

The Magnetic Field Of An Electromagnetic Wave In A Vacuum Is $B_z = (4.0\text{T})\sin((9.50106)x t)$, Where X Is In

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Understanding the behavior of electromagnetic waves in a vacuum is fundamental to many areas of physics and engineering. The magnetic field component of such waves plays a crucial role in their propagation, interaction, and applications. The given expression for the magnetic field, $B_z = (4.0\text{ T})\sin((9.50106) \times x \times t)$, provides a detailed snapshot of how the magnetic field varies both in space and time. In this article, we will analyze this expression comprehensively, exploring its physical significance, mathematical properties, and implications in the context of electromagnetic wave theory.

Introduction to Electromagnetic Waves in a Vacuum

Electromagnetic (EM) waves are solutions to Maxwell's equations, describing oscillating electric and magnetic fields that propagate through space at the speed of light, $c \approx 3.00 \times 10^8\text{ m/s}$. In a vacuum, these waves are characterized by their electric field E and magnetic field B components, which are perpendicular to each other and to the direction of propagation.

Key features of electromagnetic waves in a vacuum include:

- Speed: The wave travels at the speed of light, c .
- Perpendicular fields: The electric and magnetic fields are mutually perpendicular and perpendicular to the direction of wave propagation.
- Energy transfer: EM waves carry energy and momentum through space.
- Sinusoidal nature: The fields vary sinusoidally with space and time, reflecting their oscillatory behavior.

The specific form of the magnetic field component provides insight into the wave's characteristics, including its amplitude, frequency, wavelength, and phase.

Analyzing the Magnetic Field Expression

The given magnetic field expression is:

$$B_z = (4.0\text{ T}) \sin(9.50106 x t)$$

This can be viewed as a sinusoidal wave with specific parameters. To interpret it fully, we should examine each component:

1. Amplitude of the Magnetic Field

- Amplitude (B_0): 4.0 Tesla (T)

This value indicates the maximum magnetic field strength of the wave at any point in space and time. In practical terms, 4.0 T is an extremely strong magnetic field, comparable to those generated in MRI machines. In typical electromagnetic waves (like light), the magnetic field amplitudes are much lower, but for theoretical or specialized applications, such high amplitudes can be considered.

2. The Argument of the Sine Function: $(9.50106x, t)$

This term encapsulates the wave's oscillatory nature, coupling space and time variables. It indicates that the magnetic field oscillates sinusoidally with a phase that depends on position (x) and time (t) .

- Wave number (k): 9.50106 (units depend on the context of x and t)

- Angular frequency (ω): Also 9.50106 in this expression, implying a specific relationship between frequency and wavelength.

3. Units and Variables

- x : The position coordinate, typically measured in meters (m).

- t : Time, measured in seconds (s).

The product (xt) suggests the wave propagates through space over time, with the phase of the wave depending on both variables.

Understanding the Wave Parameters

To fully characterize the wave, we need to determine its wavelength (λ) and frequency (f) . These are related to the wave number (k) and angular frequency (ω) :

$$k = \frac{2\pi}{\lambda}$$

$$\omega = 2\pi f$$

Given that the argument of the sine function is $(\omega t - kx)$ (assuming a propagation in the positive x -direction), the expression:

$$\sin(kx - \omega t)$$

matches our form if we interpret the coefficient as (ω) and the spatial coefficient as (k) .

Note: The given expression is:

$$B_z = 4.0 \sin(9.50106 x, t)$$

which suggests the phase depends on $(x t)$. Usually, in wave equations, the phase depends on $(k x - \omega t)$, but the form provided implies a product $(x t)$, which is unusual. For standard wave solutions, the argument should be linear in either (x) or (t) separately, such as $(k x - \omega t)$.

Possible interpretation: The expression might be simplified or written in a specific context, or perhaps the notation is shorthand for the combined phase $(\omega t - k x)$.

Wave Propagation and Direction

Assuming the form aligns with standard wave equations, the wave propagates in the positive x -direction with a phase:

$$\phi(x, t) = k x - \omega t$$

If the argument is $(\omega t - k x)$, the wave moves in the positive x -direction; if it's $(k x - \omega t)$, the same applies.

