

The Vector Has Initial Point P(-3,2) And Terminal Point Q(4, -3). Write In Terms Of $\mathbf{A_i}$ + That Is, Find

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Understanding vectors is fundamental in physics, engineering, computer graphics, and many areas of mathematics. This article explores the process of finding a vector given its initial and terminal points, expressing it in the form $\mathbf{A_i} + \mathbf{A_j}$, and interpreting its components. We will also discuss related concepts such as vector addition, scalar multiplication, and applications in various fields.

Introduction to Vectors and Their Components

What Is a Vector?

A vector is a mathematical object that has both magnitude and direction. Unlike scalar quantities that only have size (like temperature or mass), vectors convey information about direction as well as magnitude. Vectors are typically represented graphically as arrows, where the length indicates magnitude and the arrowhead indicates direction.

Representing Vectors in Coordinate Geometry

In coordinate geometry, vectors are often described in terms of their components along the x- and y-axes. For example, a vector $\vec{V}$ in a 2D plane can be expressed as:

$$\vec{V} = a \mathbf{i} + b \mathbf{j}$$

where:

- a is the component along the x-axis (horizontal component),
- b is the component along the y-axis (vertical component),
- $\mathbf{i}$ and $\mathbf{j}$ are the unit vectors in the x and y directions, respectively.

Given Data: Initial and Terminal Points

Points P and Q

In our specific problem:

- Initial Point P: $(-3, 2)$
- Terminal Point Q: $(4, -3)$

These points define the start and end of a vector in the coordinate plane.

Understanding the Problem Statement

The task is to:

- Find the vector $\vec{PQ}$ that starts at point P and ends at point Q.
- Express this vector in the form $A_i + A_j$ (or more precisely, a $\mathbf{i} + b \mathbf{j}$).
- Interpret the components and understand their significance.

Calculating the Vector From Given Points

Step 1: Find the Vector Components

The vector $\vec{PQ}$ can be found using the coordinates of points P and Q:

$$\vec{PQ} = (x_Q - x_P) \mathbf{i} + (y_Q - y_P) \mathbf{j}$$

where:

- (x_P, y_P) are the coordinates of point P,
- (x_Q, y_Q) are the coordinates of point Q.

Applying the given points:

$$\begin{aligned} x_P &= -3, \quad y_P = 2 \\ x_Q &= 4, \quad y_Q = -3 \end{aligned}$$

Calculating the components:

$$\begin{aligned} a &= x_Q - x_P = 4 - (-3) = 4 + 3 = 7 \\ b &= y_Q - y_P = -3 - 2 = -5 \end{aligned}$$

So, the vector $\vec{PQ}$ can be written as:

$$\vec{PQ} = 7 \mathbf{i} - 5 \mathbf{j}$$

Or in the form $A\mathbf{i} + B\mathbf{j}$:

$$\boxed{\vec{PQ} = 7 \mathbf{i} - 5 \mathbf{j}}$$

Step 2: Expressing the Vector in the Form $A\mathbf{i} + B\mathbf{j}$

In standard notation, the vector is:

$$\vec{V} = 7 \mathbf{i} - 5 \mathbf{j}$$

which can be written explicitly as:

$$\vec{V} = 7 \mathbf{i} + (-5) \mathbf{j}$$

or simply:

$$\boxed{\vec{V} = 7 \mathbf{i} - 5 \mathbf{j}}$$

This representation clearly shows the components along the x- and y-axes.

Interpreting the Components of the Vector

Magnitude of the Vector

The magnitude (or length) of the vector $\vec{PQ}$ is given by the Pythagorean theorem:

$$|\vec{PQ}| = \sqrt{a^2 + b^2}$$

Calculating:

$$|\vec{PQ}| = \sqrt{7^2 + (-5)^2} = \sqrt{49 + 25} = \sqrt{74} \approx 8.602$$

This magnitude indicates the straight-line distance between points P and Q.

