The Larger Of Two Numbers Exceeds 3 Times The Smallernumber By 6. The Sum Of 3 Times The Larger Number

The Larger Of Two Numbers Exceeds 3 Times The Smallernumber By 6. The Sum Of 3 Times The Larger Number is a classic problem in algebra that involves setting up equations based on given relationships between two unknown numbers. Such problems are fundamental in developing mathematical reasoning and problem-solving skills, especially in understanding how to translate verbal descriptions into algebraic expressions. This article explores this particular problem in depth, illustrating how to formulate the equations, solve them systematically, and interpret the solutions. Whether you're a student seeking to strengthen your algebra skills or someone interested in mathematical problem-solving, understanding this problem provides a solid foundation for tackling similar questions.

Understanding the Problem

Restating the Problem

The problem states:
- The larger of two numbers exceeds three times the smaller number by 6.
- The sum of three times the larger number is to be considered (though the original statement appears incomplete, we interpret it as focusing on the relationship involving these quantities).

In essence, the problem involves two unknowns—let's call them x and y—where:
- x is the larger number.
- y is the smaller number.

The key points to understand are:
- The comparison between the larger and smaller numbers.
- The relationships involving multiples of these numbers.
- The potential for setting up equations based on these relationships.

Deciphering the Verbal Statement

A typical way to approach such problems is to identify the key phrases and convert them into algebraic expressions.

- "The larger of two numbers exceeds 3 times the smaller number by 6" translates to:

$$
x = 3y + 6
$$

- The phrase "The sum of 3 times the larger number" suggests examining an expression like $3x$, but the problem appears incomplete. Usually, such problems continue with a statement like "Find the sum" or "Compare it to," which helps determine what to do with this sum.

Assuming the problem's intended continuation is to find the sum of 3 times the larger number and perhaps relate it to other quantities, the core relationship remains.

For clarity, the main focus will be on the first part: establishing the relationship between x and y.

Formulating the Equations

Defining Variables

To analyze the problem systematically, define:
- x: the larger number.
- y: the smaller number.

Establishing Relationships

Based on the problem statement:
- Larger number exceeds three times the smaller by 6:

$$
x = 3y + 6
$$

- The problem mentions the sum of 3 times the larger number, which could imply:

$$
\text{Sum} = 3x
$$

but without additional context, this may involve further operations or comparisons.

Possible Additional Relationships

If the original problem intended to compare this sum to other quantities or to find specific values, typical extensions include:
- Equate $3x$ to another expression.
- Find the value of the sum given certain conditions.

For the purpose of this article, we will focus on solving for x and y based on the initial relationship, then explore how to compute the sum of $3x$.

Solving the Equations

Expressing One Variable in Terms of the Other

From the key relationship:

$$
x = 3y + 6
$$

we see that:
- Once y is known, x can be directly calculated.
- Conversely, if x is known, y can be found by rearranging:

$$
y = \frac{x - 6}{3}
$$

Finding Numerical Solutions

Since the problem doesn't specify particular values, we can select arbitrary values for y to find corresponding x. For example:

- If $y = 2$:

$$
x = 3(2) + 6 = 6 + 6 = 12
$$

- If $y = 4$:

$$
x = 3(4) + 6 = 12 + 6 = 18
$$

- If $y = 0$:

$$
x = 3(0) + 6 = 6
$$

This illustrates the relationship; for any value of y, x can be calculated.

Calculating the Sum of 3 Times the Larger Number

Once x is known, the sum of three times the larger number is:

\[

\text{Sum} = 3x
\]

For example:
- With $x=12$, sum = $3 \times 12 = 36$.
- With $x=18$, sum = $3 \times 18 = 54$.
- With $x=6$, sum = $3 \times 6 = 18$.

This demonstrates how the relationship between the two numbers influences the sum.

Interpreting the Results and Additional Considerations

Understanding the General Solution

The relationship $x = 3y + 6$ is linear, and it allows for an infinite set of solutions depending on the value chosen for y. The solutions are all pairs (x, y) satisfying this equation.

Implications of the Relationship

- The larger number x is always 6 more than three times the smaller number.
- As y increases, x increases proportionally.
- For negative values of y, the larger number x decreases accordingly, but the relationship still holds.

