The Points (4, 3) And (3, Y) Lie On The Same Line. If The Slope Of The Line Is 2, What Is The Value Of

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Introduction

Understanding the relationship between points on a line is fundamental in coordinate geometry. When two points lie on the same straight line, the slope of that line remains constant. Given the points (4, 3) and (3, Y), and knowing that the slope of the line passing through these points is 2, our goal is to find the value of Y. This problem not only reinforces the concept of slope but also demonstrates how algebraic techniques are applied to geometric problems.

In this article, we will explore the concept of slope, derive the necessary formula, and apply it step-by-step to determine the unknown coordinate Y. We will also discuss related concepts and problem-solving strategies to deepen understanding.

Understanding the Concept of Slope

Definition of Slope

The slope of a line measures its steepness or inclination. It is typically denoted by the letter m and is calculated as the ratio of the change in the y-coordinate to the change in the x-coordinate between two points on the line:

$$
m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}
$$

where:
- (x_1, y_1) and (x_2, y_2) are two points on the line.

Significance of the Slope

- A positive slope indicates the line rises from left to right.
- A negative slope indicates the line falls from left to right.
- A zero slope indicates a horizontal line.
- An undefined slope (vertical line) occurs when the change in x is zero.

In our problem, the slope is given as 2, which means the line rises two units vertically for every one unit horizontally.

Applying the Slope Formula to the Given Points

Given Data

- First point: $(x_1, y_1) = (4, 3)$
- Second point: $(x_2, y_2) = (3, Y)$
- Slope: $m = 2$

Setting Up the Equation

Using the slope formula:

$$
m = \frac{y_2 - y_1}{x_2 - x_1}
$$

Substituting the known values:

$$
2 = \frac{Y - 3}{3 - 4}
$$

Note that $x_2 = 3$, $x_1 = 4$, $y_2 = Y$, and $y_1 = 3$.

Solving for Y

Step-by-Step Solution

1. Simplify the denominator:

$$
3 - 4 = -1
$$

2. Rewrite the equation:

$$
2 = \frac{Y - 3}{-1}
$$

3. Multiply both sides by -1 to isolate $Y - 3$:

$$
2 \times (-1) = Y - 3
$$

$$
-2 = Y - 3
$$

4. Add 3 to both sides to solve for Y:

$$
-2 + 3 = Y
$$

$$
Y = 1
$$

Result: The value of Y is 1.

Verification of the Solution

To ensure the correctness of our result, we can verify by calculating the slope between the points $(4, 3)$ and $(3, 1)$:

$$
m = \frac{1 - 3}{3 - 4} = \frac{-2}{-1} = 2
$$

Since the calculated slope matches the given slope, our solution is verified.

Additional Insights and Related Concepts

Alternative Method: Using the Slope-Intercept Form

The slope-intercept form of a line is:

$$
y = mx + c
$$

where:
- m is the slope,
- c is the y-intercept.

Given a point (x_1, y_1) and slope m, we can find c:

$$
c = y_1 - m x_1
$$

Applying this to point $(4, 3)$:

$$
c = 3 - 2 \times 4 = 3 - 8 = -5
$$

The equation of the line becomes:

$$
y = 2x - 5
$$

To find Y when $x = 3$:

$$
Y = 2 \times 3 - 5 = 6 - 5 = 1
$$

which confirms our previous calculation.

Implications of the Result

- The point $(3, 1)$ lies on the line passing through $(4, 3)$ with a slope of 2.

- The line extends infinitely in both directions, containing all points satisfying the equation $y = 2x - 5$.

Practical Applications

Understanding how to find values of unknown points on a line with a known slope is essential in various fields such as:

- Physics (motion analysis)
- Engineering (structural design)
- Computer graphics (line rendering)
- Economics (trend analysis)

Common Mistakes and Tips

- **Incorrectly swapping points:** Remember to keep track of which point is which when applying the slope formula.
- **Dividing by zero:** The slope formula is undefined when $x_2 = x_1$; this indicates a vertical line.
- **Sign errors:** Pay attention to the signs when subtracting coordinates, especially with negative values.

Tip: Always verify your solutions by substituting back into the slope formula to ensure the calculation aligns with the given slope.

