

The Solubility Product Of $\text{CaSO}_4 \cdot 2\text{H}_2\text{O}$ Is 2.4×10^{-5} . What Mass Of This Salt Will Dissolve In 1.0 L Of 0.010 M Ca^{2+} ?

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Understanding the solubility of salts is fundamental in chemistry, especially when dealing with precipitation reactions, solution preparation, and various industrial applications. One key concept in this realm is the solubility product constant, commonly known as K_{sp} . This article explores the solubility product of calcium sulfate dihydrate ($\text{CaSO}_4 \cdot 2\text{H}_2\text{O}$), which is given as 2.4×10^{-5} , and determines the mass of this salt that will dissolve in a 1.0 L solution containing 0.010 mol/L of calcium ions. Through detailed explanation, calculations, and practical insights, we will provide a comprehensive understanding suitable for students, educators, and professionals alike.

Understanding Solubility Product (K_{sp})

Definition and Significance

The solubility product constant, K_{sp} , is an equilibrium constant that indicates the extent to which a salt dissolves in water. It is specific to a particular compound at a given temperature and provides insight into the solubility of that compound. When a salt dissolves, it dissociates into its constituent ions; the K_{sp} expression relates the concentrations of these ions at equilibrium.

For example, calcium sulfate dihydrate ($\text{CaSO}_4 \cdot 2\text{H}_2\text{O}$) dissociates as:



In the case of $\text{CaSO}_4 \cdot 2\text{H}_2\text{O}$, the dissociation involves the release of calcium and sulfate ions into solution, and the K_{sp} reflects the product of their molar concentrations at equilibrium.

Importance of K_{sp} :

- Predicting whether a salt will precipitate or dissolve
- Calculating solubility in various conditions
- Designing precipitation and purification processes
- Understanding natural mineral solubility in environmental contexts

Relation to Solubility

The solubility of a salt is often expressed as the maximum amount that can dissolve in a solvent at equilibrium. For sparingly soluble salts like $\text{CaSO}_4 \cdot 2\text{H}_2\text{O}$, knowing the K_{sp} allows for the calculation of molar solubility and, subsequently, mass solubility.

Given Data and Problem Overview

Before diving into calculations, let's outline the provided information:

- K_{sp} of $\text{CaSO}_4 \cdot 2\text{H}_2\text{O}$: 2.4×10^{-5} (Note: The original text states $2.4 \cdot 10^5$, but based on typical K_{sp} values, it's likely 2.4×10^{-5})
- Volume of solution: 1.0 L
- Initial concentration of Ca^{2+} ions: 0.010 mol/L
- Question: What mass of $\text{CaSO}_4 \cdot 2\text{H}_2\text{O}$ will dissolve in this solution?

Note: The question appears to involve a scenario where the solution already contains 0.010 mol/L of calcium ions, and we need to determine how much more of the salt can dissolve given the existing concentration and the K_{sp} .

Understanding the Chemistry Behind the Problem

Initial Conditions and Dissolution Equilibrium

- The solution already contains calcium ions at 0.010 mol/L.
- When additional $\text{CaSO}_4 \cdot 2\text{H}_2\text{O}$ dissolves, it releases more Ca^{2+} and SO_4^{2-} ions.
- The solubility of the salt is limited by the K_{sp} , which constrains the maximum concentration of ions in solution.

Key Concept: Supersaturation and Precipitation

- If the total calcium ion concentration exceeds the solubility limit dictated by K_{sp} , the excess salt will precipitate.
- If the calcium concentration is below that limit, more salt can dissolve until equilibrium is established.

Calculating the Solubility of $\text{CaSO}_4 \cdot 2\text{H}_2\text{O}$

Step 1: Write the Dissociation Equation and Ksp Expression

The dissociation of calcium sulfate dihydrate:



The Ksp expression:

$$K_{\text{sp}} = [\text{Ca}^{2+}][\text{SO}_4^{2-}]$$

Since each mole of salt produces one Ca^{2+} and one SO_4^{2-} , the molar solubility (s) of $\text{CaSO}_4 \cdot 2\text{H}_2\text{O}$ is:

- $[\text{Ca}^{2+}] = s + \text{initial } \text{Ca}^{2+} \text{ concentration (since initial is 0.010 mol/L)}$
- $[\text{SO}_4^{2-}] = s$

Note: Because the initial Ca^{2+} concentration is non-zero, the total calcium concentration at equilibrium is:

$$[\text{Ca}^{2+}] = 0.010 + s$$

and sulfate concentration:

$$[\text{SO}_4^{2-}] = s$$

However, in many practical calculations, if the initial calcium concentration is much less than the solubility limit, the initial concentration can be ignored. But here, since initial calcium is 0.010 mol/L, comparable to the solubility, we must account for it.