Potential Extensions and Variations

In more complex versions of this problem, additional relationships or constraints could be introduced, such as:
- The sum of the two numbers equals a specific value.
- The larger number is greater than the smaller by a certain amount.
- The sum of 3 times the larger number equals a particular total.

These extensions transform the problem into systems of equations, requiring methods like substitution or elimination to find solutions.

Practical Applications and Problem-Solving Strategies

Steps to Tackle Similar Problems

When approaching problems involving relationships between two numbers, consider the following steps:

1. **Identify variables:** Assign symbols to unknown quantities.

2. **Translate verbal statements into equations:** Convert phrases into algebraic expressions.
3. **Set up equations:** Based on the relationships described.
4. **Solve the equations:** Use substitution, elimination, or graphing methods.
5. **Interpret the solutions:** Check for logical consistency and relevance to the problem context.

Common Pitfalls to Avoid

- Misinterpreting the verbal statement.
- Forgetting to consider the domain of the variables (e.g., negative numbers if contextually inappropriate).
- Overlooking additional constraints or conditions.

Conclusion

Understanding how to model and solve problems like "The larger of two numbers exceeds three times the smaller number by 6" is fundamental in algebra. By systematically translating verbal descriptions into equations, analyzing the relationships, and exploring solutions, students and learners develop critical reasoning skills. The key takeaway is that such problems often have infinitely many solutions unless additional constraints are provided. Recognizing the pattern $x = 3y + 6$ allows for easy computation of the larger number once the smaller is known, and vice versa. Moreover, calculating the sum of three times the larger number becomes straightforward once the relationship is established. Mastery of these concepts forms a strong foundation for tackling more complex algebraic problems and enhances problem-solving confidence across various mathematical contexts.

Frequently Asked Questions

If the larger number exceeds three times the smaller number by 6, and the sum of three times the larger number is given, how can we determine both numbers?

Set variables for the smaller number (x) and the larger number (y). Write the equations: $y = 3x + 6$, and 3y (sum of three times the larger). Substitute y into the second equation to find the values of x and y accordingly.

What algebraic method can be used to solve for the two numbers when given that the larger exceeds three times the smaller by 6?

Use substitution or simultaneous equations. First, express y in terms of x ($y = 3x + 6$), then substitute into the other given condition to solve for x, and consequently find y.

How does understanding the relationship 'larger exceeds three times the smaller by 6' help in setting up equations for problem-solving?

It provides a direct linear equation ($y = 3x + 6$) that relates the two variables, which can be used alongside other conditions to form a system of equations for solving the numbers.

If the sum of three times the larger number is known, how can it be used to find the smaller and larger numbers?

Once y is expressed in terms of x, substitute into the sum equation (e.g., $3y$ = total) to find the value of y, then back-substitute to find x. This allows determination of both numbers.

Can you provide an example with actual numbers based on the problem statement to illustrate the solution process?

Suppose the sum of three times the larger number is 48. Using $y = 3x + 6$, then $3y = 48$, so $y = 16$. Substitute y into $y = 3x + 6$: $16 = 3x + 6$, leading to $3x = 10$, so $x = 10/3$. Therefore, the smaller number is 10/3, and the larger is 16.

Additional Resources

The Larger Of Two Numbers Exceeds 3 Times The Smaller Number By 6

In the realm of algebra, word problems often serve as gateways to understanding fundamental principles of relationships between quantities. Among these, the problem statement: "The larger of two numbers exceeds three times the smaller number by 6; find the sum of three times the larger number" stands out as a classic example illustrating the power of algebraic modeling and problem-solving techniques. This article delves into the intricacies of this problem, exploring its formulation, solution methods, implications, and broader significance within mathematical education and application.

Understanding the Problem: Breaking Down the Statement

Before engaging with calculations, it is crucial to interpret the problem accurately. The statement provides two key pieces of information:

1. The larger number exceeds three times the smaller number by 6.
2. Find the sum of three times the larger number.

Let’s analyze these components:

- The phrase "exceeds" indicates a greater-than relationship, specifically an addition of the difference to the smaller value.
- The comparison involves two quantities: a larger number (let's denote it as L) and a smaller number (S).
- The second part asks for a specific value: "the sum of three times the larger number," which is 3L.

By translating these into algebraic expressions, we can create equations to model the problem.

