpret the given information correctly.

Given Data:
- The height of the door is 1.8 feet longer than its width.
- The front area (likely the surface area of the door's front face) is 1496.9 square feet.

What is asked:
- Find the width of the door.
- Find the height of the door.

This problem involves two key relationships:
1. The relationship between height and width.
2. The calculation of the area based on these dimensions.

Breaking Down the Known Data

1. Relationship Between Height and Width

Let’s denote:
- Width of the door = w (in feet)
- Height of the door = h (in feet)

From the problem:

$h = w + 1.8$

This simple equation relates height and width directly.

2. Area of the Front Face

The front face of the door is a rectangle, so its area is:

$\text{Area} = \text{height} \times \text{width} = 1496.9 \text{ ft}^2$

Using the notation:
Area = h × w

Formulating the Mathematical Equation

Since we know that h = w + 1.8, substitute this into the area equation:

$$ (w + 1.8) \times w = 1496.9 $$

Expanding the equation:

$$ w^2 + 1.8w = 1496.9 $$

This is a quadratic equation in terms of w:

$$ w^2 + 1.8w - 1496.9 = 0 $$

Solving the Quadratic Equation

To find w, we can apply the quadratic formula:

$$ w = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} $$

where:
- $ a = 1 $
- $ b = 1.8 $
- $ c = -1496.9 $

Calculating the discriminant:

$$ D = b^2 - 4ac $$
$$ D = (1.8)^2 - 4 \times 1 \times (-1496.9) $$
$$ D = 3.24 + 5987.6 $$
$$ D = 5990.84 $$

Calculate the square root:

$$ \sqrt{D} = \sqrt{5990.84} \approx 77.4 $$

Now, plug into the quadratic formula:

$$ w = \frac{-1.8 \pm 77.4}{2} $$

This yields two solutions:

1. $ w = \frac{-1.8 + 77.4}{2} = \frac{75.6}{2} = 37.8 $
2. $ w = \frac{-1.8 - 77.4}{2} = \frac{-79.2}{2} = -39.6 $

Since width cannot be negative, the valid solution is:

$$ w = 37.8 \text{ feet} $$

Finding the Height of the Door

Recall:

$$ h = w + 1.8 $$
$$ h = 37.8 + 1.8 = 39.6 \text{ feet} $$

Therefore:
- The width of the door is approximately 37.8 feet.
- The height of the door is approximately 39.6 feet.

Verifying the Solution

To ensure our calculations are correct, verify by calculating the area:

$$ \text{Area} = w \times h = 37.8 \times 39.6 $$

Calculate:

$$ 37.8 \times 39.6 \approx 1496.88 $$

This is extremely close to the given area of 1496.9 square feet, confirming the accuracy of our solution.

Implications and Practical Considerations

While the problem yields a large door size—about 37.8 feet wide and 39.6 feet tall—these are not typical dimensions for standard doors, which often measure around 3 feet in width and 7 feet in height. This suggests the problem might be theoretical or related to a large structure such as a gate or an industrial door.

Key takeaways:
- The relationship between dimensions can be modeled mathematically to solve for unknowns.
- Quadratic equations are common in geometric problems involving areas and dimensions.
- Always verify solutions to ensure they make sense within the problem context.

Conclusion

In this detailed exploration, we started with a problem involving a door's dimensions and area, formulated an equation based on the given relationships, and applied algebraic methods to find the unknown measurements. The key steps included:

- Interpreting the problem statement accurately
- Establishing algebraic relationships
- Formulating and solving a quadratic equation
- Verifying the solution with the original data

This approach exemplifies how mathematical reasoning can be applied to solve practical measurement problems, emphasizing the importance of understanding relationships between dimensions and the power of algebraic techniques.

Whether you’re tackling real-world construction challenges or engaging in mathematical exercises, mastering these steps will enhance your ability to analyze and solve diverse problems involving geometric measurements.

Summary:
- Width of the door ≈ 37.8 feet
- Height of the door ≈ 39.6 feet

By following a systematic approach—interpreting data, forming equations, solving quadratics, and verifying results—you can confidently determine unknown dimensions of objects based on given relationships and measurements.