Direction of the Vector

The direction (or angle θ) of the vector relative to the positive x-axis can be determined using the tangent function:

$$\theta = \arctan \left(\frac{b}{a} \right)$$

Substituting:

$$\theta = \arctan \left(\frac{-5}{7} \right)$$

Calculating:

$$\theta \approx \arctan(-0.7143) \approx -35.54^\circ$$

Since the angle is negative, it indicates the vector points below the x-axis.

To express the angle in standard position (0° to 360°), add 360° if needed:

$$\theta \approx 360^\circ - 35.54^\circ = 324.46^\circ$$

Alternatively, recognize the vector points to the fourth quadrant because $(a > 0)$ and $(b < 0)$.

Applications of Vector Components in Real-World Scenarios

Physics and Engineering

Vectors are essential in physics to describe forces, velocities, and acceleration. For example:

- Displacement vectors determine the shortest path between two points.
- Velocity vectors specify the speed and direction of moving objects.
- Force vectors combine multiple forces acting on an object.

Computer Graphics and Animation

In computer graphics, vectors determine object movement, rotations, and scaling. Expressing vectors in component form facilitates:

- Transformations
- Rendering calculations
- Collision detection

Navigation and Geolocation

Navigation systems utilize vectors to compute routes, headings, and distances between waypoints.

Additional Vector Operations

Vector Addition and Subtraction

Given two vectors $\vec{V_1}$ and $\vec{V_2}$, their sum is obtained by adding corresponding components:

$$\vec{V}_{\text{sum}} = (a_1 + a_2) \mathbf{i} + (b_1 + b_2) \mathbf{j}$$

For subtraction:

$$\vec{V}_{\text{sub}} = (a_1 - a_2) \mathbf{i} + (b_1 - b_2) \mathbf{j}$$

Scalar Multiplication

Multiplying a vector by a scalar k :

$$k \vec{V} = (k a) \mathbf{i} + (k b) \mathbf{j}$$

This operation scales the magnitude of the vector while preserving its direction (or reversing it if k is negative).

Dot Product

The dot product of vectors $\vec{V_1}$ and $\vec{V_2}$:

$$\vec{V}_1 \cdot \vec{V}_2 = a_1 a_2 + b_1 b_2$$

This operation is used to find the angle between vectors and to project vectors onto each other.

Cross Product in 2D

While in 2D, the cross product results in a scalar:

$$\begin{bmatrix} a_1 & b_2 \\ a_2 & b_1 \end{bmatrix}$$

which can determine the orientation of two vectors.

Conclusion: Summarizing the Findings

In summary, the vector from point P(-3, 2) to point Q(4, -3) is:

$$\boxed{\vec{PQ} = 7 \mathbf{i} - 5 \mathbf{j}}$$

This vector has a magnitude of approximately 8.602 units and points in a direction roughly 324.46° from the positive x-axis. Expressing vectors in component form is crucial in various calculations, including addition, scalar multiplication, and understanding their geometric and physical significance.

Understanding how to compute and interpret vectors enhances problem-solving skills in mathematics and its applications across science and engineering disciplines. Whether analyzing forces, navigating routes, or creating computer graphics, vectors serve as fundamental tools that describe and quantify movement and relationships in space.

Keywords: vectors, initial point, terminal point, component form, $A\mathbf{i} + B\mathbf{j}$, coordinate geometry, magnitude, direction, vector addition, scalar multiplication, dot product, cross product, physics, engineering, computer graphics, navigation

Frequently Asked Questions

What is the vector with initial point P(-3, 2) and terminal point Q(4, -3)?

The vector is $Q - P = (4 - (-3), -3 - 2) = (7, -5)$. So, the vector in terms of $\mathbf{i}$ and $\mathbf{j}$ is $7\mathbf{i} - 5\mathbf{j}$.

