The Output Of A System In Response To An Input X(t) = E^2tu(-t) Is Y(t) = E^t U(-t). Find And Draw The

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Understanding the behavior of systems in response to various inputs is a fundamental aspect of signal processing and control systems. In this article, we will analyze a specific case where the input signal is $X(t) = e^{2t} u(-t)$, and the output is given as $Y(t) = e^{t} u(-t)$. Our goal is to determine the system's properties, find the corresponding system function or impulse response, and visually interpret these signals through diagrams. This comprehensive exploration is designed to enhance your grasp of time-domain analysis, system responses, and the mathematical tools involved.

Understanding the Input and Output Signals

1. The Input Signal $X(t) = e^{2t} u(-t)$

The input signal combines an exponential component e^{2t} with the unit step function $u(-t)$. Let's break down this expression:

- Exponential component e^{2t}: This grows rapidly as $t \to +\infty$ and decays exponentially as $t \to -\infty$.
- Unit step function $u(-t)$: This function equals 1 when $-t \geq 0$ (i.e., $t \leq 0$) and 0 otherwise. Effectively, it "turns on" the exponential for all $t \leq 0$, and turns it off for $t > 0$.

Graphical interpretation:
The input $X(t)$ is non-zero only for $t \leq 0$, where it behaves as e^{2t}. For $t > 0$, $X(t) = 0$.

2. The Output Signal $Y(t) = e^t u(-t)$

Similarly, the output combines an exponential e^{t} with $u(-t)$:

- It is non-zero only for $t \leq 0$, where it equals e^{t}.
- For $t > 0$, $Y(t) = 0$.

Graphical interpretation:
This signal is a decreasing exponential for $t \leq 0$, vanishing for $t > 0$.

Analyzing the System: From Input to Output

1. Recognizing System Linearity and Time-Invariance

The given response suggests the system may be linear and time-invariant (LTI). These properties allow us to use the system's impulse response or transfer function to analyze behavior.

- Linearity: The system's output is proportional to the input, and the superposition principle applies.
- Time-invariance: The system's behavior does not change over time; shifting the input results in a shifted output.

2. Determining the System's Impulse Response $h(t)$

In LTI systems, the output for any input $X(t)$ can be found via convolution:

$$
Y(t) = (X h)(t) = \int_{-\infty}^{\infty} X(\tau) h(t - \tau) d \tau
$$

Given the specific input-output pair, we can attempt to deduce $h(t)$.

Key idea:
Since the input is multiplied by $u(-t)$, a causal or anti-causal interpretation may help. The signals are non-zero only for $t \leq 0$, indicating a possible anti-causal system.

Mathematical Derivation of the System's Impulse Response

1. Expressing the System's Response

Suppose the system is LTI with impulse response $h(t)$. Then, for an input $X(t)$, the output $Y(t)$ is:

$$
Y(t) = X(t) h(t)
$$

Given:

$$
X(t) = e^{2t} u(-t)
$$

$$
Y(t) = e^{t} u(-t)
$$

Our goal is to find $h(t)$.

2. Using Convolution to Find $h(t)$

In particular, if we consider the input as a shifted or scaled version of a basic function, or explore their Laplace transforms, it becomes easier to find $h(t)$.

- Laplace Transform of $X(t)$:

$$
\mathcal{L}\{X(t)\} = \int_{-\infty}^{\infty} e^{2t} u(-t) e^{-st} dt
$$

Since $u(-t)$ restricts the integral to $t \leq 0$:

$$
\mathcal{L}\{X(t)\} = \int_{-\infty}^{0} e^{2t} e^{-st} dt = \int_{-\infty}^{0} e^{(2 - s)t} dt
$$

Evaluating:

$$
\mathcal{L}\{X(t)\} = \left[\frac{e^{(2 - s)t}}{2 - s} \right]_{t=-\infty}^{t=0} = \frac{1}{2 - s} \left(e^{0} - 0 \right) = \frac{1}{2 - s}
$$

(assuming $\operatorname{Re}(s) > 2$ for convergence).

- Laplace Transform of $Y(t)$:

Similarly,

$$
\mathcal{L}\{Y(t)\} = \int_{-\infty}^{0} e^{t} e^{-st} dt = \int_{-\infty}^{0} e^{(1 - s)t} dt = \left[\frac{e^{(1 - s)t}}{1 - s} \right]_{-\infty}^{0} = \frac{1}{1 - s}
$$

(assuming $\operatorname{Re}(s) > 1$).

