The Quadratic Parent Function Has Been Stretchedby A Factor Of 3, Translated Up 6 And Right 4, Andthen

The Quadratic Parent Function Has Been Stretchedby A Factor Of 3, Translated Up 6 And Right 4, Andthen to understand the transformations applied to the basic quadratic function, we need to analyze each modification carefully. These transformations change the graph's size, position, and orientation, providing valuable insights into how quadratic functions behave under various transformations. This article will explore the nature of these transformations, their mathematical representations, and their effects on the graph of the quadratic parent function, $y = x^2$. We will also discuss real-world applications of these transformations and how to graph them accurately.

Understanding the Quadratic Parent Function

What Is the Quadratic Parent Function?

The quadratic parent function is the simplest quadratic function, represented by the equation:

$$y = x^2$$

This function creates a parabola that opens upward, with its vertex at the origin $(0, 0)$. Its graph is symmetric about the y-axis, and it serves as the basis for understanding more complex quadratic functions.

Key Features of $y = x^2$:

- Vertex at $(0, 0)$
- Axis of symmetry along the y-axis
- Opens upward
- Parabola shape
- The y-value grows quadratically as x moves away from zero

Transformations of the Quadratic Function

Transformations manipulate the graph of the basic quadratic function through various operations such as stretching, compressing, shifting, and reflecting. These changes can be combined to produce a wide array of parabola shapes and positions.

Types of Transformations:

- Vertical and horizontal stretches/compressions
- Translations (shifts) up/down and left/right
- Reflections across axes
- Rotations (less common for quadratic functions)

In this discussion, we'll focus on the specific transformations:

1. Stretching by a factor of 3
2. Translating upward by 6 units
3. Translating rightward by 4 units

Mathematical Representation of the Transformations

To understand how these transformations affect the quadratic function, let's analyze each step mathematically.

1. Vertical Stretch by a Factor of 3

A vertical stretch of the graph $y = x^2$ by a factor of 3 is represented as:
$$y = 3x^2$$
This transformation makes the parabola "steeper," increasing the y-values for each x.

2. Translation Upward by 6 Units

Moving the graph up by 6 units involves adding 6 to the entire function:
$$y = 3x^2 + 6$$
This shifts the parabola vertically upward, raising its vertex from $(0, 0)$ to $(0, 6)$.

3. Translation Rightward by 4 Units

Shifting the graph to the right by 4 units involves replacing x with $x - 4$:
$$y = 3(x - 4)^2 + 6$$
This moves the parabola horizontally to the right, positioning its vertex at $(4, 6)$.

Combined Transformation: The Final Equation

Combining all the transformations, the equation of the parabola becomes:
$$ y = 3(x - 4)^2 + 6 $$

This single equation encapsulates all the described transformations applied to the quadratic parent function.

Effects of Each Transformation on the Graph

Vertical Stretching

- Increases the parabola's steepness
- Causes y-values to grow faster as x moves away from the vertex
- The parabola becomes "narrower"

Vertical Translation Upward

- Raises the entire parabola vertically
- The vertex shifts from $(0, 0)$ to $(0, 6)$
- Maintains the shape but changes the position

Horizontal Translation Rightward

- Moves the vertex from $(0, 6)$ to $(4, 6)$
- The parabola’s axis of symmetry shifts to $x = 4$

Graphing the Transformed Quadratic Function

Step-by-Step Guide to Graph the Function $y = 3(x - 4)^2 + 6$:

1. Identify the vertex: $(4, 6)$
2. Determine the axis of symmetry: $x = 4$
3. Plot the vertex on the coordinate plane.
4. Choose x-values around the vertex (e.g., 3, 5, 2, 6) to find corresponding y-values.
5. Calculate y-values:
- For $x = 3$:
$$ y = 3(3 - 4)^2 + 6 = 3(-1)^2 + 6 = 3(1) + 6 = 9 $$
- For $x = 5$:

$$ y = 3(5 - 4)^2 + 6 = 3(1)^2 + 6 = 3(1) + 6 = 9 $$
- For $x = 2$:
$$ y = 3(2 - 4)^2 + 6 = 3(-2)^2 + 6 = 3(4) + 6 = 12 + 6 = 18 $$
- For $x = 6$:
$$ y = 3(6 - 4)^2 + 6 = 3(2)^2 + 6 = 3(4) + 6 = 12 + 6 = 18 $$
6. Plot these points and sketch the parabola.

