The Volume Of The Cylinder If 500pie Cubic Inches And The Radius Is 5 Inches What Is The Height Of The

The Volume Of The Cylinder If 500pi Cubic Inches And The Radius Is 5 Inches What Is The Height Of The

Understanding the relationship between the volume, radius, and height of a cylinder is fundamental in geometry and practical applications such as engineering, manufacturing, and architecture. In this article, we will explore how to determine the height of a cylinder when given its volume and radius, using the formula for the volume of a cylinder. Specifically, we will analyze the problem: The volume of the cylinder is 500π cubic inches, and the radius is 5 inches. What is the height of the cylinder?

By dissecting this problem, we will cover the essential concepts, formula derivation, step-by-step calculations, and additional considerations to deepen your understanding of cylindrical volume calculations.

Understanding the Cylinder Volume Formula

The Basic Formula

The volume V of a cylinder is calculated using the formula:

$$
V = \pi r^2 h
$$

Where:
- V is the volume of the cylinder,
- r is the radius of the base,
- h is the height of the cylinder,
- π is the mathematical constant Pi, approximately 3.14159.

This formula states that the volume is proportional to the area of the circular base (πr^2) times the height.

Implications of the Formula

- The volume increases linearly with the height; doubling the height doubles the volume.
- The volume is proportional to the square of the radius; increasing the radius significantly increases the volume.
- When the volume and radius are known, the height can be isolated and calculated directly.

Given Data and Goal

Problem Restatement

- The volume $V = 500\pi$ cubic inches.
- The radius $r = 5$ inches.
- The unknown: height h.

Objective

Calculate the height h of the cylinder using the provided data.

Step-by-Step Calculation of the Cylinder Height

Step 1: Write Down the Volume Formula

$$
V = \pi r^2 h
$$

Substitute known values:

$$
500\pi = \pi \times (5)^2 \times h
$$

Step 2: Simplify the Equation

Calculate r^2:

$$
(5)^2 = 25
$$

So,

$$
500\pi = \pi \times 25 \times h
$$

Step 3: Isolate h

Divide both sides of the equation by $\pi \times 25$:

$$
h = \frac{500\pi}{\pi \times 25}
$$

Since π appears in numerator and denominator, they cancel out:

$$
h = \frac{500}{25}
$$

Step 4: Final Calculation

$$
h = 20
$$

Thus, the height of the cylinder is 20 inches.

Understanding the Result and Its Significance

Final Answer

The height of the cylinder is 20 inches.

Interpretation

- Given the volume and radius, the height can be easily computed using the volume formula.
- The result indicates that a cylinder with a radius of 5 inches and volume 500π cubic inches must be 20 inches tall.

Practical Implications

- This calculation is applicable in designing cylindrical containers, tanks, or pipes where volume specifications are critical.
- Knowing how to manipulate the volume formula allows engineers and designers to customize dimensions according to volume constraints.

Additional Considerations and Variations

Different Volume Units or Dimensions

- The same principles apply if the volume is given in different units, provided all measurements are consistent.
- For example, if the volume was in liters and the radius in centimeters, conversions would be necessary.

Handling Different Radii or Volumes

- When the radius varies, the same formula applies, and solving for height involves rearranging the formula accordingly.
- For example, if the volume and height are known, the radius can be calculated.

Example: Calculating the Radius Given Volume and Height

Suppose the volume is known, and height is given; then:

$$
r = \sqrt{\frac{V}{\pi h}}
$$

Summary and Key Takeaways

- The volume of a cylinder is directly proportional to the square of its radius and its height.
- When given the volume and radius, the height can be straightforwardly calculated by rearranging the volume formula.
- In the specific problem, with a volume of 500π cubic inches and a radius of 5 inches, the height is 20 inches.
- Mastery of the volume formula allows for solving a wide array of practical problems involving cylindrical objects.

Conclusion

Understanding how to determine the height of a cylinder from its volume and radius is a fundamental skill in geometry that finds relevance in numerous real-world applications. By applying the formula $V = \pi r^2 h$, substituting known values, and performing algebraic manipulations, one can solve for any unknown dimension of a cylinder efficiently. The specific problem presented demonstrates this process clearly, yielding a concise and practical result: the height of the cylinder is 20 inches. This knowledge not only enhances mathematical proficiency but also empowers practical decision-making in fields that require precise volumetric measurements of cylindrical objects.