Frequently Asked Questions

What is the height of the door if its width is W and the front area is 1496.9 sq ft?

The height of the door is (W + 1.8) feet, where W is the width in feet.

How can we find the width of the door given the front area and the height relation?

Using the area formula: Area = Width × Height. Since Height = W + 1.8, the equation becomes 1496.9 = W × (W + 1.8).

What quadratic equation is formed to find the width of the door?

The quadratic equation is W^2 + 1.8W - 1496.9 = 0.

How do you solve for the width W in the given problem?

Apply the quadratic formula: W = [-b ± sqrt(b^2 - 4ac)] / 2a, where a=1, b=1.8, c=-1496.9.

What is the approximate width of the door based on the quadratic solution?

Calculating yields W ≈ 38.4 feet (positive root), which is the feasible width.

What is the height of the door corresponding to the width found?

The height is approximately 38.4 + 1.8 = 40.2 feet.

Why is it necessary to use the quadratic formula in this problem?

Because the area equation results in a quadratic in W, requiring the quadratic formula to find W.

Can the door's width be negative based on the quadratic

solution?

No, a negative width is not physically possible; only the positive root is valid.

What real-world applications can this problem relate to?

It relates to designing doors and understanding spatial dimensions in architecture or construction projects.

How can this problem be generalized for similar area and dimension problems?

Set up the area as a product of dimensions with one depending on the other, form a quadratic equation, then solve accordingly.

Additional Resources

The Height Of A Door Is 1.8 Feet Longer Than Its Width, Its Front Area Is 1496.9 Ft. Find The Width And

Understanding the dimensions of everyday objects such as doors might seem straightforward at first glance, but when presented with specific measurements and relationships, the problem becomes an interesting exercise in mathematical reasoning. In this article, we delve into a particular problem involving the height and width of a door, along with its front area, to uncover the unknown dimensions. This analytical exploration aims to clarify the problem, demonstrate the step-by-step solution process, and highlight the practical applications of such mathematical modeling.

Introduction: The Significance of Precise Measurements in Architecture and Design

Accurate measurements are pivotal in architecture, construction, interior design, and even in manufacturing. Knowing the exact dimensions of doors, windows, and other fixtures ensures structural integrity, aesthetic appeal, and functional efficiency. Misjudgments or inaccuracies can lead to costly errors, safety concerns, or design flaws.

The problem under discussion involves calculating the width and height of a door based on its area and a specified relationship between its dimensions. While the problem appears simple, it encapsulates fundamental principles of algebra and geometry, which are essential tools for professionals involved in designing and constructing physical spaces.

Problem Restatement and Initial Observations

Let's restate the problem to understand what is being asked:

> "The height of a door is 1.8 feet longer than its width. Its front area is 1496.9 square feet. Find the width and height."

From this, we identify key elements:

- Relationship between height and width: Height = Width + 1.8 feet
- Area of the door (front area): 1496.9 sq. ft.
- Unknowns to find: Width (w) and Height (h)

With these points in mind, we can proceed to develop mathematical expressions to model the problem accurately.

Mathematical Modeling of the Problem

To solve for the unknowns, we first need to translate the problem statement into algebraic equations.

2.1 Defining Variables

Let:

- w = width of the door (in feet)
- h = height of the door (in feet)

2.2 Expressing the Relationship Between Dimensions

Given that:

> "The height of the door is 1.8 feet longer than its width."

We can write:

Equation 1:

h = w + 1.8

2.3 Expressing the Area

The front area of the door is given as 1496.9 sq. ft., which is the product of its width and height:

Equation 2:

Area = w h = 1496.9

Substituting Equation 1 into Equation 2 allows us to express everything in terms of a single variable.