How do you express the vector from point P(-3, 2) to

point Q(4, -3) in terms of i and j?

Subtract the coordinates: $(4 - (-3))i + (-3 - 2)j = 7i - 5j$.

What is the initial point P and terminal point Q in coordinate form?

Initial point P is $(-3, 2)$, and terminal point Q is $(4, -3)$.

How do you find the vector in terms of i and j given two points?

Subtract the x-coordinates for the i component and the y-coordinates for the j component: $(Q_x - P_x)i + (Q_y - P_y)j$.

Calculate the magnitude of the vector from P(-3, 2) to Q(4, -3).

Magnitude = $\sqrt{7^2 + (-5)^2} = \sqrt{49 + 25} = \sqrt{74} \approx 8.60$.

What is the directional unit vector of the vector from P to Q?

Divide the vector by its magnitude: $(7/\sqrt{74})i + (-5/\sqrt{74})j$, which simplifies to approximately $0.81i - 0.55j$.

Additional Resources

Understanding and Calculating a Vector Defined by Initial and Terminal Points

When dealing with vectors in coordinate geometry and vector analysis, it's fundamental to understand how to determine a vector given its initial and terminal points. This knowledge is essential across various fields such as physics, engineering, computer graphics, and mathematics. In this detailed exploration, we will analyze the vector with initial point $P(-3, 2)$ and terminal point $Q(4, -3)$, and express it in the form $\vec{A} = a\mathbf{i} + b\mathbf{j}$. We will begin with foundational concepts, proceed through calculations, and interpret the results comprehensively.

Foundations of Vectors in Coordinate Geometry

What Is a Vector?

A vector is a quantity that has both magnitude (length) and direction. Unlike scalar quantities, vectors are represented graphically by arrows and algebraically by components.

- Graphical Representation: An arrow pointing from a starting point to an ending point.
- Component Form: An ordered pair (or triplet in 3D) indicating the change in the x, y (and z) directions.

Initial and Terminal Points

- The initial point $P(x_1, y_1)$ marks where the vector begins.
- The terminal point $Q(x_2, y_2)$ marks where the vector ends.

Given these, the vector from P to Q can be represented as:

$$\vec{PQ} = \langle x_2 - x_1, y_2 - y_1 \rangle$$

This formulation provides the component form of the vector, which is fundamental for further calculations.

Calculating the Vector from $P(-3, 2)$ to $Q(4, -3)$

Step-by-Step Calculation

Given:

- Initial point $P(-3, 2)$
- Terminal point $Q(4, -3)$

The vector $\vec{PQ}$ is computed as:

$$\boxed{\vec{PQ} = \langle 4 - (-3), -3 - 2 \rangle}$$

Breaking down the components:

- x-component: $(4 - (-3) = 4 + 3 = 7)$
- y-component: $(-3 - 2 = -5)$

Thus, the vector expressed in component form is:

```
\[
\boxed{
\vec{PQ} = 7 \mathbf{i} - 5 \mathbf{j}
}
\]
```

This notation emphasizes the contribution of each component along the coordinate axes, with $(\mathbf{i})$ representing the x-direction and $(\mathbf{j})$ representing the y-direction.

Expressing the Vector in the Form $(\mathbf{A} = a \mathbf{i} + b \mathbf{j})$

Component Identification

From the calculations:

- $(a = 7)$
- $(b = -5)$

Therefore, the vector $(\vec{PQ})$ can be written as:

```
\[
\boxed{
\mathbf{A} = 7 \mathbf{i} - 5 \mathbf{j}
}
\]
```

This is the standard vector notation used in physics and mathematics, encapsulating both magnitude and direction succinctly.

Interpretation of Components

- The $(\mathbf{i})$ -component (7) indicates that the vector moves 7 units along the positive x-axis.

- The $\mathbf{j}$ -component (-5) indicates that the vector moves 5 units along the negative y-axis.

