The Sum Of Two Numbers Is 0. Twice The Smaller Number Subtracted From 3 Times The Larger Number Is 10.

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Understanding and solving algebraic problems involving two variables can seem daunting at first, but with a structured approach, these problems become manageable and even straightforward. This particular problem involves two numbers with specific relationships expressed through algebraic equations. By carefully translating words into algebraic expressions and then solving these equations step-by-step, we can find the values of these two numbers.

In this comprehensive guide, we will explore the problem in detail, analyze the given conditions, formulate the corresponding equations, and solve for the unknown numbers systematically. Whether you're a student seeking to improve your algebra skills or someone interested in problem-solving techniques, this article will provide clarity and insight into tackling similar problems.

Understanding the Problem

The problem states:

"The sum of two numbers is 0. Twice the smaller number subtracted from 3 times the larger number is 10."

Let's break this down:

- There are two numbers involved, which we will denote as variables.
- The first statement, "The sum of two numbers is 0," indicates a relationship between these numbers.
- The second statement involves a more complex relation involving multiples of these numbers.

Our goal: Find the values of these two numbers that satisfy both conditions.

Translating the Word Problem into Mathematical Equations

To solve the problem systematically, we need to convert the given statements into algebraic

equations.

Step 1: Assign Variables

Let:

- x = the larger number
- y = the smaller number

Note: It's essential to identify which number is larger and which is smaller because the problem references "the smaller number" and "the larger number."

Step 2: Write Equations from the Statements

1. Sum of the two numbers is 0:

$$
x + y = 0
$$

2. "Twice the smaller number subtracted from 3 times the larger number is 10":

This phrase can be interpreted as:

- 3 times the larger number: $3x$
- Twice the smaller number: $2y$
- Subtracting twice the smaller number from 3 times the larger number: $3x - 2y$

Set equal to 10:

$$
3x - 2y = 10
$$

Formulating the System of Equations

Based on the above translations, our system of equations is:

$$
\begin{cases}
x + y = 0 \quad (1) \\
3x - 2y = 10 \quad (2)
\end{cases}
$$

\]

Our task is to solve these equations simultaneously to find the values of \(x \) and \(y \).

Solving the System of Equations

To find the solutions, various methods can be used: substitution, elimination, or graphical methods. Here, the substitution method is straightforward given the first equation.

Step 1: Express one variable in terms of the other

From equation (1):

\[
x + y = 0 \Rightarrow y = -x
\]

Step 2: Substitute into the second equation

Replace \(y \) with \(-x\) in equation (2):

\[
3x - 2(-x) = 10
\]

Simplify:

\[
3x + 2x = 10
\]
\[
5x = 10
\]

Step 3: Solve for \(x \)

\[
x = \frac{10}{5} = 2
\]

Step 4: Find y

Recall that $y = -x$:

$$
y = -2
$$

Thus, the solutions are:

$$
x = 2 \quad \text{and} \quad y = -2
$$

Interpreting the Solution

- The larger number is 2.
- The smaller number is -2.

It's important to verify whether these values satisfy the original conditions:

1. Sum check:

$$
x + y = 2 + (-2) = 0
$$

2. Condition check:

$$
3x - 2y = 3(2) - 2(-2) = 6 + 4 = 10
$$

Both conditions are satisfied, confirming the correctness of the solution.

Additional Insights and Variations

Understanding the core problem allows learners to adapt similar problems with different conditions or more variables. Here are some points to consider:

1. Significance of the Variables

- The problem explicitly distinguishes between the larger and smaller numbers, which influences the variable assignments.
- In more complex problems, additional constraints or relations may specify which variable is larger or smaller.

2. Alternative Approaches

While substitution works well here, other methods such as elimination or graphical solutions can also be effective:

- Elimination Method: Multiply equations to align coefficients and eliminate variables.
- Graphical Method: Plot the equations to identify the intersection point visually.

3. Extending the Problem

- Introduce new conditions, such as the difference between the numbers or their product.
- Consider problems involving quadratic equations when the relationships involve squares or higher powers.

Practical Applications of Solving Such Problems

Understanding how to translate word problems into algebraic equations and solve them is fundamental in various real-world scenarios, including:

- Financial calculations, such as determining investments or loans.
- Physics problems involving distances, velocities, and accelerations.
- Engineering design calculations requiring specific constraints.
- Data analysis where relationships between variables need to be established and quantified.

