The Sum Of Two Numbers Is 21. The Second Number Is Six Times The First Number Work Out The Two Numbers

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When faced with an algebraic problem involving two numbers, it's often helpful to translate the words into mathematical expressions. In this case, the problem states that the sum of two numbers is 21, and that the second number is six times the first number. Solving such problems not only enhances your algebraic skills but also improves your logical thinking. This article will guide you through understanding the problem, setting up the equations, solving for the unknowns, and verifying your solutions.

Understanding the Problem

Before diving into the calculations, it's essential to interpret the problem carefully. The key information provided includes:

- The sum of two numbers is 21.
- The second number is six times the first number.

These clues are straightforward but require precise translation into algebraic terms to solve the problem accurately.

Breaking Down the Information

The problem involves two unknown numbers, which we can denote as:

- Let the first number be x.
- Let the second number be y.

Based on the problem statement, we have two main conditions:

1. The sum of the two numbers is 21:

$$
x + y = 21
$$

2. The second number is six times the first:

$$
y = 6x
$$

Our goal is to find the values of x and y that satisfy these conditions.

Setting Up the Equations

The problem can be approached by forming a system of equations, which is a common method in algebra to solve for multiple unknowns.

Equation 1: Sum of the Numbers

$$
x + y = 21
$$

Equation 2: Relationship Between the Numbers

\[

y = 6x

\]

Substituting Equation 2 into Equation 1 allows us to solve for \(x \).

Solving the System of Equations

Let's substitute \(y \) from Equation 2 into Equation 1:

\[

x + 6x = 21

\]

Combine like terms:

\[

7x = 21

\]

Divide both sides by 7 to isolate \(x \):

\[

x = \frac{21}{7} = 3

\]

Now that we have \(x \), we can find \(y \) using Equation 2:

\[

$$
y = 6x = 6 \times 3 = 18
$$

Solution:

- The first number x is 3.
- The second number y is 18.

Verifying the Solution

Verification is an essential step to ensure the solution satisfies all the given conditions.

- Check the sum:

$$
3 + 18 = 21 \quad \text{(True)}
$$

- Check the relationship:

$$
y = 6x \Rightarrow 18 = 6 \times 3 \quad \text{(True)}
$$

Since both conditions are satisfied, the solution is correct.

Additional Insights and Variations

While this particular problem has a straightforward solution, there are variations and extensions that can deepen understanding.

1. General Form of the Problem

Suppose the sum of two numbers is S, and the second number is k times the first. The equations would be:

$$
x + y = S
$$

$$
y = kx
$$

Substituting:

$$
x + kx = S \Rightarrow x(1 + k) = S \Rightarrow x = \frac{S}{1 + k}
$$

$$
y = k \times \frac{S}{1 + k}
$$

This general approach allows solving similar problems with different values of S and k.

2. Exploring Different Ratios and Sums

- What happens if the second number is three times the first?
- How does the solution change if the sum is different, say 30 or 50?

By adjusting the parameters, students can practice and understand the relationships between variables more deeply.

Practical Applications of the Concept

Understanding how to set up and solve such problems has real-world applications, including:

- Budgeting: Distributing amounts based on ratios.
- Engineering: Calculating parts in a system based on proportional relationships.
- Business: Determining quantities of products or resources based on customer demand or supply ratios.

These applications highlight the importance of algebraic problem-solving skills in everyday life.

Summary

To summarize, solving the problem where the sum of two numbers is 21 and the second is six times the first involves:

- Defining variables to represent unknowns.
- Translating words into algebraic equations.
- Substituting and solving the equations systematically.
- Verifying the solutions to ensure accuracy.

The key steps included:

1. Assigning x to the first number.
2. Expressing the second number as $y = 6x$.
3. Using the sum condition to form an equation.
4. Solving for x, then y.

The solutions obtained are $x = 3$ and $y = 18$, which satisfy all the problem conditions.

