The Position Of An Object At Any Time T Is Given Bys(t)=3t440t3+126t29.1 Determine The Velocity Of The

Understanding the Position Function: Analyzing the Object’s Motion

The Position Of An Object At Any Time T Is Given By s(t) = 3t^4 - 40t^3 + 126t^2 + 9.1. Determine The Velocity Of The – this statement sets the foundation for understanding how the position of an object changes over time and how to derive its velocity. In physics and calculus, the position function s(t) describes where an object is at any given moment, and the velocity function v(t) indicates how fast and in which direction the object is moving at that moment.

This article aims to explore the process of deriving the velocity function from the position function, analyze the behavior of the object over time, and understand the significance of velocity in motion studies. Whether you're a student, a physics enthusiast, or someone interested in the mathematical modeling of motion, this comprehensive guide will help clarify these concepts.

The Position Function s(t): Breaking It Down

Given Position Function

The position function provided is:

$s(t) = 3t^4 - 40t^3 + 126t^2 + 9.1$

where:

- t represents time (usually in seconds),
- s(t) indicates the position of the object at time t (could be in meters, feet, etc., depending on the context).

This polynomial function suggests a complex motion pattern, with different terms contributing to acceleration, deceleration, and possibly changes in direction.

Understanding Each Term

- $3t^4$: Dominates at large t, indicating the position increases rapidly over time.
- $-40t^3$: Introduces a decreasing component, possibly causing the object to slow down or change direction.
- $126t^2$: Adds curvature to the motion, influencing acceleration.
- 9.1: Constant term, representing the initial position when t=0.

By analyzing these terms, we can interpret the overall motion pattern, but to understand the speed and direction at any moment, we need to compute the derivative of s(t).

Calculating the Velocity Function v(t)

What Is Velocity?

Velocity is the rate of change of position with respect to time:

$v(t) = ds(t)/dt$

Calculating v(t) involves differentiating the position function term-by-term using basic rules of calculus.

Derivative of the Position Function

Applying the power rule to each term:

- Derivative of $3t^4$: 4 3 $t^{(4-1)}$ = $12t^3$
- Derivative of $-40t^3$: 3 -40 $t^{(3-1)}$ = $-120t^2$
- Derivative of $126t^2$: 2 126 $t^{(2-1)}$ = 252t
- Derivative of 9.1 (constant): 0

Thus, the velocity function is:

$v(t) = 12t^3 - 120t^2 + 252t$

Analyzing the Velocity Function

Expression for Velocity

$v(t) = 12t^3 - 120t^2 + 252t$

This cubic function describes how the velocity changes over time.

Factoring the Velocity Function

To understand the behavior better, factor $v(t)$:

$v(t) = 12t(t^2 - 10t + 21)$

Factor the quadratic:

$t^2 - 10t + 21$

Find roots:

$t = [10 \pm \sqrt{(100 - 84)}] / 2 = [10 \pm \sqrt{16}] / 2$

$t = [10 \pm 4] / 2$

So:

- $t = (10 + 4) / 2 = 14 / 2 = 7$
- $t = (10 - 4) / 2 = 6 / 2 = 3$

Therefore, the fully factored form:

$v(t) = 12t(t - 3)(t - 7)$

Key points:

- The velocity is zero at $t = 0$, $t = 3$, and $t = 7$.
- The sign of $v(t)$ in different intervals indicates whether the object is moving forward or backward.

Interpreting the Velocity and Motion Behavior

Direction of Motion

- For $t < 0$, assuming $t \geq 0$ in real-world scenarios, the velocity is positive or negative depending on the sign of $v(t)$.
- For $0 < t < 3$: $v(t) > 0$ (since all factors are positive, i.e., $t > 0$, $t - 3$

$< 0, t - 7 < 0)$

Wait, let's check the sign:

At $t = 1$:

$$v(1) = 12\ 1\ (1 - 3)\ (1 - 7) = 12\ 1\ (-2)\ (-6) = 12\ 1\ 12 = 144 > 0$$

So, $v(t) > 0$ between $t=0$ and $t=3$.

