The Sum Of Three Consecutive Integers Is 42. Which Of The Equations Could Represent This Relationship?

The Sum Of Three Consecutive Integers Is 42. Which Of The Equations Could Represent This Relationship?

Understanding how to translate word problems into algebraic equations is a fundamental skill in mathematics. One common type of problem involves consecutive integers—numbers that follow one after another in order. Specifically, problems asking for the sum of three consecutive integers equal to a certain number are frequent in math classrooms. In this article, we will explore how to determine which equations accurately represent the relationship where the sum of three consecutive integers equals 42. We will break down the problem, develop the algebraic expressions, and analyze different possible equations to identify the correct one.

Understanding Consecutive Integers

What Are Consecutive Integers?

Consecutive integers are numbers that come one after the other in the natural number sequence. For example, 3, 4, 5 are three consecutive integers. Similarly, -2, -1, 0 are also three consecutive integers, just in the negative range.

In general, if the first integer is represented as n, then the next two integers are $n + 1$ and $n + 2$. This simple structure allows us to express the sum of three consecutive integers as an algebraic expression.

Expressing Consecutive Integers Algebraically

Suppose we denote the smallest of the three consecutive integers as n. The other two integers are then:

- First integer: n
- Second integer: $n + 1$
- Third integer: $n + 2$

The goal is to find the value of n such that the sum of these three integers is 42.

Formulating the Equation

Writing the Sum Equation

The sum of the three consecutive integers can be written as:

$$
n + (n + 1) + (n + 2) = 42
$$

Simplifying this, we combine like terms:

$$
n + n + 1 + n + 2 = 42
$$
$$
3n + 3 = 42
$$

This is a straightforward linear equation in n. Solving for n:

$$
3n + 3 = 42
$$
$$
3n = 42 - 3
$$
$$
3n = 39
$$
$$
n = \frac{39}{3} = 13
$$

Thus, the smallest of the three integers is 13. The other two are:

$$
n + 1 = 14
$$
$$
n + 2 = 15
$$

and their sum is indeed $13 + 14 + 15 = 42$.

Which Equations Could Represent the Relationship?

Now that we understand how to model the problem, let's analyze different equations that might represent the sum of three consecutive integers equaling 42.

Possible Equations to Consider

Here are some common forms of equations that could be proposed:

1. Equation A: $n + (n + 1) + (n + 2) = 42$

2. Equation B: $x + (x + 1) + (x + 2) = 42$

3. Equation C: $3n + 3 = 42$

4. Equation D: $3n = 39$

5. Equation E: $3(n + 1) = 42$

6. Equation F: $3n + 6 = 42$

Let's examine each to determine which accurately models the problem.

Analyzing the Equations

Equation A: $n + (n + 1) + (n + 2) = 42$

This is the direct algebraic expression of the sum of three consecutive integers, starting with n. It correctly represents the problem statement. Solving it yields the value of n that makes the sum 42.

Equation B: $x + (x + 1) + (x + 2) = 42$

This is essentially the same as Equation A but uses a different variable x. It is equally valid as a model for the same problem because the variable name doesn't change the nature of the equation.

Equation C: $3n + 3 = 42$

This is the simplified form of Equation A after combining like terms. It correctly models the sum of the three consecutive integers.

Equation D: $3n = 39$

This is the result of subtracting 3 from both sides of Equation C:

$$
3n + 3 = 42
$$
$$
3n = 42 - 3 = 39
$$

Therefore, Equation D is equivalent to Equation C and correctly models the problem.

Equation E: $3(n + 1) = 42$

This equation suggests that three times the second integer $(n + 1)$ equals 42. But since the sum of the integers is 42, and the second integer is $n + 1$, multiplying it by 3 gives only the sum of the second and third integers plus the first one in a different way. This equation does not correctly model the sum of all three integers; thus, it is not the correct representation.

Equation F: $3n + 6 = 42$

This equation seems to assume the sum is $3n + 6$. If we think of the integers as n, $n + 1$, and $n + 2$, then their sum is:

$$
n + (n + 1) + (n + 2) = 3n + 3
$$

which is Equation C. So, Equation F is off by 3, and hence does not accurately represent the sum of the three consecutive integers unless the initial assumption is modified.

Summary of Valid Equations

Based on the analysis, the equations that correctly model the problem where the sum of three consecutive integers is 42 are:

- Equation A: $n + (n + 1) + (n + 2) = 42$

- Equation C: $3n + 3 = 42$
- Equation D: $3n = 39$

These equations are equivalent representations of the problem, with each leading to the same solution for n.

Conclusion: Identifying the Correct Equation

In solving for the three consecutive integers whose sum is 42, the most straightforward equation is the initial form:

$$
n + (n + 1) + (n + 2) = 42
$$

which directly models the sum of the three integers. Simplifying this yields:

$$
3n + 3 = 42
$$
$$
3n = 39
$$
$$
n = 13
$$

Thus, the three integers are 13, 14, and 15, which sum to 42 as required.