- Relation in Laplace domain:

Since the system is LTI,

$$
\mathcal{L}\{Y(t)\} = H(s) \times \mathcal{L}\{X(t)\}
$$

$$
\Rightarrow H(s) = \frac{\mathcal{L}\{Y(t)\}}{\mathcal{L}\{X(t)\}} = \frac{\frac{1}{1 - s}}{\frac{1}{2 - s}} = \frac{2 - s}{1 - s}
$$

Finding the System's Impulse Response $h(t)$

1. Inverse Laplace Transform of $H(s)$

Given:

$$
H(s) = \frac{2 - s}{1 - s}
$$

Rewrite:

$$
H(s) = \frac{-(s - 2)}{-(s - 1)} = \frac{s - 2}{s - 1}
$$

Alternatively,

$$
H(s) = 1 + \frac{-1}{s - 1}
$$

since:

$$
\frac{s - 2}{s - 1} = 1 - \frac{1}{s - 1}
$$

Inverse Laplace Transform:

$$
h(t) = \delta(t) - e^{t} u(t)
$$

where $\delta(t)$ is the Dirac delta function, and $e^{t} u(t)$ is the exponential response for $t \geq 0$.

Interpretation:

The impulse response consists of an instantaneous impulse at $t=0$, plus a decaying exponential for $t \geq 0$.

Visualizing the Signals and System Response

1. Graphs of Input $X(t)$ and Output $Y(t)$

- Input $X(t)$:

- Zero for $t > 0$.
- Exponentially decays as $t \to -\infty$.
- Plot: exponential decay for $t \leq 0$, zero elsewhere.

- Output $Y(t)$:

- Zero for $t > 0$.
- Exponentially decays as $t \to -\infty$.
- Plot: similar exponential decay for $t \leq 0$.

- Remarks:

- Both signals are non-zero only for $t \leq 0$.
- The output is a scaled version of the input, with different exponential rates.

2. Graph of the Impulse Response $h(t) = \delta(t) - e^{t} u(t)$

- Impulse at $t=0$:
Represents an instantaneous response.

- Exponential decay $e^{t} u(t)$:
Starts at $t=0$, decaying for $t > 0$.

- Interpretation:
The system's response to an impulse involves an immediate

Frequently Asked Questions

What is the main goal when analyzing the output Y(t) of a system given the input X(t) = e^{2t} u(-t)?

The main goal is to determine the system's response to the input, verify the given output Y(t) = e^{t} u(-t), and understand the system's behavior by analyzing the relationship between input and output.

How do you verify if Y(t) = e^{t} u(-t) is the correct output for the given input X(t)?

You verify by applying the system's properties—such as linearity and time invariance—and using convolution or Laplace transform methods to check if the proposed output satisfies the system's response to X(t).

What is the significance of the unit step function u(-t) in the input and output signals?

The unit step function u(-t) indicates that the signals are active for $t \leq 0$, essentially representing signals that are non-zero in the negative time domain, which affects how the system responds and how the signals are plotted.

How can the convolution integral be used to find the output Y(t)?

The convolution integral is used by integrating the product of the input signal X(τ) and the system's impulse response h(t - τ) over all τ, which helps determine the output Y(t) in the time domain.

What steps are involved in plotting the output Y(t) based on the given functions?

To plot Y(t), identify the region where Y(t) is non-zero ($t \leq 0$), plot e^{t} in that region, and ensure the step function u(-t) is incorporated, resulting in a graph that shows exponential decay for negative t.

What are common methods to analyze such signals and system responses in signal processing?

Common methods include Laplace and Fourier transforms, convolution integrals, and system response analysis techniques like impulse response and frequency response, which help in understanding and visualizing the output.

Why is it important to verify the given output Y(t) with the system's input X(t)?

Verifying ensures that the proposed output is consistent with the system's characteristics and the input signal, confirming the correctness of the response and understanding the system's behavior.

Additional Resources

The Output Of A System In Response To An Input $X(t) = E^{2t} u(-t)$ Is $Y(t) = E^{t} U(-t)$. Find And Draw The

Introduction

In the realm of signals and systems, understanding how a system responds to various inputs is fundamental. Whether in electrical engineering, control systems, or signal processing, predicting the output for a given input allows engineers to design, analyze, and optimize systems efficiently. Today, we delve into a specific problem: given an input signal $x(t) = e^{2t} u(-t)$ and an observed output $y(t) = e^{t} u(-t)$, our goal is to determine the system's characteristics, particularly its impulse response $h(t)$, and then visualize the response. This exploration will involve a detailed analysis of the signals involved, the convolution process, and the graphical interpretation, all explained in a clear and accessible manner.

Understanding the Given Signals and System Response

Before jumping into calculations, it's crucial to understand the signals involved and what they represent.

The Input Signal $x(t) = e^{2t} u(-t)$

This function combines an exponential component e^{2t} with the unit step function $u(-t)$. The unit step function $u(-t)$ is defined as:

$$
u(-t) = \begin{cases}
1, & t \leq 0 \\
0, & t > 0
\end{cases}
$$

Therefore, $x(t)$ is non-zero only for $t \leq 0$, where it takes the form e^{2t}. Graphically, this is a decaying exponential as $t \to -\infty$ approaching zero, with the maximum value at $t=0$, where $x(0) = 1$.

Visual Insight: The input signal exists only in the negative time domain, gradually decreasing as $t \to -\infty$.

The Output Signal $y(t) = e^{t} u(-t)$

Similarly, the output is given as $y(t) = e^{t} u(-t)$. This signal is also non-zero only for $t \leq 0$, with the exponential e^{t} that decays as $t \to -\infty$ and reaches 1 at $t=0$.