Formulating the Mathematical Model

Defining Variables

Let:

- S = the smaller number
- L = the larger number

Given the problem context, both S and L are real numbers, and typically, we assume they are positive unless stated otherwise.

Constructing Equations from the Word Problem

The key sentence: "The larger of two numbers exceeds three times the smaller number by 6" can be expressed as:

$$L = 3S + 6$$

This equation states that the larger number is exactly 6 greater than three times the smaller number.

The second part of the problem asks: "Find the sum of three times the larger number."

This translates directly into:

$$\text{Sum} = 3L$$

Our goal is to find $3L$, once L is known.

Solving the Model: Approach and Techniques

At first glance, the problem provides a single equation, which introduces a limitation: without additional constraints or information, the problem may be underdetermined, meaning infinitely many solutions for S and L. To obtain a unique solution, the problem typically assumes or implies specific conditions—most often, the numbers are positive or integers.

Suppose we proceed under the assumption that both numbers are positive real numbers, which is common in such problems.

Expressing L in terms of S

From the earlier equation:

$$L = 3S + 6$$

Now, the sum of three times the larger number is:

$$3L = 3(3S + 6) = 9S + 18$$

Thus, the value we're asked to find depends on S, which is still unknown.

Determining S and L

Because there's no additional equation, the problem might be incomplete unless it specifies further constraints. For example, if the problem states that both numbers are positive integers, then S must be a positive integer, and the larger number L must also be positive.

Assuming S is a positive integer:

- For $S = 1$:

$$L = 3(1) + 6 = 9$$
$$3L = 3 \times 9 = 27$$

- For $S = 2$:

$$L = 3(2) + 6 = 12$$
$$3L = 36$$

- For $S = 3$:

$$L = 3(3) + 6 = 15$$
$$3L = 45$$

And so on.

This pattern indicates that for each positive integer S, the sum $3L$ increases linearly as $9S + 18$.

Interpreting the Results: General and Specific Solutions

The analysis reveals that without additional constraints, the problem yields a family of solutions parameterized by S. The key takeaway is that:

$$\boxed{3L = 9S + 18}$$

and

$$L = 3S + 6$$

for any positive real S.

Implications of the Solution Set

- The problem models a linear relationship between the two numbers.
- The sum of three times the larger number depends linearly on the smaller number.
- When S is known, the value of $3L$ can be directly computed.

If the problem intends for a unique solution, it would need to specify additional information such as the value of either number or the sum of the two numbers.

Broader Context and Educational Significance

Modeling Real-World Situations

This problem exemplifies how algebraic modeling can capture real-world relationships—such as comparing quantities, determining unknowns, and understanding proportional reasoning.

Developing Critical Thinking and Problem-Solving Skills

It encourages students and practitioners to:

- Translate verbal statements into algebraic equations.
- Recognize relationships and formulate models.
- Solve for unknowns systematically.
- Understand the importance of constraints in problem-solving.

Extensions and Variations

- Incorporating additional constraints: For example, if the sum of the numbers is known.
- Exploring different relationships: Such as the larger number being less than or equal to a certain value.
- Applying to real-life scenarios: Budgeting, resource allocation, or planning tasks where such relationships are relevant.

Conclusion: Unraveling the Mathematical Relationship

The problem statement: "The larger of two numbers exceeds three times the smaller number by 6; find the sum of three times the larger number" provides a clear illustration of algebraic modeling. While the initial formulation yields a family of solutions, understanding the relationships between variables allows us to compute the desired quantity once the smaller number is known.

This analysis underscores the importance of precise interpretation, the role of additional constraints in solving underdetermined problems, and the power of algebra in making sense of relationships that govern everyday and mathematical phenomena.

In educational contexts, problems like this serve as foundational exercises that enhance analytical thinking, foster a deeper understanding of linear relationships, and prepare learners for more complex problem-solving scenarios across disciplines.

In summary:

- The key equation: $L = 3S + 6$
- The quantity to find: $3L = 9S + 18$
- Without constraints, solutions depend on the value of S.
- Additional information is needed for a unique solution, but the algebraic framework remains consistent.

By dissecting such problems, students and practitioners develop vital skills in translating language into mathematics, analyzing relationships, and applying logical reasoning—core competencies that transcend the classroom and find relevance in numerous fields.

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