Summary and Key Takeaways

- Assign variables carefully based on the problem description.
- Translate words into algebraic equations accurately.
- Use substitution or elimination methods to solve systems of equations.
- Verify solutions by substituting back into original equations.
- Recognize the importance of understanding the context (larger vs. smaller number).

By mastering these steps, students and learners can confidently approach and solve various algebraic word problems involving multiple variables and conditions.

Conclusion

The problem "The sum of two numbers is 0. Twice the smaller number subtracted from 3 times the larger number is 10" demonstrates the power of algebra in solving real-world problems. By systematically translating words into equations, applying algebraic methods, and verifying solutions, we find that the two numbers are 2 and -2, respectively. This process not only reinforces algebraic skills but also enhances problem-solving confidence, preparing learners for more complex mathematical challenges.

If you're interested in further practice, consider exploring problems involving systems of equations, quadratic equations, or word problems with multiple steps. Developing proficiency in these areas is essential for academic success and real-life applications.

Frequently Asked Questions

What are the algebraic expressions representing the given problem?

Let the smaller number be x and the larger number be y. The given conditions translate to: $x + y = 0$ and $3y - 2x = 10$.

How do you solve for the two numbers based on the equations?

From $x + y = 0$, we get $y = -x$. Substituting into $3y - 2x = 10$ gives $3(-x) - 2x = 10$, leading to $-3x - 2x = 10$, so $-5x = 10$, and $x = -2$. Then $y = -(-2) = 2$.

What are the two numbers that satisfy the given conditions?

The two numbers are -2 (smaller number) and 2 (larger number).

Can the larger number be negative in this problem?

No, since the problem states 'twice the smaller number subtracted from three times the larger number is 10', implying the larger number is positive in the solution, which is y=2.

How do these equations relate to real-world problem solving?

They demonstrate how to set up and solve linear equations based on word problems involving relationships between quantities, which is essential in various real-world scenarios like budgeting or measurements.

What is the significance of the sum of the two numbers being zero?

It indicates the numbers are additive inverses of each other, meaning one is the negative of the other, which simplifies solving the system of equations.

Are there alternative methods to solve this problem besides substitution?

Yes, methods like the elimination method or graphing can also be used to solve the system of equations and find the two numbers.

Additional Resources

Mathematical Problem Solving: Exploring the Sum and Difference of Two Numbers

Understanding the relationships between two numbers through algebraic expressions is a fundamental aspect of mathematical reasoning. The problem statement, "The sum of two numbers is 0. Twice the smaller number subtracted from three times the larger number is 10," offers a rich context for exploring algebraic concepts, problem formulation, and solution strategies. This review aims to dissect this problem comprehensively, providing insights into the underlying principles, solution methods, and broader applications.

Deciphering the Problem: Breaking Down the Statement

The initial step in tackling any word problem is to interpret the given information accurately. The statement contains two key pieces of information:

1. The sum of two numbers is 0.
2. Three times the larger number, minus twice the smaller number, equals 10.

At first glance, the problem involves two unknown quantities, which we will denote as variables for clarity. The challenge lies in translating the natural language into precise algebraic expressions.

Establishing Variables and Notation

To analyze the problem systematically, it’s important to assign variables that represent the unknown numbers:

- Let x = the smaller number
- Let y = the larger number

This choice simplifies the process of formulating the equations and solving for the variables.

Formulating the Equations: Translating Words into Math

Based on the problem statement, two equations can be established:

Equation 1: Sum of Two Numbers

"The sum of two numbers is 0."

This translates to:

- x + y = 0

This simple linear equation states that the two unknowns sum up to zero, which implies that they are additive inverses of each other.

Equation 2: The Second Condition

"Twice the smaller number subtracted from three times the larger number is 10."

Breaking down this phrase:

- "Three times the larger number" → 3y
- "Twice the smaller number" → 2x
- "Subtracted from" indicates the order of subtraction: 3y minus 2x

Thus, the algebraic expression:

- 3y - 2x = 10

Understanding the System of Equations

The problem now reduces to solving the following system:

$$
\begin{cases}
x + y = 0 \\
3y - 2x = 10
\end{cases}
$$

This is a linear system with two equations and two unknowns, which can be approached via substitution, elimination, or graphical methods.