Key Observation: Both input and output signals are causal in the reverse time domain, existing only for $t \leq 0$. This suggests the system may be time-invariant and possibly linear, with the response being a scaled or transformed version of the input.

System Characterization: Finding the Impulse Response $h(t)$

In linear time-invariant (LTI) systems, the output $y(t)$ is the convolution of the input $x(t)$ with the system's impulse response $h(t)$:

$$
y(t) = (x h)(t) = \int_{-\infty}^{\infty} x(\tau) h(t - \tau) d \tau
$$

Our goal is to find $h(t)$. Once $h(t)$ is known, the response to any input can be predicted.

Approach:

- Use the convolution integral.
- Recognize the nature of the signals.
- Exploit the Laplace transform as a powerful tool to simplify convolution to multiplication.

Applying the Laplace Transform

The Laplace transform of a signal $f(t)$ is:

$$
F(s) = \mathcal{L}\{f(t)\} = \int_{0}^{\infty} f(t) e^{-st} dt
$$

However, since our signals are non-zero only for $t \leq 0$, it’s more convenient to consider the bilateral (two-sided) Laplace transform:

$$
F(s) = \int_{-\infty}^{\infty} f(t) e^{-st} dt
$$

Given the signals' support in $t \leq 0$, the bilateral transform applies well.

Transforming the Input and Output Signals

- For $x(t) = e^{2t} u(-t)$:

$$
X(s) = \int_{-\infty}^{0} e^{2t} e^{-st} dt = \int_{-\infty}^{0} e^{(2 - s)t} dt
$$

- For $y(t) = e^{t} u(-t)$:

$$
Y(s) = \int_{-\infty}^{0} e^{t} e^{-st} dt = \int_{-\infty}^{0} e^{(1 - s)t} dt
$$

Calculating these integrals:

$$
X(s) = \left[\frac{e^{(2 - s)t}}{2 - s} \right]_{t = -\infty}^{t=0} = \frac{1}{2 - s} \left(1 - 0 \right) = \frac{1}{2 - s}
$$

(assuming $\operatorname{Re}(s) > 2$ for convergence)

Similarly,

$$
Y(s) = \left[\frac{e^{(1 - s)t}}{1 - s} \right]_{t = -\infty}^{t=0} = \frac{1}{1 - s}
$$

(assuming $\operatorname{Re}(s) > 1$)

Deriving the Impulse Response $H(s)$

Since $y(t) = x(t) h(t)$, in the Laplace domain:

$$
Y(s) = X(s) H(s)
$$

Thus,

$$
H(s) = \frac{Y(s)}{X(s)} = \frac{\frac{1}{1 - s}}{\frac{1}{2 - s}} = \frac{2 - s}{1 - s}
$$

Simplify:

$$
H(s) = \frac{2 - s}{1 - s}
$$

Express as:

$$
H(s) = \frac{(1 - s) + 1}{1 - s} = 1 + \frac{1}{1 - s}
$$

Inverse Laplace Transform to Find $h(t)$

Recall that:

- $\mathcal{L}^{-1}\{1\} = \delta(t)$, the Dirac delta function.
- $\mathcal{L}^{-1}\left\{\frac{1}{1 - s}\right\} = e^{t} u(t)$, for $t \geq 0$.

Since our signals are supported in $t \leq 0$, and the inverse Laplace transform of the second term is $e^{t} u(t)$, which is non-zero only for $t \geq 0$, but in our case, the system is causal with support in $t \leq 0$, the impulse response $h(t)$ must be zero for $t > 0$.

Therefore, the impulse response is:

$$
h(t) = \delta(t) + e^{t} u(t)
$$

But because the system acts in the negative time domain, and the signals are supported on $t \leq 0$, the physically meaningful impulse response in this context is:

$$
h(t) = \delta(t) + e^{t} u(t)
$$

This confirms that the system's response comprises an instantaneous component (the delta function) and an exponential response starting at $t=0$.

Graphical Representation and Interpretation

Understanding the mathematical derivation is vital, but visualizing the signals provides an intuitive grasp.

The Impulse Response $h(t)$

- The delta function $\delta(t)$ corresponds to an instantaneous impulse at $t=0$.
- The exponential $e^{t} u(t)$ starts at $t=0$, rising exponentially for $t \geq 0$.
Since the original signals are supported on $t \leq 0$, this indicates the system's response has a 'mirror' or causal extension in the positive domain, which aligns with the mathematical form.

Graph:

- At $t=0$, a spike representing $\delta(t)$.
- For $t > 0$, the exponential curve starting at 1 and increasing exponentially.

The Input and Output Signals

- Both $x(t)$ and $y(t)$ are non-zero only for $t \leq 0$.
- $x(t)$ starts from 0 at $t \to -\infty$, rising to 1 at $t=0$.
- $y(t)$ also peaks at 1 at $t=0$ and decays for $t<0$.

Graphical Interpretation:

- In the negative time domain, the input signal dec

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