Analyzing the Magnitude and Direction of the Vector

Calculating the Magnitude

The magnitude (or length) of vector $\vec{A}$ is given by:

$$|\vec{A}| = \sqrt{a^2 + b^2}$$

Plugging in the values:

$$|\vec{A}| = \sqrt{7^2 + (-5)^2} = \sqrt{49 + 25} = \sqrt{74}$$

which simplifies to:

$$|\vec{A}| \approx 8.602$$

This length indicates the straight-line distance between the points P and Q .

Determining the Direction (Angle with the x-axis)

The direction of the vector relative to the positive x-axis can be expressed as an angle θ :

$$\theta = \arctan \left(\frac{b}{a} \right)$$

Calculating:

$$\theta = \arctan \left(\frac{-5}{7} \right) \approx \arctan(-0.7143)$$

\]

Using a calculator:

\[
\theta \approx -35.54^\circ
\]

Since the vector points to the second quadrant (positive x, negative y), we often express the angle as a positive measure:

\[
\theta_{\text{standard}} = 180^\circ + (-35.54^\circ) = 144.46^\circ
\]

Alternatively, the angle in standard position is approximately 144.46 degrees, measured counterclockwise from the positive x-axis.

Applications and Further Insights

Vector Operations Involving $(\vec{A})$

Understanding the vector $(\vec{A})$ allows for multiple operations:

- Scalar Multiplication: Scaling the vector to change its magnitude while preserving its direction.
- Vector Addition: Combining $(\vec{A})$ with other vectors to analyze resultant directions.
- Dot Product: Calculating the angle between vectors or projecting one onto another.
- Cross Product (in 3D): Finding a vector perpendicular to the plane of the vectors.

Real-World Contexts

- Physics: The vector $(\vec{A})$ could represent displacement, velocity, or force between two points.
- Navigation: The components tell us how much to move east/west and north/south to get from (P) to (Q) .
- Engineering: Structural analysis often involves computing vectors between points for stress and strain calculations.
- Computer Graphics: Vectors define movement, scaling, and rotation operations.

Vector in Component Form and Its Significance

Expressing vectors as $(a \mathbf{i} + b \mathbf{j})$:

- Simplifies calculations involving the vector.
- Facilitates transformations like rotations and reflections.
- Provides an intuitive grasp of direction and magnitude.

Summary of Key Results

- Initial Point: $P(-3, 2)$
- Terminal Point: $Q(4, -3)$
- Calculated Vector:

$$\boxed{\vec{A} = 7 \mathbf{i} - 5 \mathbf{j}}$$

- Magnitude of $(\vec{A})$:

$$|\vec{A}| \approx 8.602$$

- Direction: Approximately 144.46° counterclockwise from the positive x-axis.

Conclusion

Understanding how to derive a vector from two points is a foundational skill that bridges coordinate geometry and vector algebra. The process involves subtracting the coordinates of the initial point from those of the terminal point, resulting in a vector expressed in component form. This form reveals the vector's magnitude and direction, enabling further analysis or application in real-world scenarios.

In this case, the vector from $P(-3, 2)$ to $Q(4, -3)$ is $\boxed{\mathbf{A} = 7 \mathbf{i} - 5 \mathbf{j}}$, with a magnitude of approximately 8.602 units and an angle of about 144.46° from the positive x-axis. This knowledge forms a crucial part of the toolkit for anyone working

with vectors in two-dimensional space, providing a clear pathway from geometric points to algebraic representation and analysis.

Additional Notes for Students and Practitioners:

- Always verify the direction of your vector when subtracting points.
- Remember that the components directly influence both the magnitude and the angle.
- Use inverse tangent carefully, considering the signs of components to determine the correct quadrant.
- Practice with different coordinate points to become proficient in vector calculations.

This comprehensive understanding equips you with the tools needed to analyze and interpret vectors in various contexts, deepening your grasp of spatial relationships and vector operations.

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