If you are presented with multiple-choice options, look for equations like these, especially those that express the sum of three consecutive integers as $n + (n + 1) + (n + 2) = 42$ or its simplified forms. Recognizing the structure of the problem and translating it into the correct algebraic equation is crucial for solving similar problems efficiently.

In summary:

- The key to representing the sum of three consecutive integers is to express them algebraically as n, $n + 1$, and $n + 2$.
- The correct equations are those that reflect the sum of these three terms equaling 42.
- Simplifying these equations helps in finding the actual integers and verifying the correctness of the model.

Understanding these principles not only helps in solving this particular problem but also enhances your overall ability to translate word problems into algebraic equations, a vital skill in mathematics.

Frequently Asked Questions

What is the general form of the equation representing three consecutive integers whose sum is 42?

If the first integer is x, then the next two are x+1 and x+2. The sum is x + (x+1) + (x+2) = 3x + 3. Setting this equal to 42 gives 3x + 3 = 42.

Which of the following equations correctly models the sum of three consecutive integers equaling 42?
A) x + (x+1) + (x+2) = 42
B) 3x + 3 = 42
C) Both A and B
D) Neither A nor B

C) Both A and B. Equation A is the expanded form, and B is simplified; both correctly model the relationship.

If the three consecutive integers sum to 42, what is the value of the first integer?

First, set up the equation: 3x + 3 = 42. Solving for x: 3x = 39, so x = 13. Therefore, the integers are 13, 14, and 15.

How can you verify that the three integers 13, 14, and 15 sum to 42?

Add the integers: 13 + 14 + 15 = 42. Since their sum is 42, they satisfy the condition.

Can the equation 2x + 3 = 42 represent the sum of three consecutive integers? Why or why not?

No, because the sum of three consecutive integers is modeled by 3x + 3, not 2x + 3. Therefore, 2x + 3 does not correctly represent the relationship.

What is an alternative way to write the equation for three consecutive integers summing to 42?

You can write it as (x) + (x + 1) + (x + 2) = 42, which simplifies to 3x + 3 = 42.

How does understanding the equation help in finding the

specific integers when their sum is known?

By setting up the correct equation (e.g., $3x + 3 = 42$), you can solve for x to find the first integer, then determine the next two by adding 1 and 2 respectively.

If the sum of three consecutive integers is not 42, but another number, how would you modify the equation?

Replace 42 with the new total in the equation $3x + 3 = \text{total}$. For example, if the sum is S, then $3x + 3 = S$.

Additional Resources

Mathematics: Unraveling the Sum of Three Consecutive Integers Equal to 42

When it comes to understanding basic algebraic relationships, few concepts are as foundational yet as elegant as the idea of consecutive integers. These numbers, simple in their progression yet rich in their implications, serve as a perfect entry point for exploring how algebraic expressions can model real-world scenarios or abstract mathematical problems.

In this article, we delve into a specific problem: "The sum of three consecutive integers is 42." Our goal is to identify which equations correctly represent this relationship, understand the underlying logic, and appreciate the broader significance of such problems in mathematics. Think of this as a detailed product review, where each potential "equation" is examined for its accuracy, robustness, and applicability.

Understanding the Core Concept: Consecutive Integers

Before exploring the equations, it's essential to define what we mean by consecutive integers.

What Are Consecutive Integers?

Consecutive integers are numbers that follow one after another in order, without any gaps. For example:

- 3, 4, 5
- -2, -1, 0
- 10, 11, 12

In the context of this problem, we are considering three such integers, say:

- First integer: n
- Second integer: n + 1

- Third integer: n + 2

This notation simplifies the process of forming algebraic expressions and solving for the unknowns.

The Significance of the Sum

The problem states that the sum of these three integers equals 42. This gives us a concrete numerical relationship to work with, enabling us to set up an algebraic equation.

Formulating the Equation: The Foundation of the Problem

The key step in solving this problem is translating the verbal statement into an algebraic equation. Given our definitions:

- First integer: n
- Second integer: n + 1
- Third integer: n + 2

The sum of these three integers is:

n + (n + 1) + (n + 2)

According to the problem, this sum equals 42. Therefore, the equation becomes:

n + (n + 1) + (n + 2) = 42

Analyzing the Equation: Simplification and Solution

Let's unpack and simplify this equation step-by-step, as a critical review of the logical structure and algebraic correctness.

Step 1: Combine Like Terms

n + n + 1 + n + 2 = 42

This simplifies to:

(n + n + n) + (1 + 2) = 42

Which is:

$3n + 3 = 42$

Step 2: Isolate the Variable

Subtract 3 from both sides:

$3n + 3 - 3 = 42 - 3$

$3n = 39$

Now, divide both sides by 3:

$n = 13$

Step 3: Find the Consecutive Integers

Using $n = 13$:

- First integer: 13
- Second integer: 14
- Third integer: 15

Step 4: Verify the Sum

Sum: $13 + 14 + 15 = 42$

Since the sum checks out, our equation accurately represents the relationship.