Conclusion

Mastering the process of translating word problems into algebraic expressions and solving them is fundamental in mathematics. The specific problem of finding two numbers with a sum of 21, where one is six times the other, is a perfect example of how algebra can simplify complex word problems into manageable calculations. Practice with similar problems will enhance your problem-solving skills and prepare you for more advanced mathematical challenges. Remember, always verify your solutions to ensure they meet all the original conditions.

Frequently Asked Questions

What are the two numbers if their sum is 21 and the second is six times the first?

Let the first number be x. Then the second number is 6x. Their sum is x + 6x = 7x = 21. So, x = 3. The first number is 3, and the second number is 6 × 3 = 18.

How do you set up an equation for the problem where the sum of two numbers is 21 and one is six times the other?

Let the first number be x. Since the second number is six times the first, it is 6x. The sum is then x + 6x = 21, which simplifies to 7x = 21.

What is the value of the first number in the problem?

The first number is 3, obtained by dividing the total sum 21 by 7 (since 7x=21).

What is the second number in this problem?

The second number is 18, which is six times the first number (6 × 3).

Can this problem be solved using algebra? If so, how?

Yes. Assign a variable to the first number, set up an equation based on the given conditions, and solve for the variable. In this case, $x + 6x = 21$ leads to $x = 3$.

What would change if the sum of the numbers was different, say 30?

You would set up the same equation: $x + 6x = 30$, leading to $7x = 30$, and then find $x = 30/7 \approx 4.29$, with the second number being $6 \times 4.29 \approx 25.71$.

Is there a visual way to understand this problem?

Yes. Imagine dividing 21 into parts: one part is the first number x, and the remaining 6 parts are the second number. Since total parts are 7, each part is 3, making the first number 3 and the second 18.

What real-life scenarios can this problem represent?

It could represent sharing resources where one person gets a certain amount, and another gets six times that amount, with the total combined quantity being 21 units.

How can I verify my answer in this problem?

Add the two numbers you found: $3 + 18 = 21$. Also, check if the second is six times the first: $6 \times 3 = 18$. Both conditions are satisfied, confirming your solution.

Additional Resources

The Sum Of Two Numbers Is 21. The Second Number Is Six Times The First Number. Work Out The Two Numbers.

Introduction

Mathematics often involves solving equations that describe real-world scenarios or abstract problems. One classic type of problem involves two unknown numbers with specific relationships, and the goal is to find their individual values. Such problems help develop critical thinking, algebraic skills, and an understanding of how to manipulate equations effectively.

In this detailed exploration, we analyze the problem:

> The sum of two numbers is 21. The second number is six times the first number. Work out the two numbers.

This problem is a typical example of algebraic word problems that involve setting up equations based on given relationships and solving for unknowns. We will systematically approach this problem, understand the logic behind it, and work through various methods of solution, ensuring a comprehensive understanding.

Understanding the Problem

Before diving into the calculations, it’s essential to interpret the problem statement carefully and identify key information.

Problem Breakdown:

- "The sum of two numbers is 21"
This indicates that if the two numbers are x and y, then:
$$
x + y = 21
$$

- "The second number is six times the first number"

This establishes a direct relationship between the two numbers:

$$
y = 6x
$$

Objective:

Determine the values of x and y satisfying both conditions.

Step-by-Step Solution Approach

Step 1: Assign Variables

To solve the problem, assign variables to the unknown quantities:

- Let x = the first number.
- Let y = the second number.

Step 2: Write Equations Based on the Conditions

From the problem:

1. Sum condition:
$$
x + y = 21
$$
2. Relationship condition:
$$
y = 6x

\]

These two equations form a system that can be solved simultaneously.