Given the parameters:

$$k = \omega = 9.50106 \text{ (units to determine)}$$

we can find the wavelength and frequency.

1. Determining Units

In SI units, the wave number (k) is in radians per meter (rad/m), and angular frequency (ω) is in radians per second (rad/s).

The expression suggests $(k = \omega = 9.50106)$. Usually, these are in different units unless specified.

2. Calculating Wavelength (λ)

$$\lambda = \frac{2\pi}{k}$$

If $(k = 9.50106 \text{ rad/m})$:

$$\lambda = \frac{2\pi}{9.50106} \approx \frac{6.2832}{9.50106} \approx 0.661, \text{ m}$$

3. Calculating Frequency (f)

$$f = \frac{\omega}{2\pi}$$

If ($\omega = 9.50106, \text{ rad/s}$):

$$f = \frac{9.50106}{6.2832} \approx 1.512, \text{ Hz}$$

Note: These calculations depend on the units assigned to the coefficients. If the units differ, the actual frequency and wavelength would change accordingly.

The Relationship Between Electric and Magnetic Fields in an EM Wave

A key principle of electromagnetic wave theory is the relationship between the electric field E and magnetic field B :

$$\frac{E}{B} = c$$

where (c) is the speed of light in vacuum ($\sim 3.00 \times 10^8 \text{ m/s}$).

Given the magnetic field amplitude ($B_0 = 4.0, \text{ T}$), the electric field amplitude (E_0) can be estimated:

$$E_0 = c \times B_0 = (3.00 \times 10^8, \text{ m/s}) \times 4.0, \text{ T} = 1.2 \times 10^9, \text{ V/m}$$

This enormous electric field strength indicates a highly intense wave, not typical of everyday electromagnetic radiation like visible light, but perhaps relevant in specialized experimental contexts.

Implications and Applications

Understanding the magnetic field component of an electromagnetic wave allows scientists and engineers to design systems for various applications:

1. Communication Technologies

- Radio waves: The magnetic and electric fields are used to transmit information over long distances.
- Satellite communication: Accurate modeling of wave propagation in vacuum or space environments requires understanding magnetic field behavior.

2. Medical Imaging and Treatment

- MRI: Strong magnetic fields are employed, although not in the form of propagating EM waves like the one described here.

3. Scientific Research

- High-energy physics: Understanding wave behavior at high amplitudes aids in particle acceleration and plasma physics.
- Wave interaction studies: Analyzing how intense EM waves interact with matter or fields.

4. Space Physics

- Cosmic radiation: Earth's magnetosphere and space phenomena involve intense electromagnetic waves, where magnetic field analysis is crucial.

Conclusion

The given magnetic field expression, $B_z = (4.0 \text{ T}) \sin(9.50106 \times 10^6 x - 2.90131 \times 10^{15} t)$, encapsulates a sinusoidal electromagnetic wave propagating in vacuum with a significant magnetic field amplitude. By dissecting its parameters, we've gained insights into the wave's amplitude, wavelength, frequency, and potential energy density. Its analysis reinforces fundamental principles of electromagnetic theory, such as the relationship between electric and magnetic fields, wave propagation characteristics, and their applications across science and technology.

Understanding such expressions is essential for advancing fields like telecommunications, astrophysics, and experimental physics, where precise control and knowledge of electromagnetic wave behavior are paramount. Moreover, these analyses help bridge theoretical concepts with real-world phenomena, enabling innovation and discovery in various scientific domains.

References:

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Frequently Asked Questions

What does the magnetic field component B_z in an electromagnetic wave represent?

B_z represents the magnetic field component of the electromagnetic wave oscillating along the z-axis in a vacuum.

Given $B_z = (4.0 \text{ T}) \sin(9.50 \times 10^6 \text{ x t})$, what are the units of x and t?

Both x and t are in meters and seconds respectively, as indicated by the SI units used in the wave equation.

What is the significance of the amplitude 4.0 T in the magnetic field expression?

The amplitude 4.0 T indicates the maximum value of the magnetic field in the wave.

How can we determine the wave's phase velocity from the given magnetic field expression?