Applications of Transformed Quadratic Functions

Quadratic functions with various transformations are used in numerous real-world scenarios:

1. Physics and Engineering

- Projectile motion analysis involves parabolas, with transformations representing different initial velocities and angles.

2. Economics

- Cost and profit models often involve quadratic functions, where shifts and stretches model different economic conditions.

3. Computer Graphics

- Parabolas are used to model trajectories, curves, and shading effects, requiring transformations for accurate rendering.

4. Architecture

- Parabolic arches are shaped through transformations, optimizing strength and aesthetic appeal.

Key Points to Remember

- The original quadratic function is $y = x^2$.
- Stretching by a factor of 3 makes the parabola narrower.
- Moving upward by 6 units shifts the vertex from $(0, 0)$ to $(0, 6)$.
- Moving rightward by 4 units shifts the vertex to $(4, 6)$.
- The combined transformation results in the function $y = 3(x - 4)^2 + 6$.

Summary

Transformations of quadratic functions are fundamental in understanding how graphs change under various manipulations. The specific transformations of stretching by a factor of 3, shifting upward by 6, and moving rightward by 4 produce a parabola with a steeper shape, a new vertex position, and altered symmetry. Recognizing these transformations allows students and professionals to analyze and graph quadratic functions accurately, essential for applications across mathematics, physics, engineering, and beyond. Mastery of this topic enhances problem-solving skills and deepens comprehension of the behavior of quadratic relationships.

Further Resources

- Online graphing calculators for visualizing transformations
- Interactive quadratic function tutorials
- Practice problems on quadratic transformations
- Educational videos explaining quadratic graphing techniques

Understanding the transformations of quadratic functions is a key skill in algebra and calculus. By mastering how stretching and shifting affect the parabola, you can better analyze complex functions and solve real-world problems effectively.

Frequently Asked Questions

What is the original quadratic parent function?

The original quadratic parent function is $y = x^2$.

How does stretching a quadratic function by a factor of 3 affect its graph?

Stretching the graph by a factor of 3 vertically makes it steeper, increasing the y-values by a factor of 3 for each x-value.

What is the effect of translating the quadratic function up 6 units?

Translating up 6 units shifts the entire graph vertically upward by 6 units, increasing all y-values by 6.

How does shifting the graph right 4 units change its equation?

Shifting right 4 units replaces x with (x - 4), so the new equation becomes $y = 3(x - 4)^2 + 6$.

What is the combined transformation of stretching, translating up, and right for the quadratic function?

The combined transformation results in the equation $y = 3(x - 4)^2 + 6$, starting from $y = x^2$.

What is the vertex of the transformed quadratic function?

The vertex of the transformed function is at the point (4, 6).

How do these transformations affect the graph's shape and position?

The graph becomes steeper due to the stretch, shifts right by 4 units, and moves up by 6 units, altering its position but maintaining its parabolic shape.

Additional Resources

The Quadratic Parent Function Has Been Stretched by a Factor of 3, Translated Up 6, and Right 4 — these transformations represent fundamental concepts in algebra and coordinate geometry that help us understand how functions change when modified. Whether you're a student learning about functions for the first time or a teacher looking to deepen your explanation, understanding how the quadratic parent function morphs under various transformations is essential. This guide provides a comprehensive breakdown of these transformations, illustrating precisely how each impacts the graph and the equation, and offers practical examples to solidify your understanding.

Introduction to the Quadratic Parent Function

The quadratic parent function is the simplest form of a quadratic function, expressed as:

$f(x) = x^2$

This basic parabola opens upward, with its vertex at the origin (0,0), and is symmetric about the y-axis. It serves as the foundation for understanding more complex quadratic functions, which are transformations of this parent form.

Understanding Transformations of the Quadratic Function

Transformations alter the appearance and position of the graph without changing the fundamental shape. They include:

- Vertical and horizontal stretches/shrinks
- Translations (shifts) up/down or left/right
- Reflections across axes

When analyzing the transformation "The quadratic parent function has been stretched by a factor of 3, translated up 6, and right 4," it's crucial to understand how each of these affects the function's equation and graph.

Breaking Down the Transformations

Let's analyze each transformation separately and then combine them.

1. Vertical Stretch by a Factor of 3

What it does:
A vertical stretch makes the parabola narrower, increasing its steepness. Every y-value is multiplied by 3.