- For $3 < t < 7$: $v(t) < 0$

At $t=4$:

$$v(4) = 12\ 4\ (4 - 3)\ (4 - 7) = 12\ 4\ 1\ (-3) = -144 < 0$$

- For $t > 7$: $v(t) > 0$

At $t=8$:

$$v(8) = 12\ 8\ (8 - 3)\ (8 - 7) = 12\ 8\ 5\ 1 = 480 > 0$$

Summary:

- The object moves forward (positive velocity) during $0 < t < 3$ and $t > 7$.
- The object moves backward (negative velocity) during $3 < t < 7$.

Critical Points and Turning Points

- The objects change direction at times $t=3$ and $t=7$, where $v(t)=0$.
- Understanding these points helps in analyzing the motion pattern, such as when the object reaches maximum or minimum positions.

Further Analysis: Acceleration and Motion Dynamics

Calculating Acceleration

Acceleration is the derivative of velocity:

$$a(t) = dv(t)/dt$$

Differentiate $v(t)$:

$$v(t) = 12t^3 - 120t^2 + 252t$$

Derivative:

$$a(t) = 36t^2 - 240t + 252$$

Interpreting Acceleration

- The sign of $a(t)$ indicates whether the object is speeding up or slowing down.
- Points where $a(t) = 0$ are potential inflection points in the velocity graph.

Solve for $a(t)=0$:

$$36t^2 - 240t + 252 = 0$$

Divide through by 12:

$$3t^2 - 20t + 21 = 0$$

Discriminant:

$$D = (-20)^2 - 4\ 3\ 21 = 400 - 252 = 148$$

$$t = [20 \pm \sqrt{148}] / (23) = [20 \pm 12.165] / 6$$

Calculate:

$$t \approx (20 + 12.165)/6 \approx 32.165/6 \approx 5.36$$

$$t \approx (20 - 12.165)/6 \approx 7.835/6 \approx 1.31$$

Interpretation:

- The acceleration is zero at approximately $t \approx 1.31$ and $t \approx 5.36$.
- These points can indicate transitions in the rate of acceleration, affecting the motion's nature.

Practical Applications of Velocity Analysis

In Physics and Engineering

- Determining the maximum speed or minimum position.
- Predicting when the object changes direction.

- Calculating the time intervals of forward and backward motion.
- Designing systems where precise control of motion is required.

In Real-World Scenarios

- Automotive acceleration profiles.
- Robotics movement planning.
- Projectile motion analysis.
- Animation and simulation in computer graphics.

Summary and Key Takeaways

- The position function s(t) provides the location of the object at any time t.
- The velocity function v(t) = ds(t)/dt = 12t^3 - 120t^2 + 252t describes the speed and direction.
- Factoring v(t) reveals critical points at t=0, t=3, and t=7, indicating where the object changes direction.
- The acceleration a(t) helps understand how quickly the velocity changes over time.
- Analyzing these functions allows for comprehensive motion analysis and prediction.

Conclusion

Understanding the relationship between position, velocity, and acceleration is fundamental in kinematics. Starting from a polynomial position function, deriving the velocity function through differentiation offers insights into the object's motion patterns. Recognizing points where velocity is zero helps identify moments when the object changes direction or reaches maximum/minimum positions. The combined analysis of position, velocity, and acceleration functions equips engineers, physicists, and students with the

Frequently Asked Questions

What is the position function of the object at any time t?

The position function is given by s(t) = 3t^4 - 40t^3 + 126t^2 + 9.1.

How do you find the velocity of the object at any time t?

The velocity is the first derivative of the position function with respect to time t, so $v(t) = ds/dt$.

What is the expression for the velocity of the object based on the given position function?

Differentiating $s(t) = 3t^4 - 40t^3 + 126t^2 + 9.1$ gives $v(t) = 12t^3 - 120t^2 + 252t$.

At what time t does the object have zero velocity?

Zero velocity occurs when $v(t) = 0$, so solve $12t^3 - 120t^2 + 252t = 0$.

How can you determine when the object is moving forward or backward?