Solving the Equations

Method 1: Substitution Method

Since the second equation expresses y in terms of x, substitute $y = 6x$ into the first equation:

$$
x + 6x = 21
$$

Simplify:

$$
7x = 21
$$

Divide both sides by 7:

$$
x = \frac{21}{7} = 3
$$

Now, find y:

\[

y = 6x = 6 \times 3 = 18

\]

Solution:

\[

\boxed{

\text{First number } x = 3 \\

\text{Second number } y = 18

}

\]

Method 2: Equational Substitution (Alternative Approach)

Alternatively, you might prefer to express everything in terms of one variable and verify:

- From \(y = 6x \), substitute into \(x + y = 21 \):

\[

x + 6x = 21

\]

which leads to the same calculation as above.

Verifying the Solution

It's crucial to verify that the solution satisfies both conditions:

1. Sum check:

$$
3 + 18 = 21 \quad \text{(True)}
$$

2. Relationship check:

$$
18 = 6 \times 3 \quad \text{(True)}
$$

Thus, the solution is consistent and valid.

Alternative Methods of Solution

While substitution is the most straightforward here, other methods can also be employed:

Method 3: Graphical Representation

- Plotting the two equations on a graph:
- $y = 21 - x$ (from the sum condition)
- $y = 6x$ (from the relationship condition)

- The intersection point(s) of these lines give the solution.

Graphical steps:

1. Draw the line $y = 21 - x$:
- When $x = 0$, $y = 21$.

- When $x = 21$, $y = 0$.

2. Draw the line $y = 6x$:
- When $x = 0$, $y = 0$.
- When $x = 3.5$, $y = 21$.

3. The point of intersection occurs at $x=3$, $y=18$.

This visual approach can reinforce understanding, especially for visual learners.

Deeper Insights into the Problem

Understanding the Relationships

- The problem involves a linear relationship between the two numbers.
- The sum condition acts as a boundary constraint.
- The multiplicative relationship introduces a proportional link that simplifies solving.

Extensions and Variations

- What if the second number was five times the first?

The process remains similar: set $y = 5x$, then solve.

- What if the sum was a different number?

For example, if the sum was 30, then the sum equation becomes $x + y = 30$.

- What if the second number was a fraction of the first?

E.g., $y = \frac{1}{2} x$.

These variations demonstrate how the methods are adaptable.

Real-World Applications of Such Problems

Understanding these types of problems has practical significance:

- Financial contexts: Calculating investments and returns, where one amount relates proportionally to another.
- Physics: Determining quantities like velocity and acceleration with proportional relationships.
- Business: Analyzing profit-sharing or resource allocation based on ratios.

Common Mistakes and How to Avoid Them

1. Mixing up the variables:
Always keep track of which variable represents which quantity.

2. Ignoring the conditions:
Ensure both equations are satisfied after solving.

3. Incorrect algebraic manipulation:
Double-check steps like substituting and simplifying.

4. Forgetting to verify solutions:
Always plug back into original equations.

Additional Practice Problems

To reinforce understanding, consider solving similar problems:

1. The sum of two numbers is 50. The first number is three times the second. Find both numbers.

2. The difference between two numbers is 10. The larger number is twice the smaller. Find the numbers.

3. The sum of two numbers is 100. One number is four times the other. Find both numbers.

Attempting these will strengthen skills in setting up equations and solving word problems.

Summary and Key Takeaways

- The problem involves translating a word problem into algebraic equations.
- Assign variables and carefully interpret relationships.
- Use substitution or other algebraic methods to solve the system of equations.
- Verify solutions to ensure they meet all conditions.
- Recognize the importance of understanding proportional relationships.
- These problem-solving skills are fundamental in mathematics and applicable in real-life scenarios.

Final Words

Mastering the art of solving problems like "The sum of two numbers is 21, and the second number is six times the first" builds foundational skills in algebra. It demonstrates how to translate words into mathematical expressions, manipulate equations, and interpret solutions within context. Approaching such problems systematically enhances logical thinking and problem-solving confidence, essential skills both in academics and everyday life.

Whether you are a student learning algebra, a teacher designing problems, or someone interested in mathematical reasoning, understanding these techniques provides a vital toolkit for tackling similar challenges with clarity and precision.

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