Mathematically:
If the parent function is $f(x) = x^2$, then after a vertical stretch by factor 3:
$f(x) = 3x^2$

Graphical effect:
- The parabola becomes steeper.
- Points that were at (x, y) now move to (x, 3y).

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2. Horizontal Translation (Shift) Right by 4

What it does:
Shifts the graph to the right by 4 units. The input x-values are replaced with (x - 4).

Mathematically:
- To shift right by 4:
$f(x) = (x - 4)^2$
- Applying the stretch later will be combined with this.

Graphical effect:
- The vertex moves from (0,0) to (4,0).
- The entire parabola shifts rightward.

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3. Vertical Translation (Shift) Up by 6

What it does:
Raises the entire graph 6 units upward. Add 6 to the entire function.

Mathematically:
- For upward shift:
$f(x) = ... + 6$

Graphical effect:
- The vertex moves from (4, 0) to (4, 6).

Combining the Transformations

Transformations are applied in a specific order, generally:

1. Horizontal transformations (shifts)
2. Horizontal stretches/shrinks
3. Vertical transformations (scaling and shifting)

However, when combining transformations into a single equation, the order of operations is crucial. The general form for the combined transformation becomes:

$f(x) = a\,(\text{shifted } x)^2 + \text{vertical shift}$

Step-by-step combination:

- Start with the parent function: x^2
- Apply the horizontal shift: $(x - 4)$
- Apply the vertical stretch: multiply by 3
$3\,(x - 4)^2$
- Apply the upward translation: add 6
$f(x) = 3\,(x - 4)^2 + 6$

This is the final transformed function.

The Final Equation

Putting it all together, the quadratic parent function has been stretched by a factor of 3, translated up 6, and right 4 is expressed as:

$f(x) = 3(x - 4)^2 + 6$

Graphical Interpretation

Key features of the transformed parabola:

- Vertex: Located at (4, 6)
- Direction: Opens upward (since the coefficient 3 is positive)
- Width: Narrower than the parent parabola due to the stretch factor of 3
- Axis of symmetry: Vertical line $x = 4$
- Y-intercept: Find by plugging $x=0$:
$f(0) = 3(0 - 4)^2 + 6 = 3(16) + 6 = 48 + 6 = 54$

Practical Examples and Visualization

Example 1: Plot Key Points

- Vertex at (4, 6)
- Point at $x=0$:
$f(0) = 54$ (as above)
- Point at $x=2$:
$f(2) = 3(2 - 4)^2 + 6 = 3(-2)^2 + 6 = 3(4) + 6 = 12 + 6 = 18$
- Point at $x=6$:
$f(6) = 3(6 - 4)^2 + 6 = 3(2)^2 + 6 = 3(4) + 6 = 12 + 6 = 18$

These points help sketch the parabola accurately.

Example 2: Sketching the Graph

- Plot the vertex at (4, 6).
- Plot additional points at (0, 54), (2, 18), (6, 18).
- Draw the symmetric parabola passing through these points, opening upward, narrower than the original.

Additional Considerations

Reflection

- If the coefficient before the squared term were negative (e.g., $-3(x - 4)^2 + 6$), the parabola would open downward, reflecting across the x-axis.

Horizontal Compression/Stretch

- Changing the coefficient inside the parentheses affects the width:
- Larger coefficients (>1) make the parabola narrower.
- Smaller coefficients (<1) make it wider.

Impact of Multiple Transformations

- The sequence of transformations significantly affects the shape and position.
- Always consider applying shifts before stretch/shrink for clarity.

Summary

Transforming the quadratic parent function involves understanding how each change affects the graph:

Transformation	Effect	Equation Modification
Horizontal shift right by 4	Moves parabola 4 units right	Replace x with (x - 4)
Vertical stretch by factor 3	Makes the parabola steeper/narrower	Multiply entire function by 3
Vertical shift up by 6	Moves parabola 6 units upward	Add 6 to the entire function

The combined transformed function is:

$f(x) = 3(x - 4)^2 + 6$

This comprehensive understanding allows you to analyze, graph, and manipulate quadratic functions effectively, whether in academic settings or real-world applications.

Final Thoughts

Mastering the transformations of the quadratic parent function is a stepping stone toward deeper algebraic fluency. By breaking down each transformation, visualizing their effects, and combining them systematically, you'll develop a clear intuition for how quadratic graphs behave under various modifications. Practice with different coefficients, shifts, and stretches to strengthen your understanding and become proficient in function transformations.

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