By analyzing the sign of $v(t)$: if $v(t) > 0$, the object moves forward; if $v(t) < 0$, it moves backward.

What is the significance of critical points in the velocity function?

Critical points occur where $v(t) = 0$ or the derivative of $v(t)$ is zero, indicating potential changes in the object's motion.

How do you find the maximum or minimum velocity of the object?

Find the critical points of $v(t)$ by setting its derivative to zero, then evaluate $v(t)$ at those points.

Can the velocity function help determine when the object changes direction?

Yes, when $v(t)$ changes sign, it indicates the object changes direction at those times.

What is the importance of understanding the velocity at any time t?

It helps analyze the object's motion, including speed, direction, and points of motion reversal.

How would you compute the acceleration of the object from the given position function?

Acceleration is the derivative of velocity, so a(t) = d^2s/dt^2, which is obtained by differentiating v(t).

Additional Resources

The Position Of An Object At Any Time T Is Given By: An In-Depth Investigation Into Velocity Determination

---

Introduction

Understanding the motion of objects is fundamental in physics and engineering, underpinning everything from classical mechanics to modern technological applications. When analyzing an object's movement, its position as a function of time provides critical insight into its behavior and dynamics. In particular, determining the velocity at any given time is essential for predicting future positions, understanding forces at play, and designing systems that rely on precise motion control.

In this article, we examine a specific position function:

$$
s(t) = 3t^4 - 40t^3 + 126t^2 + 9.1
$$

which describes the position of an object in space as a function of time t. Our goal is to thoroughly analyze how to determine the velocity of this object at any time t, explore the behavior of its velocity over time, and interpret the physical significance of these findings.

Understanding the Position Function

Before delving into the calculus, it is crucial to understand the structure and components of the position function:

$$
s(t) = 3t^4 - 40t^3 + 126t^2 + 9.1
$$

This polynomial function is a fourth-degree polynomial, indicating that the object’s position varies non-linearly with time. Each term contributes differently to the overall motion:

- $3t^4$: Dominant at large t, representing rapid increases or decreases depending on the sign.
- $-40t^3$: Influences the curvature, especially at intermediate t.
- $126t^2$: Adds a quadratic component, affecting acceleration.
- Constant 9.1: The initial position or baseline when $t=0$.

Calculating Velocity: The Fundamental Approach

In kinematics, velocity $v(t)$ is the rate of change of position with respect to time:

$$
v(t) = \frac{ds(t)}{dt}
$$

Given $s(t)$, calculating $v(t)$ involves differentiating the polynomial term-by-term:

$$
v(t) = \frac{d}{dt} \left(3t^4 - 40t^3 + 126t^2 + 9.1 \right)
$$

Applying basic differentiation rules:

- Power rule: $\frac{d}{dt} [a t^n] = a n t^{n-1}$
- Derivative of a constant: 0

Thus:

$$
v(t) = 4 \times 3 t^{3} - 3 \times 40 t^{2} + 2 \times 126 t + 0
$$

Simplifying:

$$
v(t) = 12 t^3 - 120 t^2 + 252 t
$$

This velocity function is a cubic polynomial that describes how the velocity evolves over time.

Analyzing the Velocity Function

Understanding $v(t)$ helps predict motion characteristics such as:

- When the object is speeding up or slowing down.

- When the object changes direction.
- When the velocity reaches maximum or minimum values.

Critical Points and Sign Changes

To find when the object changes direction, examine points where $v(t) = 0$, i.e., where the velocity is zero:

$$
12 t^3 - 120 t^2 + 252 t = 0
$$

Factoring:

$$
t (12 t^2 - 120 t + 252) = 0
$$

So, either:

$$
t = 0
$$

or

$$
12 t^2 - 120 t + 252 = 0
$$

Dividing the quadratic by 12 to simplify:

$$
t^2 - 10 t + 21 = 0
$$

Applying quadratic formula:

$$
t = \frac{10 \pm \sqrt{(-10)^2 - 4 \times 1 \times 21}}{2}
$$

$$
t = \frac{10 \pm \sqrt{100 - 84}}{2} = \frac{10 \pm \sqrt{16}}{2}
$$

$$
t = \frac{10 \pm 4}{2}
$$

This yields:

- $t = \frac{10 + 4}{2} = 7$
- $t = \frac{10 - 4}{2} = 3$

Summary of critical points:

$$
t = 0, \quad t = 3, \quad t = 7
$$

These points partition the timeline into four intervals:

- $t < 0$
- $0 < t < 3$
- $3 < t < 7$
- $t > 7$

Examining the sign of $v(t)$ within these intervals reveals the motion direction.