Conclusion

In summary, given two points $(4, 3)$ and $(3, Y)$ on a line with a slope of 2, the value of Y can be found through the slope formula. By substituting the known values and solving algebraically, we determined that $Y = 1$. This problem highlights the importance of understanding the fundamental concepts of slopes and their applications in coordinate geometry. Mastering these techniques enables solving more complex problems involving lines, slopes, and equations, which are foundational skills in mathematics and its numerous applications.

Final note: Always verify your answers and understand the geometric significance behind algebraic calculations to build a robust understanding of coordinate geometry.

Frequently Asked Questions

Given two points (4, 3) and (3, y) lie on a line with a slope of 2, how do you find the value of y?

Use the slope formula: (y2 - y1) / (x2 - x1) = slope. Plugging in the points: (y - 3) / (3 - 4) = 2. Simplify: (y - 3) / (-1) = 2. Multiply both sides by -1: y - 3 = -2. Add 3 to both sides: y = 1.

What is the slope formula used to determine if two points lie on the same line?

The slope formula is (y2 - y1) / (x2 - x1). If the slope remains constant between multiple points, they lie on the same line.

If the points (4, 3) and (3, y) are on the same line with a slope of 2, what does the value of y tell us about the line's orientation?

Since y is found to be 1, it indicates the line passes through these points with a positive slope of 2, ascending from left to right.

Can the points (4, 3) and (3, y) lie on a line with a different slope? Why or why not?

No, they cannot lie on a line with a different slope if it's given that the slope is 2. The slope between these points must be 2 for the line to have that slope.

How does changing the value of y affect the position of the point (3, y) on the line with slope 2?

Changing y shifts the point vertically along the line. For the line with slope 2 passing through (4, 3), y must be 1, so any other y would mean the point does not lie on that specific line.

What is the importance of the slope value in determining the relationship between two points?

The slope indicates how steep the line is and helps determine whether two points lie on the same straight line. A consistent slope between points confirms they are colinear.

If the slope was different, say 3, how would you find the new y value for the point (3, y)?

Using the same approach: (y - 3) / (3 - 4) = 3. Simplify: (y - 3) / (-1) = 3, so y - 3 = -3, thus y = 0.

What are the key steps to determine the unknown coordinate y when two points and the slope are given?

First, write the slope formula for the two points, substitute known values, solve for y algebraically, and verify the result makes sense within the context of the line's slope.

Additional Resources

Understanding the Problem: Points, Slope, and Line Equation

Mathematics often presents us with geometric problems that seem straightforward at first glance but reveal layers of complexity upon closer inspection. The problem at hand involves two points, (4, 3) and (3, Y), which are given to lie on the same straight line with a specified slope of 2. The task is to determine the value of Y. To fully appreciate and solve this problem, we need to explore the fundamental concepts involved, such as the equation of a line, the meaning of slope, and how points relate to each other on a line.

Introduction to the Core Concepts

What Is a Point in the Coordinate Plane?

A point in the coordinate plane is defined by an ordered pair (x, y), where:

- x-coordinate: Horizontal position, representing how far left or right the point is from the origin.
- y-coordinate: Vertical position, indicating how far above or below the origin the point is located.

In this problem, the points are:

- Point 1: (4, 3)
- Point 2: (3, Y), where Y is unknown.

Understanding the Slope of a Line

The slope of a line, often denoted as m, measures the steepness or incline of the line. It is calculated as:

$$ m = \frac{\text{change in y}}{\text{change in x}} = \frac{y_2 - y_1}{x_2 - x_1} $$

- A positive slope indicates the line rises as it moves from left to right.
- A negative slope indicates the line falls.
- A slope of zero indicates a horizontal line.
- An undefined slope (division by zero) indicates a vertical line.

In this problem, the slope is given as 2, which means the line rises two units vertically for every one unit it moves horizontally.

The Equation of a Line with a Given Slope and Point

The slope-intercept form of a line's equation is:

$$ y = mx + b $$

where:

- m is the slope.
- b is the y-intercept, the point where the line crosses the y-axis.

Alternatively, the point-slope form is useful when a point on the line is known:

$$ y - y_1 = m(x - x_1) $$

This form allows us to find the line's equation directly using the slope and a known point.