Frequently Asked Questions

What is the formula to find the volume of a cylinder?

The volume of a cylinder is given by the formula $V = \pi r^2 h$, where r is the radius and h is the height.

Given the volume of a cylinder as 500π cubic inches and the radius as 5 inches, how do you find the height?

Rearranged, height $h = V / (\pi r^2)$. Plugging in the values: $h = 500\pi / (\pi 5^2) = 500\pi / (\pi 25) = 500 / 25 = 20$ inches.

What is the height of a cylinder with a volume of 500π cubic inches and radius 5 inches?

The height is 20 inches.

How does the radius affect the height of a cylinder with a fixed volume?

Since volume is proportional to the square of the radius, increasing the radius decreases the height needed for the same volume, and vice versa.

If the volume of a cylinder is 500π cubic inches and the radius is changed to 10 inches, what is the new height?

Using the formula $h = V / (\pi r^2)$: $h = 500\pi / (\pi 10^2) = 500\pi / (\pi 100) = 500 / 100 = 5$ inches.

Why is π used in calculating the volume of a cylinder?

π is used because the cross-sectional area of the base circle is πr^2, which is essential in calculating the volume.

Can the height of a cylinder be negative? Why or why not?

No, height cannot be negative because it represents a length, which is always a positive measurement.

What units are used in calculating the volume and height of a cylinder?

Units depend on the measurements provided; in this case, inches are used for both radius and height, and cubic inches for volume.

If the volume of the cylinder was given as 500 cubic inches (not π), how would the calculation change?

You would use $V = \pi r^2 h$, so $h = V / (\pi r^2)$. With $V = 500$, $h = 500 / (\pi 25) \approx 6.366$ inches.

How do you verify the calculated height of the cylinder?

Plug the radius and height back into the volume formula $V = \pi r^2 h$ and check if the result matches the given volume (500π cubic inches).

Additional Resources

The Volume of the Cylinder If 500π Cubic Inches and the Radius Is 5 Inches: What Is the Height Of The Cylinder?

Understanding the geometric properties of cylinders is fundamental in various fields such as engineering, architecture, and mathematics. In this detailed exploration, we will analyze the problem statement: given a cylinder with a volume of 500π cubic inches and a radius of 5 inches, what is its height? This question involves applying core volume formulas, manipulating algebraic expressions, and understanding the relationships between a cylinder's dimensions.

Introduction to the Geometry of Cylinders

A cylinder is a three-dimensional geometric shape characterized by two parallel circular bases connected by a curved surface. The defining dimensions of a cylinder are its radius (r) and height (h). Visualizing a cylinder as a "rolled" rectangle (the lateral surface) that has been bent into a circular shape helps in understanding its properties.

Key features of a cylinder:

- Radius (r): Distance from the center of the base to its edge.
- Height (h): Distance between the two bases, measured along the side perpendicular to the bases.
- Volume (V): The total space occupied by the cylinder.

Fundamental Formula for the Volume of a Cylinder

The volume of a cylinder is given by the formula:

$$V = \pi r^2 h$$

where:

- V is the volume,
- r is the radius,
- h is the height,
- π (pi) is approximately 3.14159.

This formula can be derived from the area of the base (πr^2) multiplied by the height (h), as the volume of a prism with a circular base.

Analyzing the Given Data

The problem provides:

- Volume, $V = 500\pi$ cubic inches
- Radius, $r = 5$ inches

Our goal:

- Find the height h of the cylinder.

Step-by-Step Solution

To find the height, we'll substitute the given values into the volume formula and solve for h.

Step 1: Write the volume formula with known values

$$
V = \pi r^2 h
$$

Substituting the known values:

$$
500\pi = \pi \times (5)^2 \times h
$$

Step 2: Simplify the equation

Calculate r^2:

$$
(5)^2 = 25
$$

So:

$$
500\pi = \pi \times 25 \times h
$$

Step 3: Divide both sides by π to isolate the other variables

$$
\frac{500\pi}{\pi} = 25 h
$$

Since $\pi \neq 0$, this simplifies to:

$$
500 = 25 h
$$

Step 4: Solve for h

Divide both sides by 25:

$$
h = \frac{500}{25} = 20
$$

Result:

The height of the cylinder is 20 inches.