The phase velocity v can be found using $v = \omega / k$, where $\omega = 9.50 \times 10^6 \text{ rad/s}$ and k is the wave number; here, k can be derived from the wave equation parameters.

What is the relationship between the magnetic field B_z and the electric field in this electromagnetic wave?

In a vacuum electromagnetic wave, the electric field E and magnetic field B are perpendicular, and their magnitudes are related by $E = c B$, where c is the speed of light.

How does the magnetic field vary with position and time in this wave?

B_z varies sinusoidally with both position x and time t as shown by the sine function, indicating a traveling wave oscillating in space and time.

What is the direction of propagation of this electromagnetic wave?

The wave propagates in the direction perpendicular to both the electric and magnetic fields; typically, if B_z is along z , the wave propagates along the x or y direction depending on the setup.

Can you find the frequency of this electromagnetic wave from the given expression?

Yes, the angular frequency ω is 9.50×10^6 rad/s; the frequency $f = \omega / (2\pi) \approx 1.51$ MHz.

What is the wavelength of the electromagnetic wave described by this magnetic field?

The wavelength λ can be calculated using $\lambda = v / f$; with $v \approx 3 \times 10^8$ m/s and $f \approx 1.51$ MHz, $\lambda \approx 198.7$ meters.

Why is the magnetic field component B_z described as sinusoidal in the wave equation?

Because electromagnetic waves are transverse and oscillate sinusoidally, representing the periodic variation of the magnetic field in space and time.

Additional Resources

Electromagnetic Wave Magnetic Field Analysis

Introduction: Understanding Electromagnetic Waves in a Vacuum

Electromagnetic (EM) waves are fundamental to our universe, responsible for phenomena ranging from light and radio signals to X-rays and gamma rays. These waves are oscillations of electric and magnetic fields that propagate through space at the speed of light. In a vacuum, where there are no particles to interfere, these fields are perfectly orchestrated, traveling in harmony and maintaining specific relationships dictated by Maxwell's equations.

This article delves into a particular case: the magnetic field component of an electromagnetic wave in a vacuum, expressed as $B_z = (4.0 \text{ T}) \sin(9.50106 \times 10^6 \times \pi \times (x - ct))$. We will explore the significance of this mathematical expression, interpret its physical meaning, and discuss how it fits into the broader framework of electromagnetic theory.

Deciphering the Mathematical Expression

Understanding the Components

The given magnetic field component is:

$$B_z = (4.0 \text{ T}) \sin(9.50106 \times x_t)$$

where:

- B_z : The magnetic field in the z-direction.
- 4.0 T: The amplitude of the magnetic field, indicating the maximum magnitude.
- $\sin$: The sine function, describing the oscillatory nature.
- 9.50106: The wave number (k) , expressed in radians per meter.
- x_t : The position variable, combining spatial coordinate (x) and temporal coordinate (t) .

At first glance, this expression resembles the standard form of a plane electromagnetic wave propagating through space, with the magnetic field oscillating sinusoidally as a function of position and time.

Clarifying the Variables: The Role of (x_t)

The notation (x_t) is crucial. Typically, in wave physics, the argument of the sine function for a wave propagating along the x-axis can be expressed as:

$$kx - \omega t$$

where:

- (k) : Wave number, related to the wavelength (λ) by $(k = 2\pi / \lambda)$.
- (ω) : Angular frequency, related to the period (T) by $(\omega = 2\pi / T)$.
- (x) : Spatial coordinate.
- (t) : Time.

Given the notation (x_t) , it is likely a shorthand for the combined variable:

$$x_t = x - vt$$

where (v) is the phase velocity of the wave, possibly the speed of light in a vacuum $(c \approx 3.0 \times 10^8 \text{ m/s})$. Alternatively, it could be a specific combination of (x) and (t) , such as $(x - vt)$, to represent a wave moving along the x-axis.

Physical Significance of the Magnetic Field Expression

Amplitude: The Peak Magnetic Field

The amplitude of 4.0 Tesla (T) indicates the maximum magnetic field strength at a point as the wave propagates. To contextualize:

- In typical electromagnetic waves in free space, the magnetic field amplitude is related to the electric field amplitude via:

$$B_{\text{max}} = \frac{E_{\text{max}}}{c}$$

where c is the speed of light.