Step-by-Step Breakdown of the Problem

1. Confirming the Points Lie on the Same Line

Since both points are on the same straight line with a slope of 2, the slope calculated between these two points must be 2. This condition provides a key equation to find the unknown Y.

2. Applying the Slope Formula

Given points:

- Point 1: $(x_1, y_1) = (4, 3)$
- Point 2: $(x_2, y_2) = (3, Y)$

Using the slope formula:

$$ m = \frac{Y - 3}{3 - 4} $$

Note that:

- $x_2 = 3$, $x_1 = 4$
- $y_2 = Y$, $y_1 = 3$

Since the points lie on the same line with slope 2:

$$ 2 = \frac{Y - 3}{3 - 4} $$

Calculating the Unknown Y Value

Step 1: Simplify the Denominator

Calculate the difference in x-coordinates:

$$ 3 - 4 = -1 $$

Step 2: Set Up the Equation

Substitute into the slope equation:

$$ 2 = \frac{Y - 3}{-1} $$

Step 3: Solve for Y

Multiply both sides by -1:

$$ 2 \times (-1) = Y - 3 $$

$$ -2 = Y - 3 $$

Add 3 to both sides:

$$ -2 + 3 = Y $$

$$ 1 = Y $$

Therefore, the value of Y is 1.

Verifying the Solution

To ensure correctness, it's prudent to verify the result by calculating the slope between the points again with $Y=1$:

$$ m = \frac{1 - 3}{3 - 4} = \frac{-2}{-1} = 2 $$

Since the calculated slope matches the given slope, the solution is validated.

Deeper Insights and Additional Considerations

Implications of the Points on the Line

- The points are (4, 3) and (3, 1).
- The line passing through these points has a slope of 2, which indicates a consistent rate of change.
- The proximity of the two points suggests a line with a predictable pattern, and the calculated Y confirms this.

Alternative Approaches for the Same Problem

While the direct use of the slope formula is straightforward here, alternative methods include:

- Using the point-slope form:

$$
y - y_1 = m(x - x_1)
$$

Substituting $(x_1, y_1) = (4, 3)$ and $m=2$:

$$
y - 3 = 2(x - 4)
$$

$$
y - 3 = 2x - 8
$$

$$
y = 2x - 8 + 3 = 2x - 5
$$

Now, plug in $x=3$:

$$
y = 2(3) - 5 = 6 - 5 = 1
$$

Confirming that $Y=1$.

- Graphical verification: Plotting the points on a graph with the line $y=2x - 5$ confirms both points lie on the same line.

Understanding the Line Equation $y = 2x - 5$

- The line has a slope of 2, consistent with the problem statement.
- The y-intercept is -5, indicating it crosses the y-axis at -5.

This equation is useful for visualizing the entire line, not just the two points.

Real-World Applications and Context

Understanding how to determine a missing coordinate based on slope and a known point has numerous practical applications:

- Engineering and Construction: Calculating the height or position of structures based on slope measurements.
- Navigation and Mapping: Determining positions when only partial data is available.
- Economics: Analyzing linear relationships between variables like supply and demand.
- Physics: Describing motion with constant acceleration or velocity, where slope might represent rate of change.

Common Mistakes and Misconceptions

When working through such problems, it's essential to be cautious of:

- Incorrect ordering of points: Remember that the order of points impacts the numerator and denominator in the slope calculation.
- Division by zero: If the x-coordinates are the same, the slope is undefined, corresponding to a

vertical line.
- Misreading the slope: Ensure that the slope value is correctly applied; a common mistake is to confuse the numerator and denominator.
- Assuming the points are on a line without verification: Always verify by substituting back into the slope formula or line equation.

Conclusion: Final Answer and Summary

The problem of determining the value of Y when points $(4, 3)$ and $(3, Y)$ lie on a line with a slope of 2 is a classic application of the slope formula and the equation of a line. By applying the fundamental formula:

$$
Y = 1
$$

we confirm that the missing coordinate is Y = 1. This solution is consistent with the proportional change dictated by the slope and aligns perfectly with the slope-intercept form of the line $y = 2x - 5$.

In essence, solving such problems reinforces the importance of understanding the relationship between points, slopes, and line equations in coordinate geometry. Mastery of these concepts enables us to analyze and interpret real-world situations with confidence and precision.

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