Deriving the Equation for the Unknowns

3.1 Substitution into the Area Equation

Replace h with (w + 1.8):

w (w + 1.8) = 1496.9

3.2 Expanding the Equation

w w + w 1.8 = 1496.9

which simplifies to:

$w^2 + 1.8w = 1496.9$

3.3 Forming a Quadratic Equation

Bring all terms to one side:

$w^2 + 1.8w - 1496.9 = 0$

This quadratic equation in standard form is:

$w^2 + 1.8w - 1496.9 = 0$

Solving the Quadratic Equation

Quadratic equations of the form $ax^2 + bx + c = 0$ can be solved using the quadratic formula:

$w = [-b \pm \sqrt{(b^2 - 4ac)}] / 2a$

where:

- a = 1
- b = 1.8
- c = -1496.9

4.1 Calculating the Discriminant

Discriminant $D = b^2 - 4ac$

$D = (1.8)^2 - 4\ 1\ (-1496.9)$

$D = 3.24 + 5987.6 = 5990.84$

4.2 Calculating the Roots

Since the discriminant is positive, there are two real solutions:

$w = [-1.8 \pm \sqrt{5990.84}] / 2$

Calculate $\sqrt{5990.84}$:

$\sqrt{5990.84} \approx 77.4$ (rounded to one decimal place)

Now, compute the two roots:

- $w_1 = (-1.8 + 77.4) / 2 \approx 75.6 / 2 \approx 37.8$ ft
- $w_2 = (-1.8 - 77.4) / 2 \approx -79.2 / 2 \approx -39.6$ ft

Since width cannot be negative, we discard the negative solution.

4.3 Determining the Dimensions

Thus, the width of the door is approximately 37.8 feet.

Now, find the height:

$h = w + 1.8 \approx 37.8 + 1.8 = 39.6$ feet

Interpreting the Results: Practical Considerations

5.1 Real-world Plausibility

A door width of approximately 37.8 feet and height of 39.6 feet are considerably larger than standard residential or commercial doors, which typically range from 3 to 8 feet in width and 6.5 to 8 feet in height. This suggests that the problem might be hypothetical or designed for a specialized architectural context, such as large industrial entrances, hangar doors, or monumental architecture.

5.2 Validation of the Solution

To verify:

- Calculate the area with the found dimensions:

w h ≈ 37.8 39.6 ≈ 1496.88 sq. ft., matching the given area closely, considering rounding errors.

- Check the relationship:

h - w = 39.6 - 37.8 = 1.8 ft, which aligns perfectly with the given relationship.

5.3 Sensitivity to Measurement Precision

Small variations in the area or the relationship could significantly alter the dimensions due to the quadratic nature of the problem. Accurate measurements are essential in practical applications to ensure safety and functionality.

Additional Analytical Insights

6.1 Effect of Changing the Area

Suppose the front area was different; the quadratic equation would change accordingly. For example, if the area were 1500 sq. ft., the calculation would be similar but with c = -1500, leading to slightly different dimensions.

6.2 Considering Alternative Scenarios

If the problem involved other relationships, such as the perimeter or volume (for doors with depth), the complexity would increase, requiring more advanced modeling.

6.3 Limitations of the Model

The model assumes:

- The door is a perfect rectangle.
- The relationship between height and width is linear.
- Measurements are exact, ignoring units like thickness or material constraints.

In real-world applications, additional factors such as material strength, aesthetic proportions, and structural support influence the actual dimensions.

Conclusion: Significance and Practical Implications

The problem of determining the width and height of a door based on its area and a specified dimensional relationship exemplifies fundamental principles of algebra and geometry. Through the process of translating word problems into mathematical models, solving quadratic equations, and interpreting results, we gain valuable insights into designing functional, safe, and aesthetically pleasing structures.

While the computed dimensions may seem unusual in everyday contexts, such exercises reinforce the importance of precise calculations in architecture and engineering. They also demonstrate how mathematical reasoning can be applied to solve real-world problems, ensuring that designs meet specified criteria and constraints.

In practice, professionals must consider additional factors beyond pure geometry, including material properties, safety standards, and environmental conditions. Nonetheless, mastering such foundational problem-solving techniques is essential for advancing in fields that require rigorous quantitative analysis.

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- "Dimensions and Standards for Commercial Doors," Building Codes and Specifications, 2021.

Author's Note: This article demonstrates a systematic approach to solving a dimensional problem involving a door, combining algebraic modeling, quadratic solutions, and practical interpretation to provide a comprehensive understanding suitable for students, architects, engineers, and enthusiasts alike.

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