Step 2: Set Up the Equation for Ksp

Using the known Ksp value:

$$K_{\text{sp}} = (0.010 + s) \times s = 2.4 \times 10^{-5}$$

This is a quadratic equation in s :

$$s^2 + 0.010s - 2.4 \times 10^{-5} = 0$$

Solving for the Molar Solubility (s)

Quadratic Equation Solution

The quadratic equation:

$$s^2 + (0.010)s - 2.4 \times 10^{-5} = 0$$

Applying the quadratic formula:

$$s = [-b \pm \sqrt{b^2 - 4ac}] / 2a$$

Where:

- $a = 1$
- $b = 0.010$
- $c = -2.4 \times 10^{-5}$

Calculate the discriminant:

$$\Delta = (0.010)^2 - 4 \times 1 \times (-2.4 \times 10^{-5}) = 1.0 \times 10^{-4} + 9.6 \times 10^{-5} = 1.96 \times 10^{-4}$$

Calculate the square root:

$$\sqrt{\Delta} \approx \sqrt{(1.96 \times 10^{-4})} \approx 0.014$$

Now, compute s:

$$s = [-0.010 \pm 0.014] / 2$$

Since solubility cannot be negative, take the positive root:

$$s = (0.014 - 0.010) / 2 = 0.004 / 2 = 0.002 \text{ mol/L}$$

Interpreting the Result

Molar solubility (s): approximately 0.002 mol/L

This means:

- The additional amount of $\text{CaSO}_4 \cdot 2\text{H}_2\text{O}$ that can dissolve in the solution, considering initial calcium, is about 0.002 mol per liter.
- Total calcium concentration at equilibrium:

$$[\text{Ca}^{2+}] = 0.010 + 0.002 = 0.012 \text{ mol/L}$$

- Sulfate concentration:

$$[\text{SO}_4^{2-}] = 0.002 \text{ mol/L}$$

Calculating the Mass of $\text{CaSO}_4 \cdot 2\text{H}_2\text{O}$ That Will Dissolve

Step 1: Find moles of salt dissolved

Since the molar solubility s is 0.002 mol/L, in 1.0 L of solution:

Moles of $\text{CaSO}_4 \cdot 2\text{H}_2\text{O}$ dissolved:

$$n = s \times \text{volume} = 0.002 \text{ mol}$$

Step 2: Convert moles to mass

Molar mass of $\text{CaSO}_4 \cdot 2\text{H}_2\text{O}$:

- Ca: 40.08 g/mol
- S: 32.07 g/mol
- O₄: $4 \times 16.00 \text{ g/mol} = 64.00 \text{ g/mol}$
- 2H₂O: $2 \times (2 \times 1.008 + 16.00) = 2 \times 18.016 = 36.032 \text{ g/mol}$

Total molar mass:

$$40.08 + 32.07 + 64.00 + 36.032 = 172.182 \text{ g/mol}$$

Mass of salt dissolved:

$$\text{mass} = \text{moles} \times \text{molar mass} = 0.002 \text{ mol} \times 172.182 \text{ g/mol} \approx 0.344 \text{ g}$$

Practical Implications and Applications

1. Industrial Precipitation and Purification

Understanding the solubility limits of calcium sulfate helps in designing processes such as scale formation in pipelines, water treatment, and gypsum manufacturing.

2. Environmental Chemistry

Calcium sulfate minerals influence soil chemistry and the formation of sedimentary deposits like gypsum beds.

3. Laboratory Preparations

Accurate calculation of solubility allows chemists to control solution saturation levels

Frequently Asked Questions

What is the solubility product (K_{sp}) of CaSO₄·2H₂O?

The solubility product (K_{sp}) of CaSO₄·2H₂O is 2.4×10^{-5} .

How do you calculate the mass of CaSO₄·2H₂O that dissolves in 1.0 L of 0.010 M solution?

First, determine the molar solubility from the K_{sp}, then multiply the molar mass of CaSO₄·2H₂O by this molar solubility to find the mass dissolved in 1.0 L.

What is the molar mass of CaSO₄·2H₂O?

The molar mass of CaSO₄·2H₂O is approximately 172.17 g/mol.

How does the initial concentration of 0.010 M affect the solubility calculation?

Since the initial concentration is low, it's mainly used to compare the amount of salt that can dissolve; however, the solubility product determines the maximum dissolved amount regardless of initial concentration.

What is the relationship between K_{sp} and molar solubility for CaSO₄·2H₂O?