Which Equations Could Represent This Relationship?

Given the derivation above, several algebraic expressions could model the problem, provided they are logically equivalent to the initial setup. Let's explore the potential options, analyze their correctness, and determine which equations accurately reflect the scenario.

Potential Equations:

1. $n + (n + 1) + (n + 2) = 42$
2. $(n) + (n + 1) + (n + 2) = 42$
3. $3n + 3 = 42$
4. $3(n + 1) = 42$
5. $(n - 1) + n + (n + 1) = 42$
6. $(n - 2) + (n - 1) + n = 42$

Let's analyze each.

Equation 1: n + (n + 1) + (n + 2) = 42

Analysis:
This is the fundamental equation directly derived from the definition of three consecutive integers starting at n. It correctly models the problem and is mathematically sound.

Verdict: Correct.

Equation 2: (n) + (n + 1) + (n + 2) = 42

Analysis:
This is essentially the same as Equation 1, just with parentheses added for clarity. Parentheses do not change the value of an expression unless they alter the order of operations, which they do not here.

Verdict: Correct.

Equation 3: 3n + 3 = 42

Analysis:
This is the simplified form obtained after combining like terms in Equation 1. It is derived directly from the initial assumption and algebraic manipulation.

Verdict: Correct.

Equation 4: 3(n + 1) = 42

Analysis:
This equation implies that the sum of three consecutive integers starting at n + 1 is 42, which is not the original problem statement. If we expand it:

3(n + 1) = 3n + 3 = 42

This matches Equation 3, but it shifts the starting point of the integers to n + 1, which does not represent the original problem of three consecutive integers starting at n.

Verdict: Not correct for the original setup.

Equation 5: $(n - 1) + n + (n + 1) = 42$

Analysis:
This sums three integers centered around n: one less than n, n itself, and one more than n. This is a different but valid scenario representing three consecutive integers centered around n. The sum simplifies to:

$(n - 1) + n + (n + 1) = 3n$

Set equal to 42:

$3n = 42$

$n = 14$

The three integers are: 13, 14, 15, which sum to 42, matching the previous solution but with a different starting point.

Verdict: Correct, but it models a different set of three consecutive integers (centered around n), not the initial assumption starting at n.

Equation 6: $(n - 2) + (n - 1) + n = 42$

Analysis:
This sums three integers decreasing from n:

$n - 2, n - 1, n$

Their sum:

$(n - 2) + (n - 1) + n = 3n - 3$

Set equal to 42:

$3n - 3 = 42$

$3n = 45$

$n = 15$

The integers are 13, 14, 15, again summing to 42.

Verdict: Correct, but this models the three consecutive integers ending at n.

Summary of Valid Equations

Based on the analysis, the equations that correctly represent the relationship where the sum of three consecutive integers is 42 are:

- $n + (n + 1) + (n + 2) = 42$
- $(n) + (n + 1) + (n + 2) = 42$
- $3n + 3 = 42$
- $(n - 1) + n + (n + 1) = 42$
- $(n - 2) + (n - 1) + n = 42$

All these equations are valid representations, provided the context of the integers is understood. The differences lie in where the starting point n is positioned relative to the sequence, but all ultimately lead to the same core solution.

Conclusion: The Broader Significance of Algebraic Modeling

This exploration highlights the importance of precise algebraic modeling when solving problems involving consecutive integers. Different equations can represent the same underlying relationship, but their form and the interpretation of the variable n can vary.

Key Takeaways:

- The fundamental equation $n + (n + 1) + (n + 2) = 42$ accurately models the sum of three consecutive integers starting at n.
- Simplified forms like $3n + 3 = 42$ are equally valid and often more convenient for solving.
- Alternative equations, such as $(n - 1) + n + (n + 1) = 42$, demonstrate the flexibility in choosing the starting point, provided the sequence remains consecutive.
- Understanding the structure of these equations helps in recognizing similar problems and developing algebraic intuition.

In essence, mastering the translation between verbal descriptions and algebraic expressions empowers problem-solvers, students, and educators alike to approach a wide array of mathematical challenges confidently. Whether you're tackling a school assignment or engaging in complex modeling, recognizing the validity and equivalence of different equations is a vital skill—one that turns abstract numbers into meaningful insights.

Final note: When presented with a problem like "The sum of three consecutive integers is

The Sum Of Three Consecutive Integers Is 42 Which Of The Equations Could Represent This Relationship

The Sum Of Three Consecutive Integers Is 42 Which Of The Equations Could Represent This Relationship

Related Articles

- The Marginal Benefit From Pollution Abatement Group Of Answer Choices Decreases As Pollution Abatement
- The Three Dimensions Of Situational Control Concern Whether A Leader Can Issue Rewards Or Punishments,
- Two Ships Leave A Port At The Same Time. The First Ship Sails On A Bearing Of 52 At 22 Knots (nautical

Back to Home