Sign Analysis of Velocity

- For $t < 0$:

Choose $t = -1$:

$$
v(-1) = 12(-1)^3 - 120(-1)^2 + 252(-1) = -12 - 120 - 252 = -384 < 0
$$

Object moves backward (negative velocity).

- For $0 < t < 3$:

Choose $t=1$:

$$
v(1) = 12 - 120 + 252 = 144 > 0
$$

Object moves forward.

- For $3 < t < 7$:

Choose $t=5$:

$$
v(5) = 12(125) - 120(25) + 252(5) = 1500 - 3000 + 1260 = -240 < 0

Object moves backward again.

- For $t > 7$:

Choose $t=8$:

$$
v(8) = 12(512) - 120(64) + 252(8) = 6144 - 7680 + 2016 = 480 < 0
$$

Object continues backward.

Physical Interpretation

The object:

- Moves backward when $t < 0$.
- Moves forward between 0 and 3.
- Moves backward again between 3 and 7.
- Continues backward for $t > 7$.

This oscillatory pattern indicates complex motion, likely involving acceleration and deceleration phases, possibly due to external forces or initial conditions.

Velocity at Specific Times

Calculating the actual velocity at key points:

- At $t=0$:

$$
v(0) = 0
$$

- At $t=3$:

$$
v(3) = 12(27) - 120(9) + 252(3) = 324 - 1080 + 756 = 0
$$

- At $t=7$:

$$
v(7) = 12(343) - 120(49) + 252(7) = 4116 - 5880 + 1764 = 0
$$

\]

Thus, at $t=0, 3, 7$, the velocity is zero, indicating moments when the object changes direction.

Acceleration and Further Insights

To fully understand the motion, examining acceleration $a(t)$:

$$
a(t) = \frac{d v(t)}{dt}
$$

From earlier:

$$
v(t) = 12 t^3 - 120 t^2 + 252 t
$$

Differentiating:

$$
a(t) = 36 t^2 - 240 t + 252
$$

Critical points for acceleration:

$$
a(t) = 0 \Rightarrow 36 t^2 - 240 t + 252 = 0
$$

Dividing by 12:

$$
3 t^2 - 20 t + 21 = 0
$$

Quadratic formula:

$$
t = \frac{20 \pm \sqrt{400 - 252}}{6} = \frac{20 \pm \sqrt{148}}{6}
$$

$$
t = \frac{20 \pm 12.165}{6}
$$

- $t \approx \frac{20 + 12.165}{6} \approx \frac{32.165}{6} \approx 5.36$
- $t \approx \frac{20 - 12.165}{6} \approx \frac{7.835}{6} \approx 1.31$

These points denote where acceleration changes sign, indicating shifts between acceleration and deceleration periods.

Summary of Motion Characteristics

Key Time t	Velocity $v(t)$	Motion Direction	Acceleration $a(t)$	Nature of Motion
0	0	Transition point	$a(0) = 252$	Rest, speeding up in positive direction

[The Position Of An Object At Any Time T Is Given Bys T 3t440t3 126t29 1 Determine The Velocity Of The]

The Position Of An Object At Any Time T Is Given Bys T 3t440t3 126t29 1 Determine The Velocity Of The

Related Articles

- The Compound Shape Below Is Formed From Rectangle ABDE And Right-angled Triangle BCD. What Is The Area
- The Type Of Interpersonal Influence Where A Client Perceives The Care That A Wellness Professional Has
- The History Of Forensic Psychology Can Be Traced Back To Late 19th-century Experiments Involving Which

Back to Home