- A 4.0 T magnetic field is extraordinarily strong compared to natural electromagnetic waves like visible light, which have magnetic fields on the order of nanoteslas (nT). Such a high field suggests this is either a theoretical or specialized laboratory-generated wave, possibly used in high-energy physics or advanced electromagnetic experiments.

Wave Number and Spatial Oscillations

The wave number ($k = 9.50106 \text{ rad/m}$) implies a wavelength (λ):

$$\lambda = \frac{2\pi}{k} \approx \frac{6.2832}{9.50106} \approx 0.661 \text{ meters}$$

This wavelength indicates the spatial periodicity of the magnetic field component—how frequently the magnetic field oscillates in space.

Propagation Characteristics of the Wave

Wave Direction and Velocity

The form of the sinusoidal function suggests a plane wave propagating along the x-axis. The phase

velocity v of the wave can be deduced from the relationship:

$$v = \frac{\omega}{k}$$

where:

- ω : Angular frequency.
- k : Wave number.

In standard electromagnetic waves in a vacuum:

$$v = c = 3.0 \times 10^8, \text{ m/s}$$

Given the wave number, if the wave propagates at the speed of light, the angular frequency ω would be:

$$\omega = c \times k = (3.0 \times 10^8, \text{ m/s}) \times 9.50106, \text{ rad/m} \approx 2.8503 \times 10^9, \text{ rad/sec}$$

which corresponds to a frequency:

$$f = \frac{\omega}{2\pi} \approx \frac{2.8503 \times 10^9}{6.2832} \approx 4.54 \times 10^8, \text{ Hz}$$

or roughly 454 MHz, within the radio-frequency domain.

Electric and Magnetic Fields: A Perfectly Perpendicular Pair

In free space, the electric and magnetic fields of an electromagnetic wave are perpendicular and in phase. The relationship is given by:

$$\mathbf{E} \perp \mathbf{B} \perp \text{direction of propagation}$$

and the magnitudes are related by:

$$E = c B$$

\]

Thus, if the magnetic field amplitude is 4.0 T, the corresponding electric field amplitude would be:

$$E_{\text{max}} = c \times B_{\text{max}} = (3.0 \times 10^8 \text{ m/s}) \times 4.0 \text{ T} = 1.2 \times 10^9 \text{ V/m}$$

which is an extraordinarily intense electric field, again indicating a highly specialized or theoretical wave.

Implications and Applications

High-Field Electromagnetic Wave Phenomena

While the magnetic field strength of 4.0 T is beyond typical natural phenomena, understanding such intense fields is crucial in certain research domains:

- Particle acceleration: High magnetic fields are used in advanced particle accelerators.
- Magnetic resonance imaging (MRI): Although MRI fields are much weaker, understanding magnetic field oscillations is foundational.
- Laser-plasma interactions: Extremely intense electromagnetic fields can influence plasma behavior, relevant in fusion research.

Potential Experimental Scenarios

This wave's parameters could be hypothetical or designed for:

- Testing the limits of electromagnetic field interactions.
- Modeling wave behavior in strong-field regimes.
- Exploring the effects of high-intensity electromagnetic radiation on matter.

Conclusion: Synthesis of the Magnetic Field Dynamics

The given expression for the magnetic field component of an electromagnetic wave in a vacuum encapsulates a wealth of information about wave behavior. It indicates a high-amplitude, high-frequency wave propagating along a specific axis, with clear spatial oscillations characterized by the wave number.

Understanding such a wave requires integrating knowledge of Maxwell's equations, wave mechanics, and electromagnetic theory. While the parameters imply an extraordinary wave, the fundamental principles remain consistent: the magnetic field oscillates sinusoidally, perpendicular to both the electric field and the direction of propagation, fulfilling the critical relationships that define electromagnetic radiation.

This detailed analysis not only clarifies the physical meaning behind the mathematical expression but also highlights the profound interplay between electric and magnetic fields that underpins much of modern physics and engineering.

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