Solution Strategies for the System of Equations

1. Substitution Method

Given that the first equation expresses y in terms of x:

- $y = -x$

Substitute $y = -x$ into the second equation:

- $3(-x) - 2x = 10$
- $-3x - 2x = 10$
- $-5x = 10$
- $x = -2$

Using $x = -2$ in $y = -x$:

- $y = -(-2) = 2$

Solution:

- Smaller number (x): -2
- Larger number (y): 2

2. Verification of the Solution

Check the solution against the original equations:

- Sum: x + y = -2 + 2 = 0 → Correct
- Second condition: 3y - 2x = 3(2) - 2(-2) = 6 + 4 = 10 → Correct

The solution set is consistent and satisfies both equations.

Deep Dive: Interpreting the Solution and Its Implications

The solution reveals that the two numbers are -2 and 2. This result is interesting because:

- The numbers are additive inverses.
- One is negative, and the other positive, summing to zero.
- The larger number is positive, as expected.

This scenario exemplifies the property that if two numbers sum to zero, then one must be the negative of the other.

Broader Mathematical Concepts Illustrated

This problem encapsulates several fundamental algebraic ideas:

- Linear Equations: The equations are linear, involving variables to the first power.
- Systems of Equations: Solving two equations simultaneously.
- Substitution Method: A straightforward approach when one equation can be easily rearranged.
- Inverse Relationships: The sum of the two numbers being zero indicates they are inverses.

Applications of Such Problems

- Financial calculations (e.g., balancing accounts or debts).
- Physics (e.g., balancing forces or velocities).
- Engineering (e.g., load distributions).

Extensions and Variations of the Problem

The problem can be modified or extended to explore different scenarios:

1. Changing the Sum

- What if the sum is a different constant, say, 5 or -3?
- How would the solution change?

2. Altering the Second Condition

- Instead of "three times the larger minus twice the smaller," consider other combinations, such as:

- "Four times the smaller minus the larger is 8."
- "Twice the larger plus the smaller equals 12."

3. Introducing Additional Conditions

- Adding constraints like positivity or specific ranges.
- Incorporating inequalities instead of equalities.

Understanding the Role of the Variables' Labels

While the initial problem labels the numbers as "smaller" and "larger," actual values depend on their relative sizes. The solution indicates:

- The smaller number is -2.
- The larger number is 2.

This labeling aligns with the initial assumption, but it’s essential to verify whether the labels match the numerical values, especially in more complex problems.

Real-World Contexts and Practical Examples

- Financial Context: Suppose x and y represent financial gains and losses, with their sum being zero indicating balanced profit and loss.
- Physics: The numbers could represent velocities in opposite directions, summing to zero, with the second condition representing an energy or force balance.
- Statistics: The numbers could be deviations from a mean, with their sum zero indicating zero mean deviation.

Key Takeaways and Educational Insights

- Formulation is Critical: Translating words into equations is a key skill. Clear variable assignment

and understanding the relationships help in solving.
- Multiple Methods: Substitution, elimination, and graphical methods are all viable; choosing depends on the problem structure.
- Verification: Always verify solutions against original conditions to ensure correctness.
- Interpretation: Understanding what the solution means in context adds depth to mathematical problem-solving.

Conclusion: Mastering Algebraic Word Problems

The problem "The sum of two numbers is 0. Twice the smaller number subtracted from three times the larger number is 10" exemplifies the essence of algebraic reasoning through word problems. It demonstrates how to translate natural language into equations, solve systems systematically, and interpret the solutions meaningfully.

By exploring such problems deeply, learners develop critical thinking, problem-solving skills, and an appreciation for the elegance of algebra. These skills are not only fundamental in mathematics but also invaluable in real-world decision-making, engineering, science, finance, and beyond.

Final Reflection: Whether tackling straightforward problems or more complex scenarios, the approach remains consistent—careful interpretation, systematic formulation, rigorous solution, and thoughtful interpretation. Mastery of these steps empowers learners to navigate the rich landscape of algebra and mathematical reasoning with confidence and clarity.

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