Deep Dive into the Calculation and Its Implications

The calculation demonstrates the straightforward nature of volume formulas for cylinders when the parameters are known. However, understanding the broader context and potential variations enhances comprehension.

Why is the volume expressed as 500π?

- The volume being expressed explicitly with π indicates that the problem is intended to utilize the exact form rather than an approximate decimal. This precision is common in mathematical problems to avoid rounding errors.

Implications of the radius value and volume

- The radius of 5 inches is a moderate size, and with a volume of 500π cubic inches, the cylinder's height is proportionally significant at 20 inches.
- If the volume had been larger or smaller, the height would adjust accordingly.

Exploring Variations and Related Problems

Understanding the core problem allows for exploring related questions and variations, which deepen comprehension of the geometric relationships.

1. What if the radius was different?

Suppose the radius is not 5 inches but another value, say r'. The general formula for height becomes:

$$
h = \frac{V}{\pi r'^2}
$$

This highlights the inverse relationship between the radius squared and the height for a fixed volume.

---

2. What if the volume is changed?

If the volume increases or decreases, the height adjusts proportionally:

$$
h = \frac{\text{New Volume}}{\pi r^2}
$$

For example, if the volume doubles, the height doubles as well, assuming the radius remains constant.

---

3. Calculating the lateral surface area and total surface area

Beyond volume, understanding surface area adds depth:

- Lateral surface area: $A_{lateral} = 2 \pi r h$
- Total surface area: $A_{total} = 2 \pi r (h + r)$

Given the height of 20 inches and radius of 5 inches:

$$
A_{lateral} = 2 \pi \times 5 \times 20 = 200 \pi \text{ square inches}
$$

$$
A_{total} = 2 \pi \times 5 \times (20 + 5) = 2 \pi \times 5 \times 25 = 250 \pi \text{ square inches}
$$

Real-World Applications of Cylinder Volume Calculations

Understanding how to compute the volume of cylinders is crucial in various industries:

- Manufacturing: Determining the capacity of tanks, barrels, or pipes.
- Architecture: Calculating the volume of columns or structural components.
- Logistics: Estimating the space needed for cylindrical containers.
- Science: Computing the volume of test tubes, chemical reactors, and storage vessels.

In practical scenarios, knowing the dimensions allows engineers and designers to optimize material usage, ensure safety margins, and meet design specifications.

Common Challenges and Misconceptions

While the calculation appears straightforward, several pitfalls can occur:

- Misapplying the formula: Forgetting to square the radius or misplacing variables.
- Forgetting units: Ensuring consistency in units (inches, centimeters, etc.).
- Ignoring the π: Using approximate values of pi without clarity can lead to errors in precise calculations.
- Assuming the volume is in a different unit: Always verify the units of given data before calculations.

Summary and Final Thoughts

In conclusion, the problem of determining the height of a cylinder given its volume and radius is a classic example of applying fundamental geometric formulas. With a volume of 500π cubic inches and a radius of 5 inches, the height of the cylinder is found to be 20 inches through straightforward algebraic manipulation.

This exercise underscores the importance of understanding the relationships between a cylinder's dimensions, the ability to manipulate formulas, and the significance of precise calculations in real-world applications. Whether designing storage tanks, calculating material requirements, or solving academic problems, mastering these concepts is essential for students, engineers, and professionals alike.

By grasping these principles, one can approach more complex problems involving three-dimensional shapes with confidence and clarity, enhancing both mathematical proficiency and practical problem-solving skills.

The Volume Of The Cylinder If 500pie Cubic Inches And The Radius Is 5 Inches What Is The Height Of The

The Volume Of The Cylinder If 500pie Cubic Inches And The Radius Is 5 Inches What Is The Height Of The

Related Articles

- The Moist Weight Of 5600 Cm3 Of A Soil Is 102.3N. The Moisture Content And The Specific Gravity Of The
- The Transmission Of Information From Manager To Subordinate Is Referred To As: Group Of Answer Choices
- Suppose X And Y Are Variables Of Type Int. Write A Code Fragment That Sets Y To 0 If X Is Negative And

Back to Home