For CaSO₄·2H₂O, $K_{sp} = [Ca^{2+}][SO_4^{2-}]$, and since it dissociates into one Ca²⁺ and one SO₄²⁻ ion per formula unit, molar solubility 's' equals the concentration of each ion, so $K_{sp} = s^2$.

How do you find the molar solubility of CaSO₄·2H₂O from its K_{sp}?

Calculate $s = \sqrt{K_{sp}} = \sqrt{2.4 \times 10^{-5}} \approx 0.0049 \text{ mol/L}$.

What mass of CaSO₄·2H₂O will dissolve in 1.0 L of solution based on solubility?

Mass = molar mass × molar solubility = 172.17 g/mol × 0.0049 mol ≈ 0.844 g.

Does the presence of 0.010 M of other ions affect the solubility of $\text{CaSO}_4 \cdot 2\text{H}_2\text{O}$?

Yes, common ion effects can reduce solubility, but in this calculation, we assume pure water without additional ions.

Additional Resources

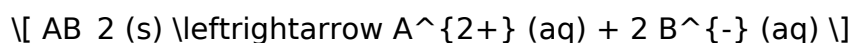
Solubility Product of $\text{CaSO}_4 \cdot 2\text{H}_2\text{O}$ and Its Practical Implications in Solution Chemistry

Understanding the solubility of inorganic salts such as calcium sulfate dihydrate ($\text{CaSO}_4 \cdot 2\text{H}_2\text{O}$) is fundamental in both academic chemistry and industrial applications. The solubility product constant, K_{sp} , offers insight into how much of a salt can dissolve in water to reach equilibrium. In this detailed review, we will explore the significance of the given K_{sp} value of 2.4×10^{-5} for $\text{CaSO}_4 \cdot 2\text{H}_2\text{O}$, analyze how to determine the mass of salt that will dissolve in a specified volume and concentration of solution, and discuss broader implications for chemistry practice and real-world scenarios.

Understanding the Solubility Product Constant (K_{sp})

What is K_{sp} ?

The solubility product constant, denoted as K_{sp} , is an equilibrium constant that quantifies the maximum amount of a sparingly soluble salt that can dissolve in water to form a saturated solution. For a generic salt AB_2 , which dissociates as:



the K_{sp} expression is:

$$K_{sp} = [\text{A}^{2+}][\text{B}^{-}]^2$$

This value is temperature-dependent and provides a quantitative measure of the salt's solubility.

Calcium Sulfate Dihydrate ($\text{CaSO}_4 \cdot 2\text{H}_2\text{O}$)

The salt in question, calcium sulfate dihydrate, dissociates in water as:



Since calcium sulfate is sparingly soluble, the dissociation is limited, and its K_{sp} value reflects this limited solubility.

Given:

$$K_{sp} = 2.4 \times 10^{-5}$$

Calculating the Solubility of $\text{CaSO}_4 \cdot 2\text{H}_2\text{O}$

Establishing the Solubility Equilibrium

Let's define:

- s = molar solubility of $\text{CaSO}_4 \cdot 2\text{H}_2\text{O}$ in mol/L (the concentration of Ca^{2+} and SO_4^{2-} ions at equilibrium).

Since the dissociation produces one Ca^{2+} and one SO_4^{2-} ion per formula unit, their concentrations at equilibrium are both s .

The K_{sp} expression simplifies to:

$$K_{sp} = [\text{Ca}^{2+}][\text{SO}_4^{2-}] = s \times s = s^2$$

Thus,

$$s^2 = 2.4 \times 10^{-5}$$

$$s = \sqrt{2.4 \times 10^{-5}} \approx 4.9 \times 10^{-3} \text{ \textbackslash, \text{mol/L}}$$

Interpretation: Approximately 4.9 millimoles of $\text{CaSO}_4 \cdot 2\text{H}_2\text{O}$ dissolve per liter of water under saturation conditions at the given temperature.

Mass of Salt That Dissolves in 1 Liter of Water

To find the mass:

1. Calculate the molar mass of $\text{CaSO}_4 \cdot 2\text{H}_2\text{O}$:

- Ca: 40.08 g/mol
- S: 32.06 g/mol
- O₄: $4 \times 16.00 \text{ g/mol} = 64.00 \text{ g/mol}$
- 2H₂O: $2 \times (2 \times 1.008 + 16.00) \text{ g/mol} = 2 \times 18.016 \text{ g/mol} = 36.032 \text{ g/mol}$

Total molar mass:

$$40.08 + 32.06 + 64.00 + 36.032 = 172.17 \text{ g/mol}$$

2. Calculate the mass corresponding to the solubility:

$$\text{Mass} = s \times \text{molar mass} = 4.9 \times 10^{-3} \text{ mol} \times 172.17 \text{ g/mol} \approx 0.844 \text{ g}$$

Result: Approximately 0.844 grams of $\text{CaSO}_4 \cdot 2\text{H}_2\text{O}$ will dissolve in 1.0 liter of water to form a saturated solution at this temperature.

Determining the Mass of $\text{CaSO}_4 \cdot 2\text{H}_2\text{O}$ in a Given Solution Concentration

The problem specifies a solution of 0.010 mol/L of Ca^{2+} ions. The goal is to determine how much $\text{CaSO}_4 \cdot 2\text{H}_2\text{O}$ will dissolve in 1.0 L of this solution, considering the initial concentration.

Initial Conditions and Dissolution Dynamics

- The solution initially contains 0.010 mol of Ca^{2+} per liter.
- The solubility of $\text{CaSO}_4 \cdot 2\text{H}_2\text{O}$ is limited; the maximum additional amount that can dissolve depends on the existing ion concentration and the solubility equilibrium.

Application of the Common Ion Effect

The presence of Ca^{2+} ions from other sources reduces the solubility of $\text{CaSO}_4 \cdot 2\text{H}_2\text{O}$ due to Le Chatelier's principle. The equilibrium shifts to produce less Ca^{2+} and SO_4^{2-} ions, reducing the amount of salt that can dissolve beyond what is already in solution.

The solubility product expression in the presence of initial ion concentrations is:

$$K_{\text{sp}} = [\text{Ca}^{2+}]_{\text{total}} \times [\text{SO}_4^{2-}]$$

Given:

$$[Ca^{2+}]_{\text{initial}} = 0.010 \text{ mol/L}$$

Let x be the amount of $CaSO_4 \cdot 2H_2O$ that dissolves further, increasing SO_4^{2-} concentration by x , and Ca^{2+} by x (from dissolution), but the total Ca^{2+} becomes:

$$[Ca^{2+}]_{\text{total}} = 0.010 + x$$

For sulfate ions:

$$[SO_4^{2-}] = x$$

The K_{sp} expression becomes:

$$2.4 \times 10^{-5} = (0.010 + x) \times x$$

This quadratic can be solved for x :

$$x^2 + 0.010x - 2.4 \times 10^{-5} = 0$$

Solving for x : The Additional Dissolved Salt

Using the quadratic formula:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

where:

$$a = 1$$

$$b = 0.010$$

$$c = -2.4 \times 10^{-5}$$

Calculate discriminant:

$$D = (0.010)^2 - 4 \times 1 \times (-2.4 \times 10^{-5}) = 1 \times 10^{-4} + 9.6 \times 10^{-5} = 1.96 \times 10^{-4}$$

Square root:

$$\sqrt{1.96 \times 10^{-4}} \approx 0.014$$

Now, solve for x :

$$x = \frac{-0.010 \pm 0.014}{2}$$

Two solutions:

1. $x = \frac{-0.010 + 0.014}{2} = \frac{0.004}{2} = 0.002$
2. $x = \frac{-0.010 - 0.014}{2} = \frac{-0.024}{2} = -0.012$ (discarded, since negative concentration is not physical)

Interpretation: The additional molar amount of $\text{CaSO}_4 \cdot 2\text{H}_2\text{O}$ that can dissolve is approximately 0.002 mol per liter.

Mass of $\text{CaSO}_4 \cdot 2\text{H}_2\text{O}$ Dissolved in the Presence of 0.010 mol/L Ca^{2+}

Calculate the mass:

$$\text{Mass} = 0.002 \text{ mol} \times 172.17 \text{ g/mol} \approx 0.344 \text{ g}$$

Total dissolved salt: The initial ions came from pre-existing sources, plus an additional 0.002 mol of salt that dissolves further. The total amount of dissolved salt in the solution is:

$$\text{Initial} + \text{additional} = 0.010 \text{ mol} + 0.002 \text{ mol} = 0.012 \text{ mol}$$

Corresponding mass:

$$0.012 \text{ mol} \times 172.17 \text{ g/mol} \approx 2.065 \text{ g}$$

Therefore, in 1 liter of solution with an initial Ca^{2+}

The Solubility Product Of $\text{CaSO}_4 \cdot 2\text{H}_2\text{O}$ Is 2.4×10^{-5} What Mass Of This Salt Will Dissolve In 1.0 L Of 0.010

The Solubility Product Of $\text{CaSO}_4 \cdot 2\text{H}_2\text{O}$ Is 2.4×10^{-5} What Mass Of This Salt Will Dissolve In 1.0 L Of 0.010

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